Confined thermal convection and phase changes

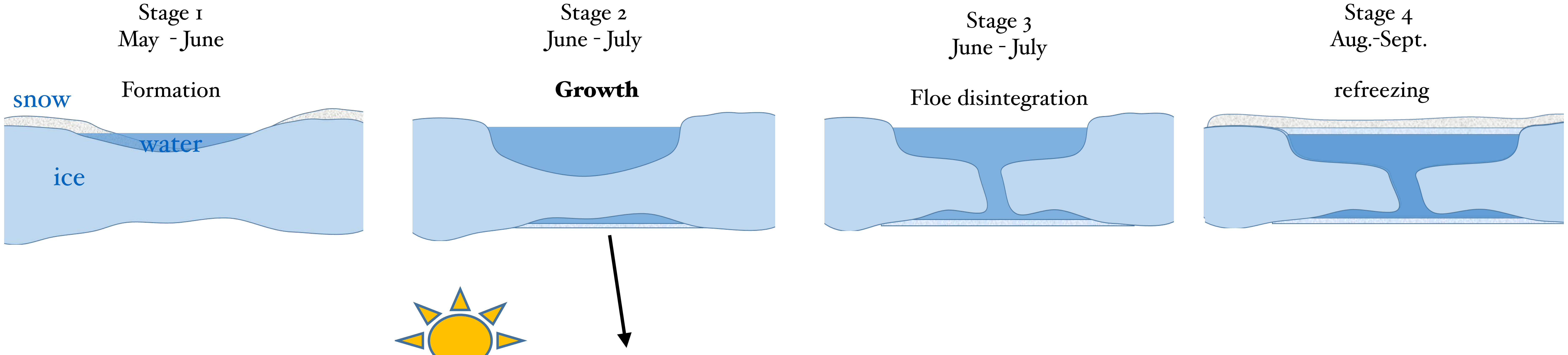
Ice in a Box

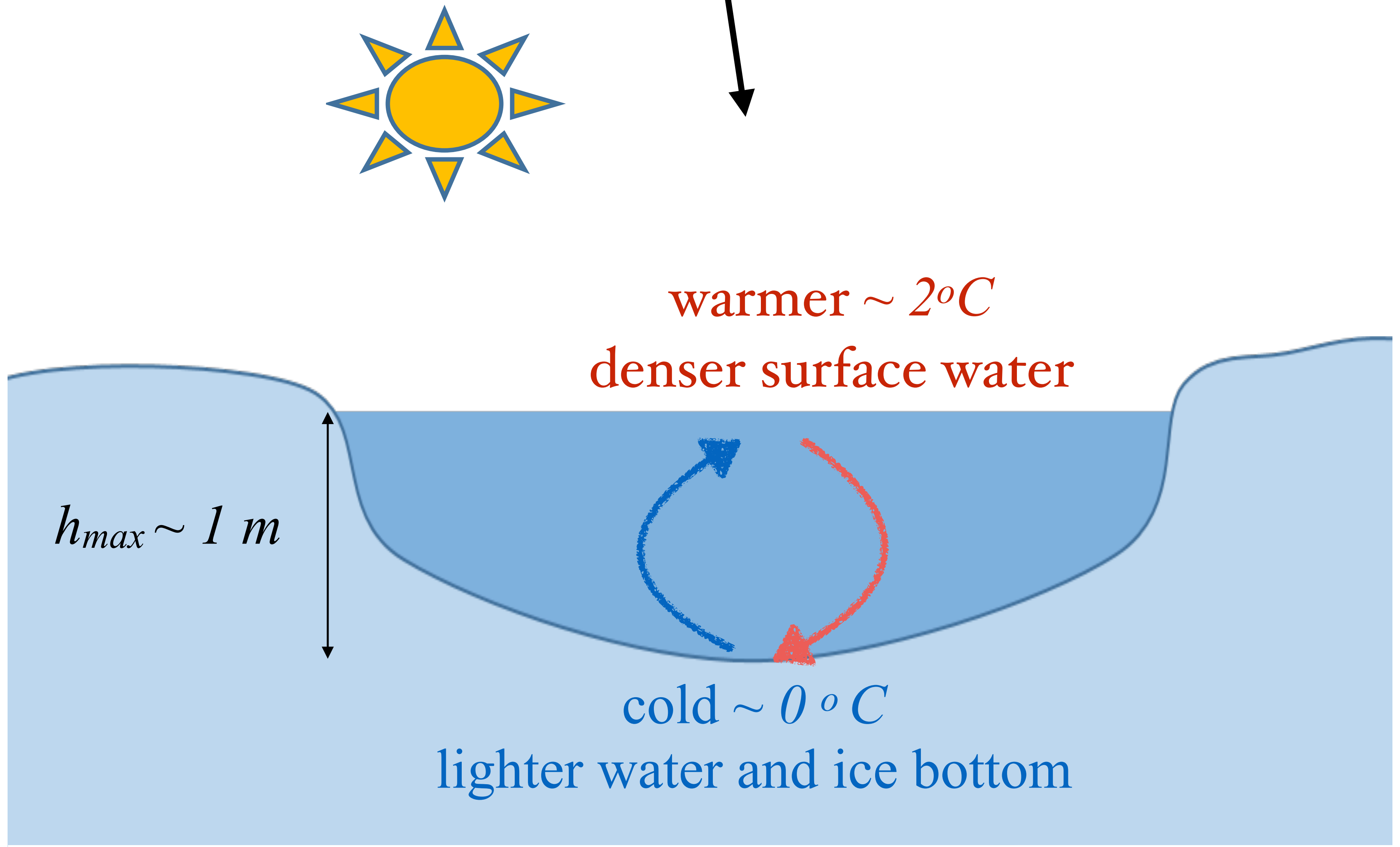
Enrico Calzavarini - University of Lille (France)

Rencontre du non-linéaire, Paris, March 24th 2026

Motivations

Global warming and Arctic melt ponds

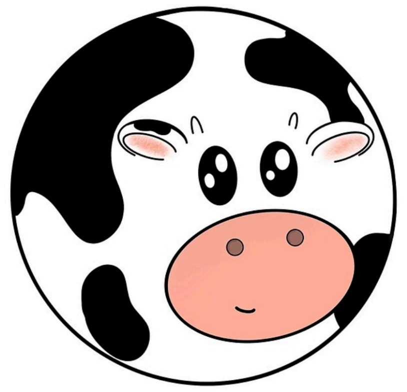


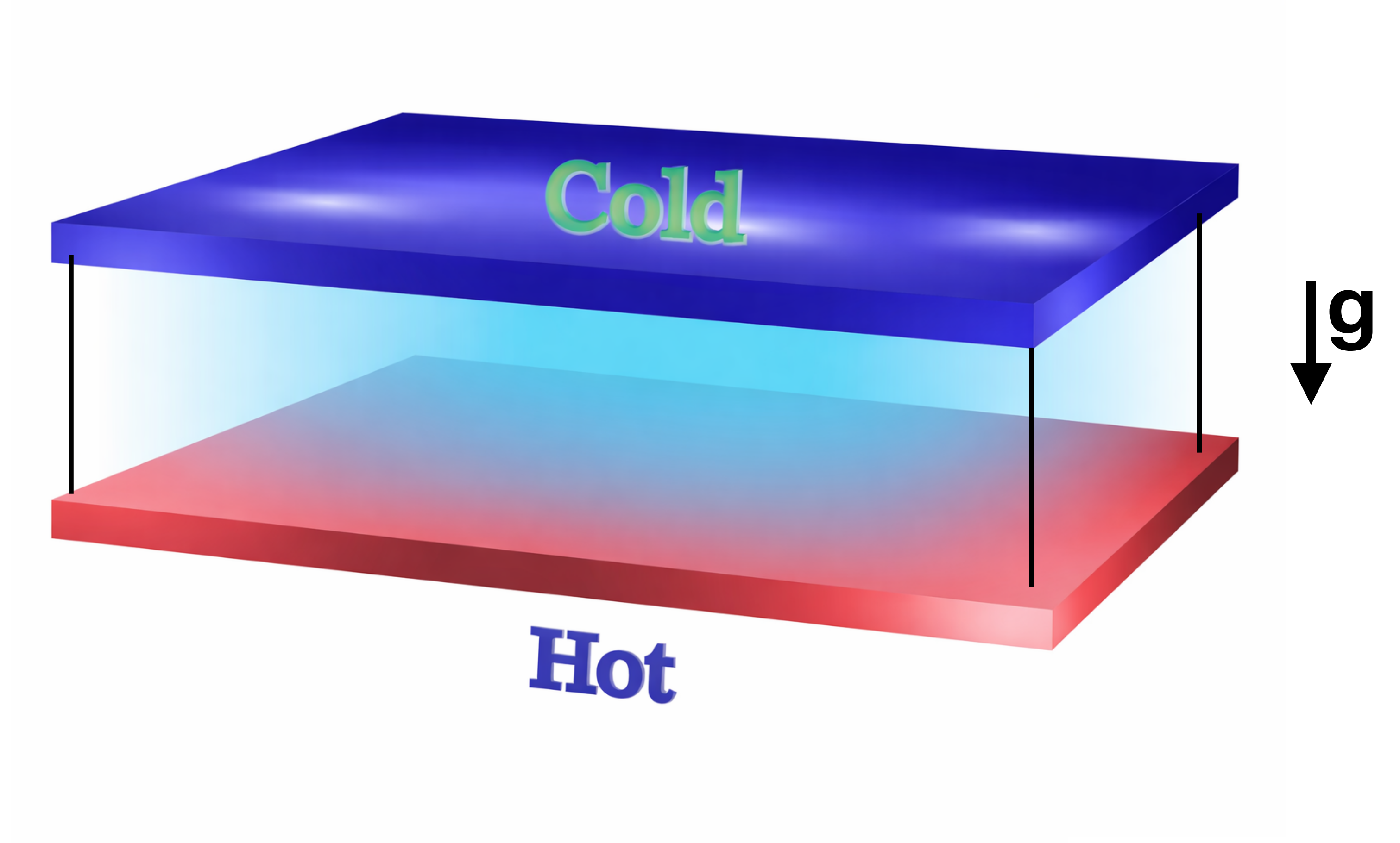


- Eicken, H. et al. Tracer studies of pathways and rates of meltwater transport through arctic summer sea ice. *J. Geophys. Res. C: Oceans, 107*(10) (2012)
- C.Polashenski et al. Percolation blockage: A process that enables melt pond formation on first year Arctic sea ice JGR Oceans (2017)

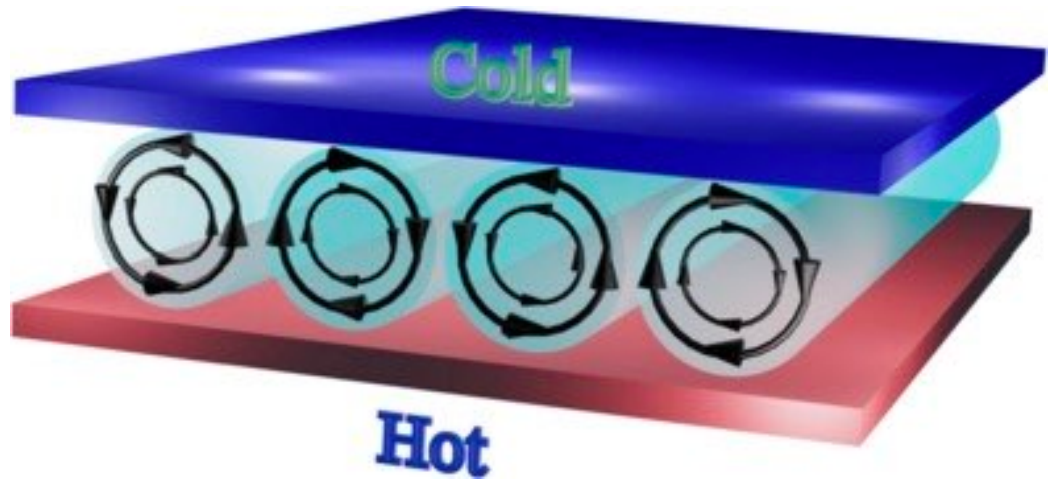
Arctic ice melt ponds are unstably stratified → Thermal convection speeds up melting

The Rayleigh-Bénard model system

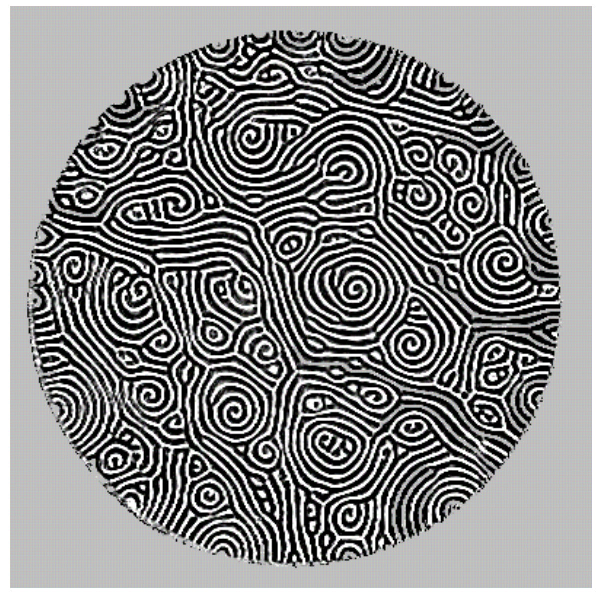
Our spherical cow  a successful prototype system

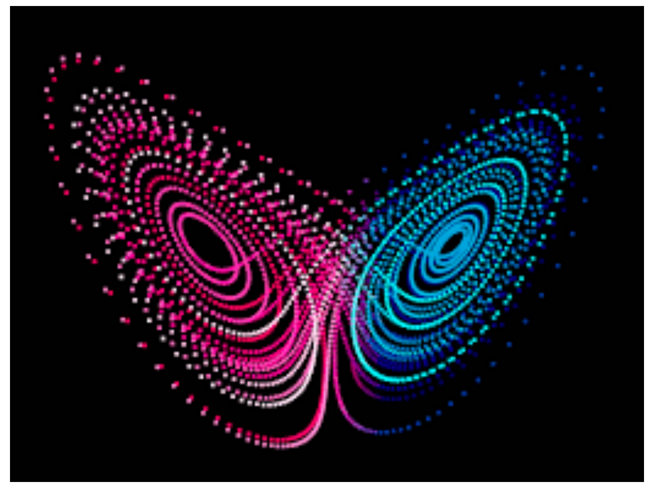


Instabilities



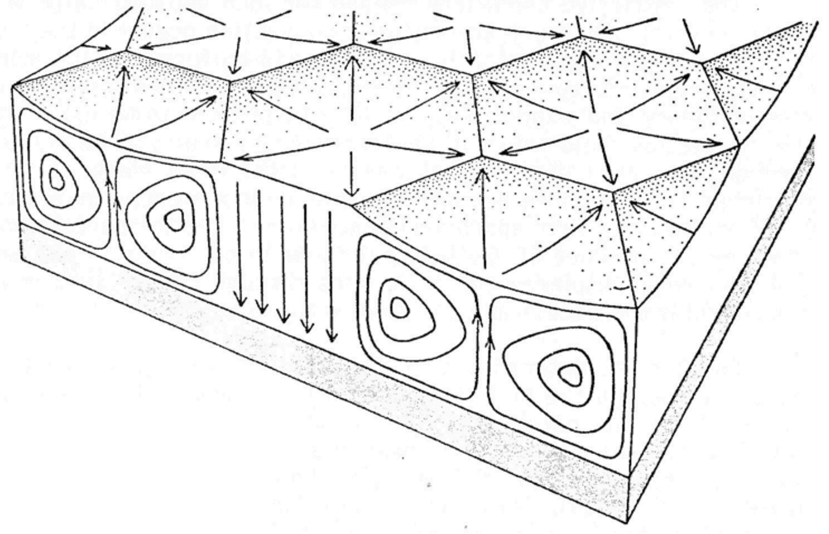
Bifurcations and transition to Chaos

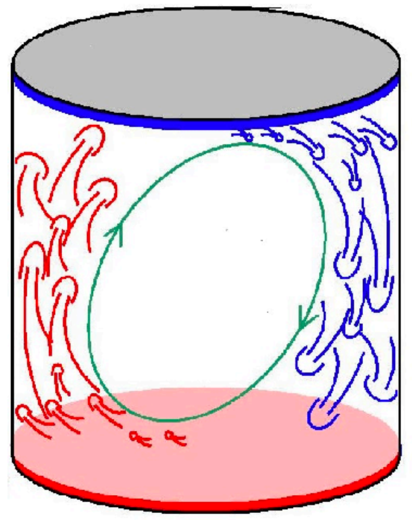


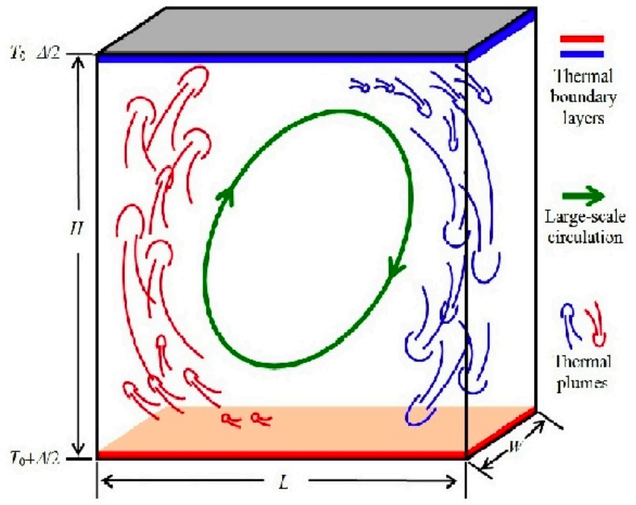


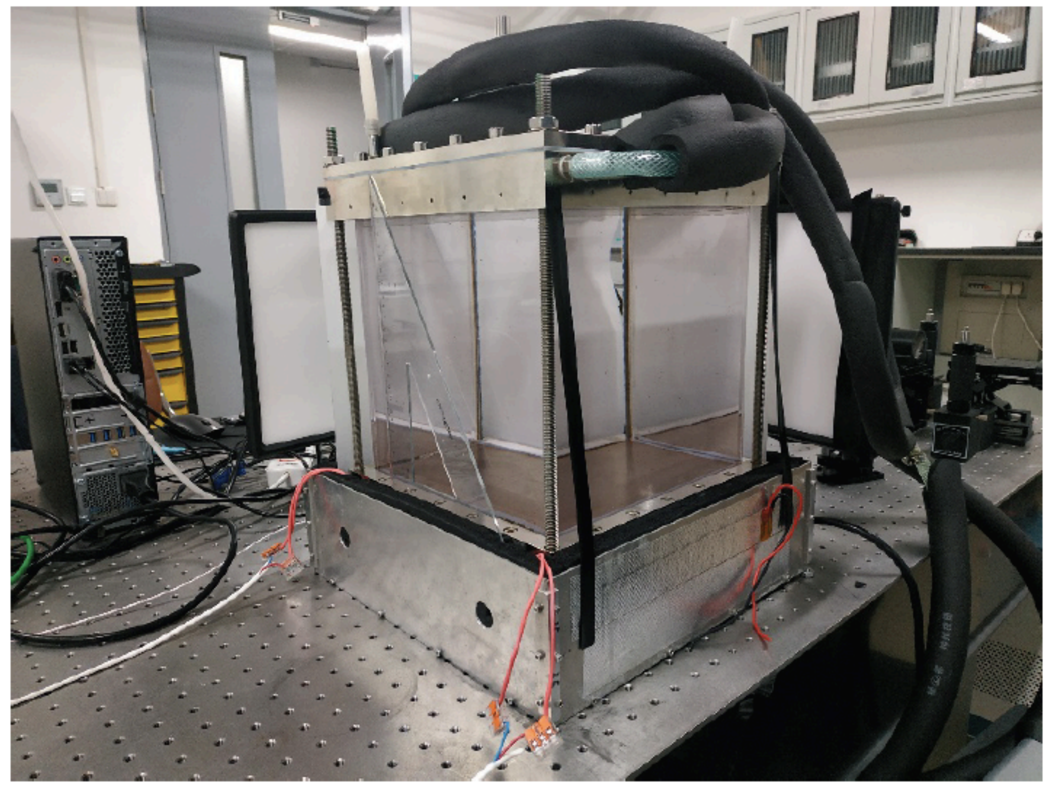
Pattern formations

self-organization

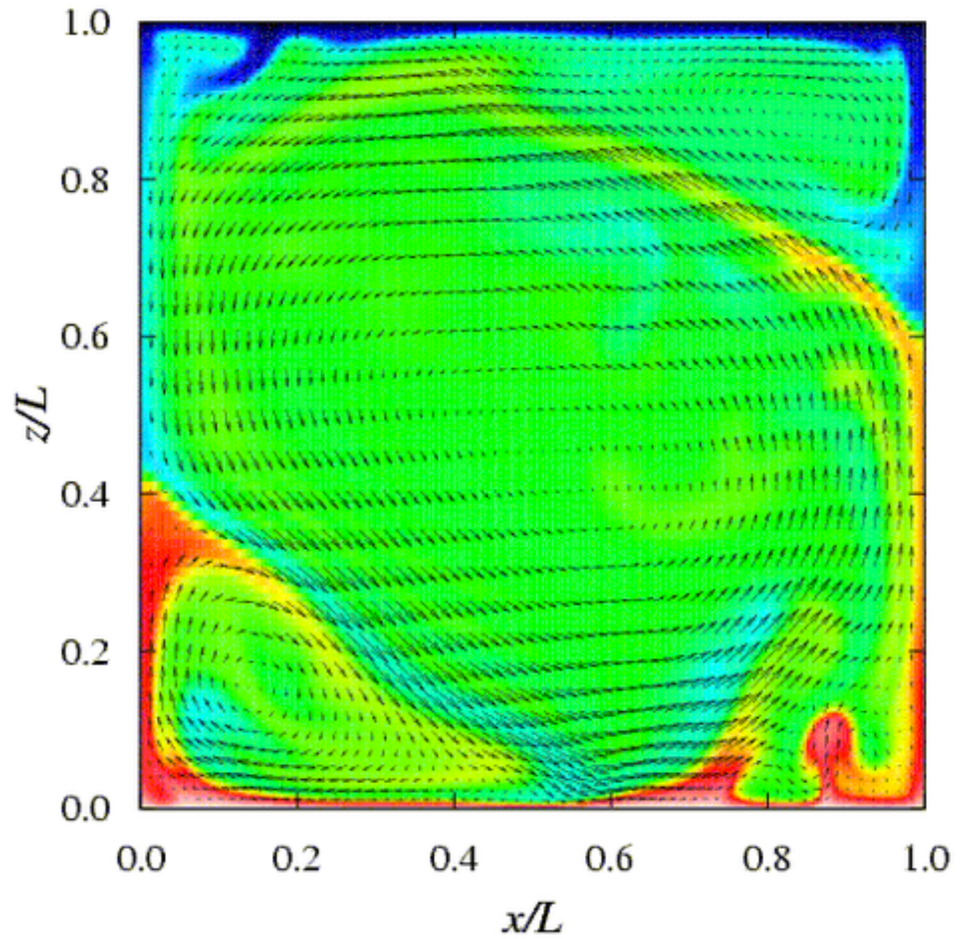






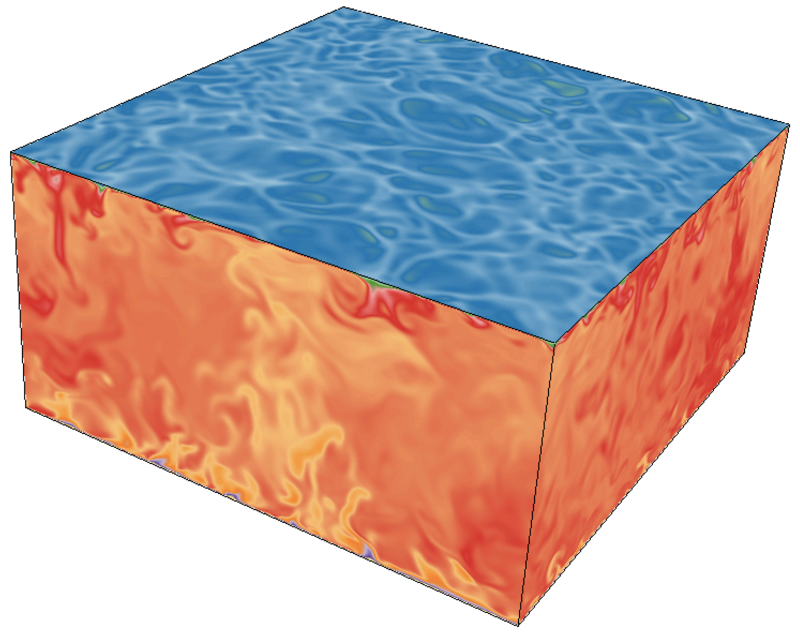


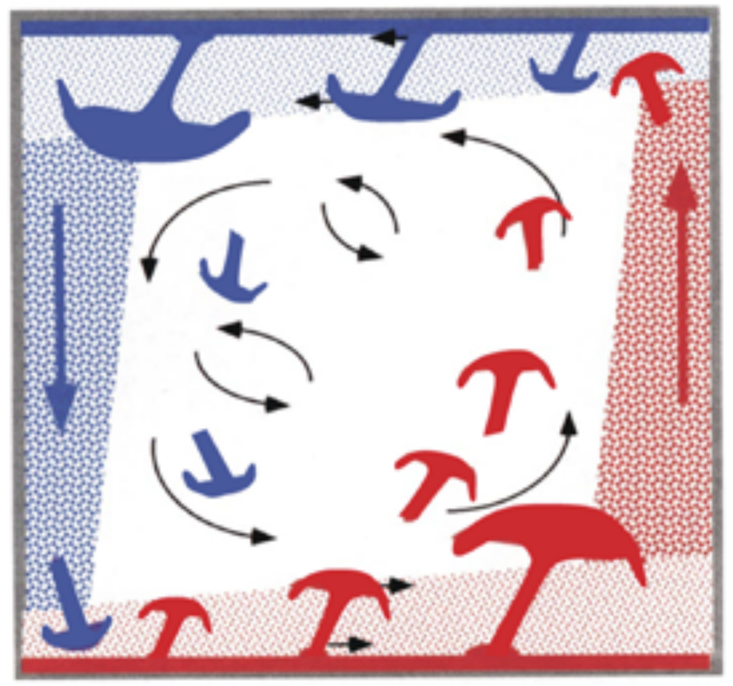
Experiment



Simulation

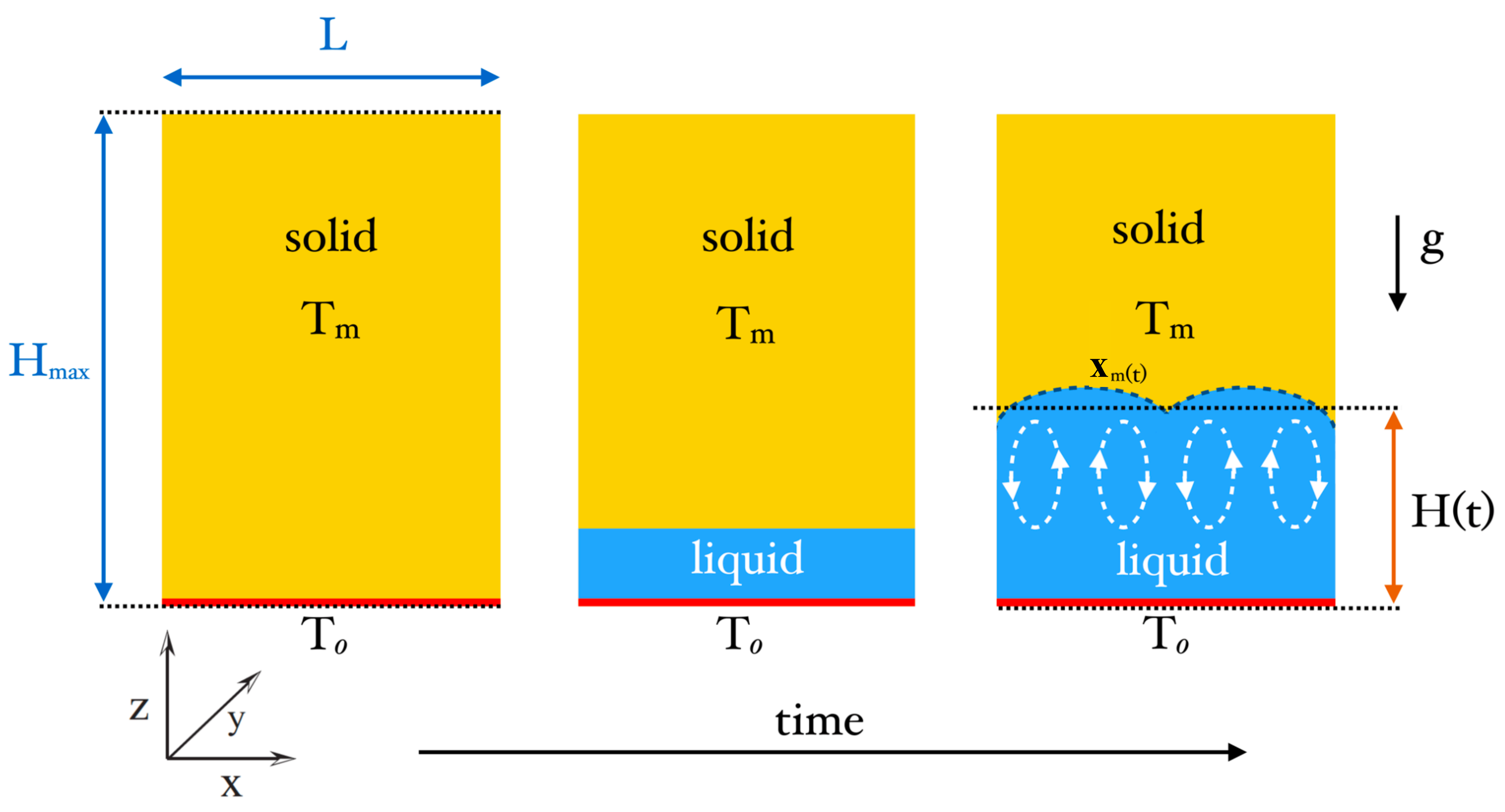
Turbulence





Convective melting

Convective melting



Equations of motion: Boussinesq system

$$\rho_0\left(\frac{\partial \mathbf{u}}{\partial t}+\mathbf{u}\cdot\boldsymbol{\nabla}\mathbf{u}\right)=-\boldsymbol{\nabla}p+\mu\,\Delta\mathbf{u}+\rho_L\mathbf{g},$$

$$\boldsymbol{\nabla}\cdot\mathbf{u}=\mathbf{0},$$

$$\rho_L=\rho_0\left(1-\beta(T-T_0)\right),$$

$$\frac{\partial T}{\partial t}+\mathbf{u}\cdot\boldsymbol{\nabla}T=\kappa\,\Delta T,$$

+ Boundary conditions

$$-\kappa\;\boldsymbol{\nabla}T|_{\boldsymbol{x}=\boldsymbol{x}_m(t)}=\frac{\mathcal{L}}{c_p}\dot{\boldsymbol{x}}_m(t)$$

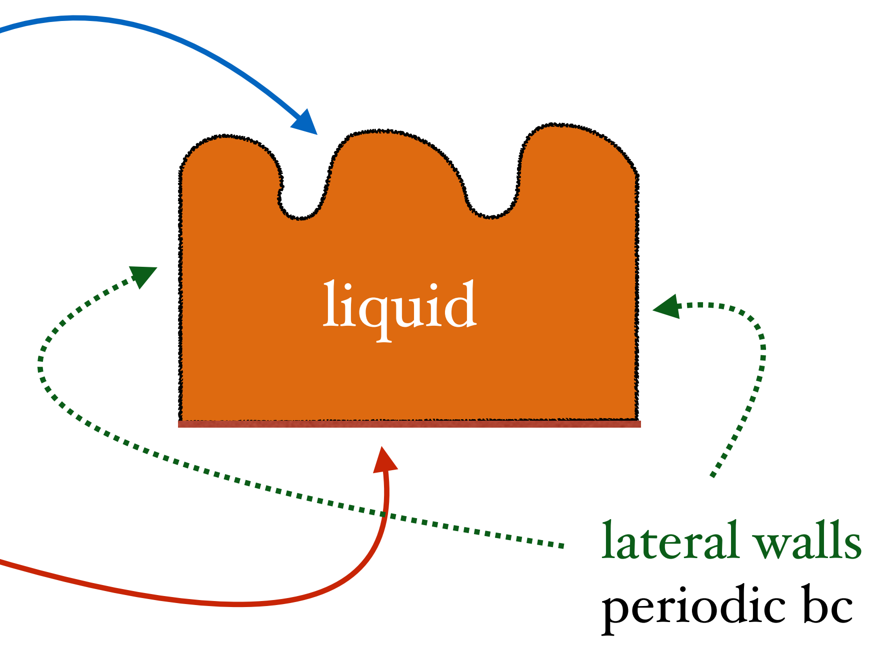
$$\mathbf{u}|_{\boldsymbol{x}=\boldsymbol{x}_m(t)}=\mathbf{0}$$

phase-change interface $\forall \boldsymbol{x}_m(t)\in\mathcal{I}(t)$

$$T|_{\boldsymbol{x}=(x,y,0)}=T_0$$

$$\mathbf{u}|_{\boldsymbol{x}=(x,y,0)}=\mathbf{0}$$

bottom wall $\forall x,y\in[0,L][0,L]$



Basal melting driven by turbulent thermal convection, B. Rabbanipour Esfahani, S. C. Hirata, S. Berti and E. Calzavarini, **Phys. Rev. Fluids** 3, 053501 (2018)

Rayleigh-Bénard convection with a melting boundary B Favier, J Purseed, L Duchemin **J. Fluid Mech** 858, 437-473 (2019)

Control parameters

INPUT

Rayleigh

$$Ra_{max} = \frac{\beta g \Delta T H_{max}^3}{\nu \kappa}$$

Prandtl

$$Pr = \frac{\nu}{\kappa}$$

Aspect ratio

$$\Gamma_{min} = \frac{L}{H_{max}}$$

Stefan

$$St = \frac{c_p \Delta T}{\mathcal{L}}$$

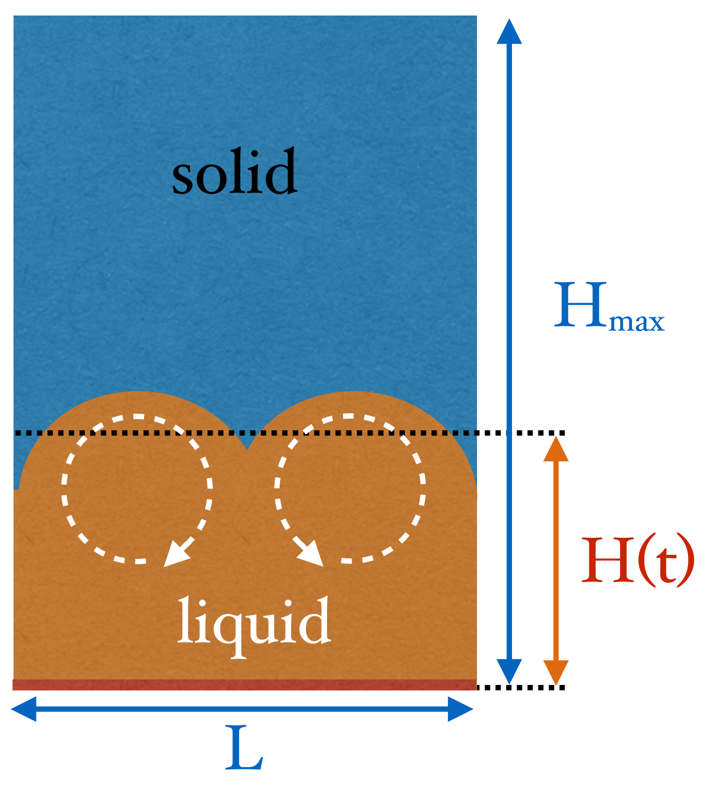
For water in melt ponds : St = $O(10^{-2})$, Pr ~ $O(10)$, $Ra_{max} = O(10^{3-9})$

For melted rocks (lava,magmas): St = $O(1\text{-}10)$, Pr ~ $10^{4\text{-}8}$, $Ra_{max} = O(10^{9-17})$

Instantaneous average melt height

$$H(t) = \frac{1}{L^2}\int_V \phi_l \, d^3x = H_{max}\langle\phi_l\rangle$$

global liquid fraction



Effective Rayleigh

$$Ra_{eff}(t) = \frac{\beta g \Delta T H(t)^3}{\nu\kappa} = Ra_{max}\langle\phi_l\rangle^3$$

Effective Aspect ratio

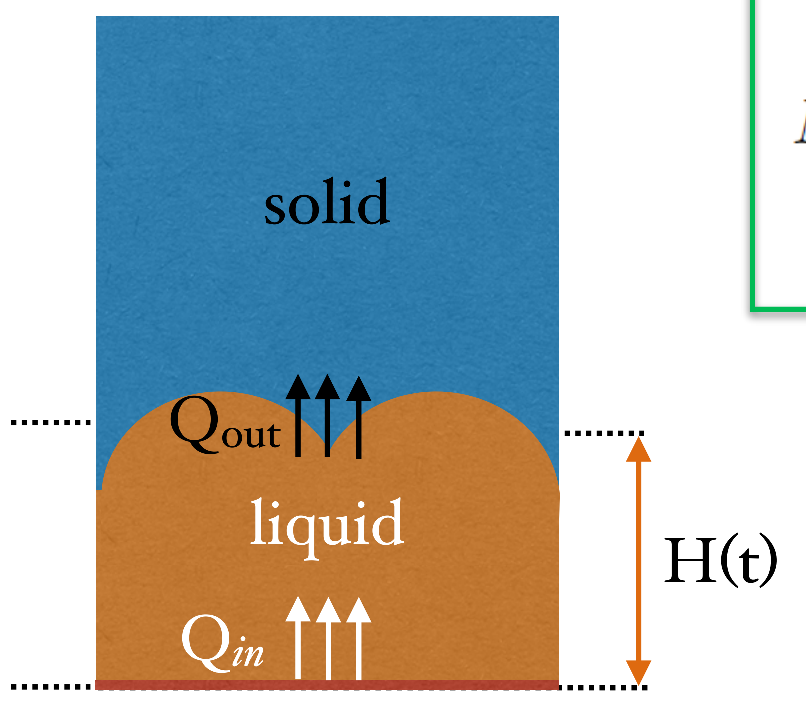
$$\Gamma_{eff}(t) = \frac{L}{H(t)} = \frac{L}{H_{max}\langle\phi_l\rangle}$$

OUTPUT

Effective Nusselt number

$$Nu_{eff}^{in}(t) = \frac{Q_{in}(t)}{k\,\Delta T/H(t)} = -\left\langle \partial_{\tilde z}\tilde T|_{z=0}\right\rangle_A \langle\phi_l\rangle$$

$$Nu_{eff}^{out}(t) = \frac{Q_{out}(t)}{k\,\Delta T/H(t)} = \frac{1}{2St}\frac{d\langle\phi_l\rangle^2}{d\tilde t}$$



$$Nu_{eff}^{in} = Nu_{eff}^{out} + \langle\phi_l\rangle^2 \langle\partial_{\tilde t}\tilde T\rangle_{V_l}$$

> 0

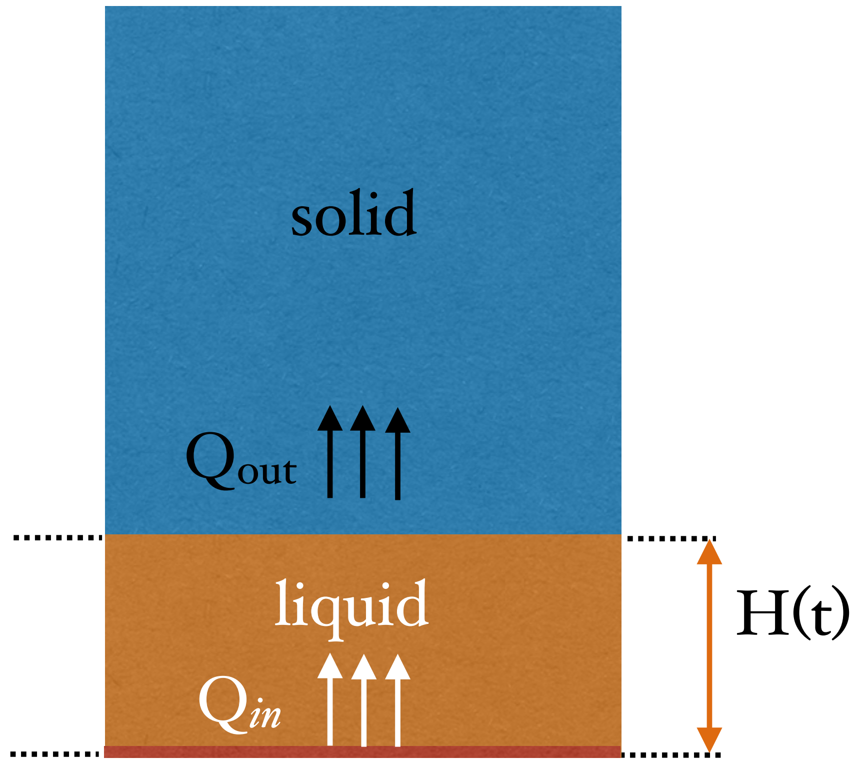
$Nu_{eff}(Ra_{eff}, Pr, \Gamma_{eff}, St) = ?$

Effective control parameters introduced in:
Davis, S.H. *et al. J. Fluid Mech. 144, 133–151 (1984)*. Pattern selection in single-component systems coupling Bénard convection and solidification

Effective Nusselt used in :
Ulvrová, Labrosse, *et al. Physics of the Earth and Planetary Interiors* 206 (2012): 51-66.

Heat flux regimes

Conductive melting



Stefan solution

$$\mathrm{H}\ (t) = 2\lambda\sqrt{\kappa t} \qquad \lambda \exp(\lambda^2)\ \mathrm{erf}\,(\lambda) = \frac{\mathrm{St}}{\sqrt{\pi}}$$

$$T_c(z,t) = T_0 - (T_0 - T_m)\frac{\mathrm{erf}\left(z/(2\sqrt{\kappa t})\right)}{\mathrm{erf}(\lambda)}$$

$$Nu^{in}_{eff} = \frac{2\lambda^2}{St}\ e^{\lambda^2} \qquad Nu^{out}_{eff} = \frac{2\lambda^2}{St}$$

independent of time

Convective melting

$$Nu_{eff}(Ra_{eff}, Pr, St)$$

$$Nu^{out}_{eff} \sim Ra^{\alpha}_{eff}\ Pr^{\delta}\ St^{\gamma} \longrightarrow \langle\phi_l\rangle \sim \tilde{t}^{\frac{1}{2-3\alpha}}\ Pr^{\frac{\delta}{2-3\alpha}}\ St^{\frac{\gamma+1}{2-3\alpha}}$$

Conductive case and St small

$$\alpha = \delta = \gamma = 0 \longrightarrow \langle\phi_l\rangle \sim \tilde{t}^{1/2}\ St^{1/2}$$

RB Classical (Malkus) scaling

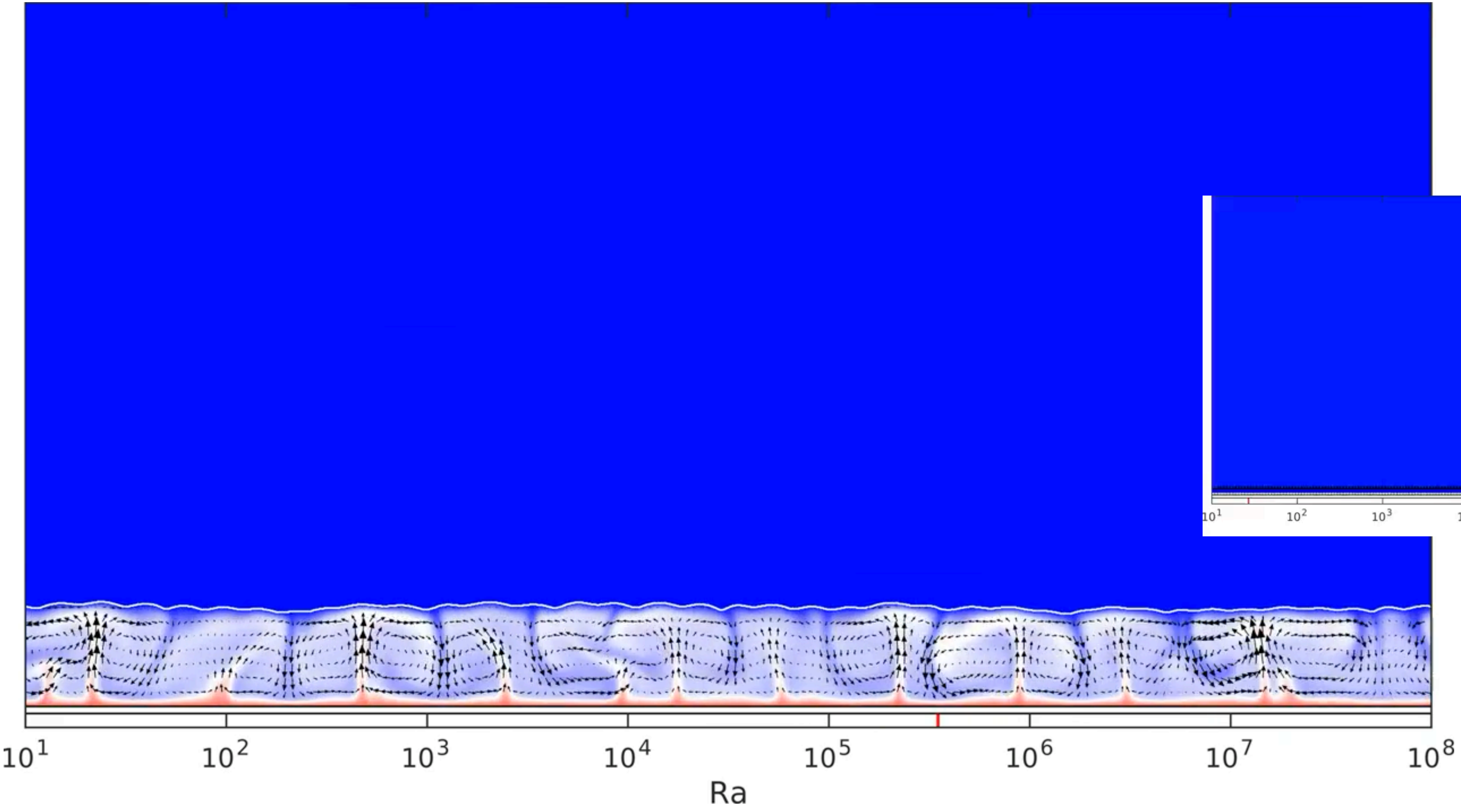
$$\alpha = 1/3 \longrightarrow \tilde{v}_m = \frac{d}{dt}\langle\phi_l\rangle = const$$

RB Ultimate regime

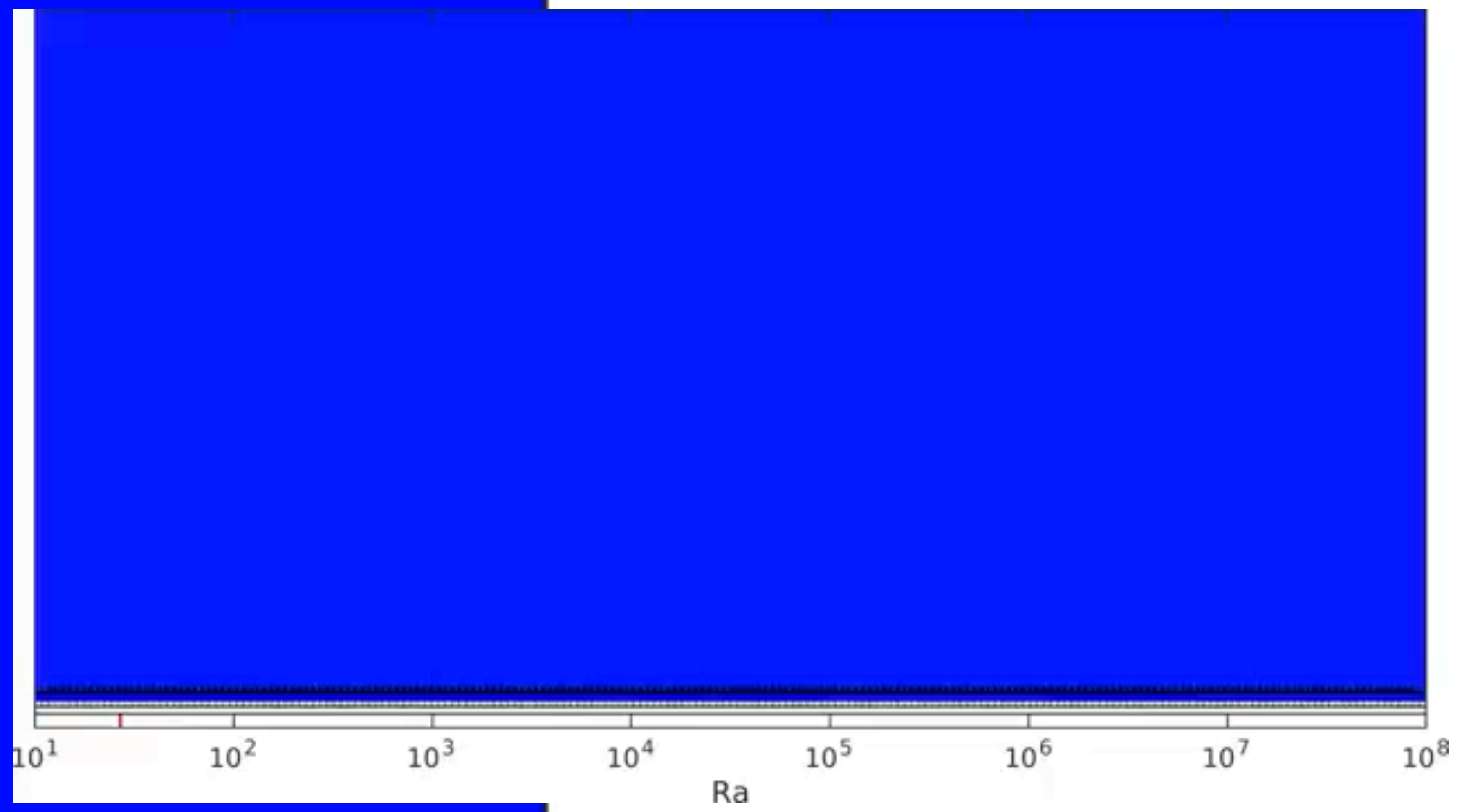
$$\alpha = 1/2 \longrightarrow \tilde{a}_m = \frac{d^2}{dt^2}\langle\phi_l\rangle = const$$

Convective melting 2D

$St=1$, $Pr= 10$, $\Gamma_{min} = 2$

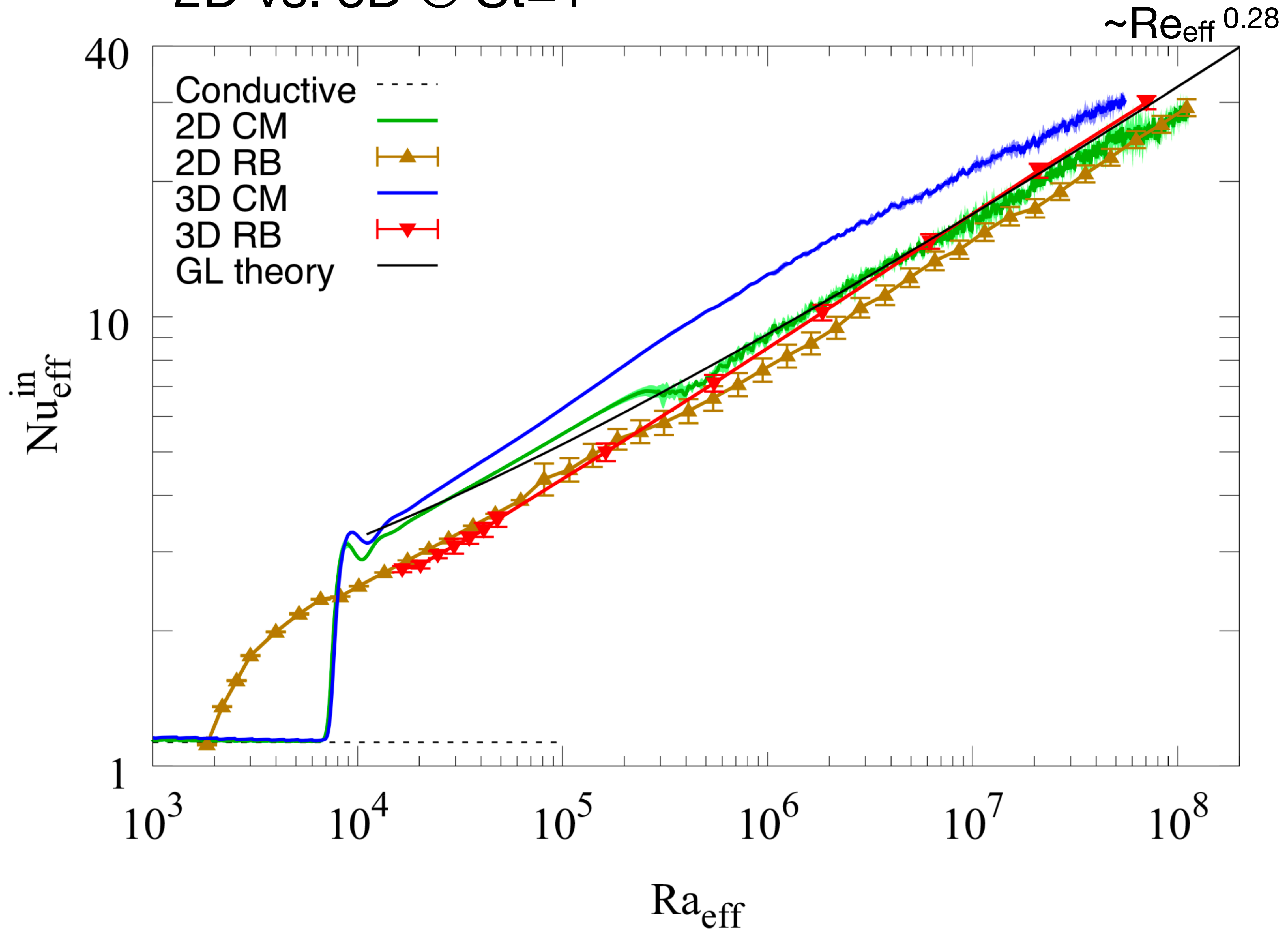


x4 faster

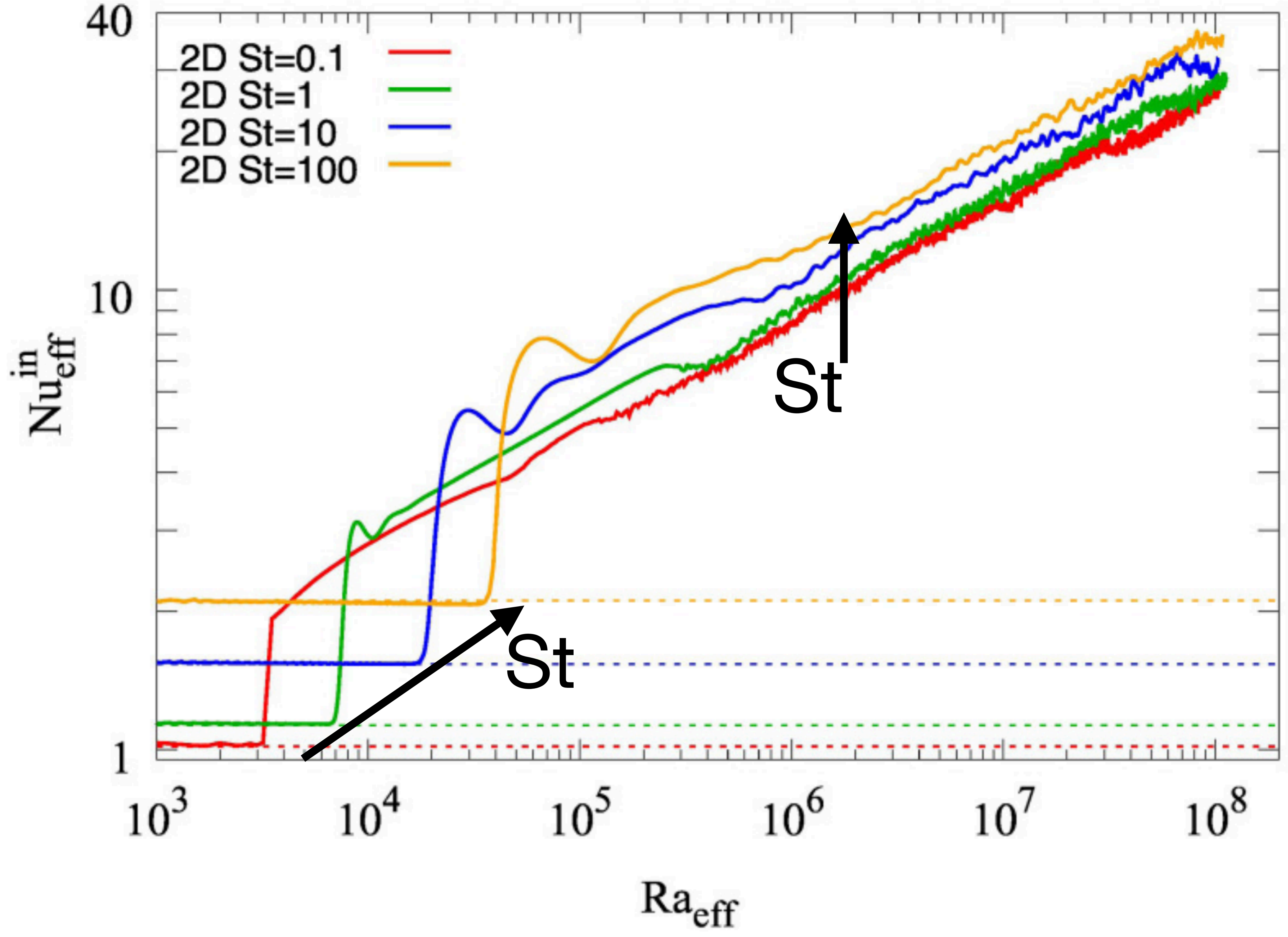


Global scaling laws

2D vs. 3D @ St=1

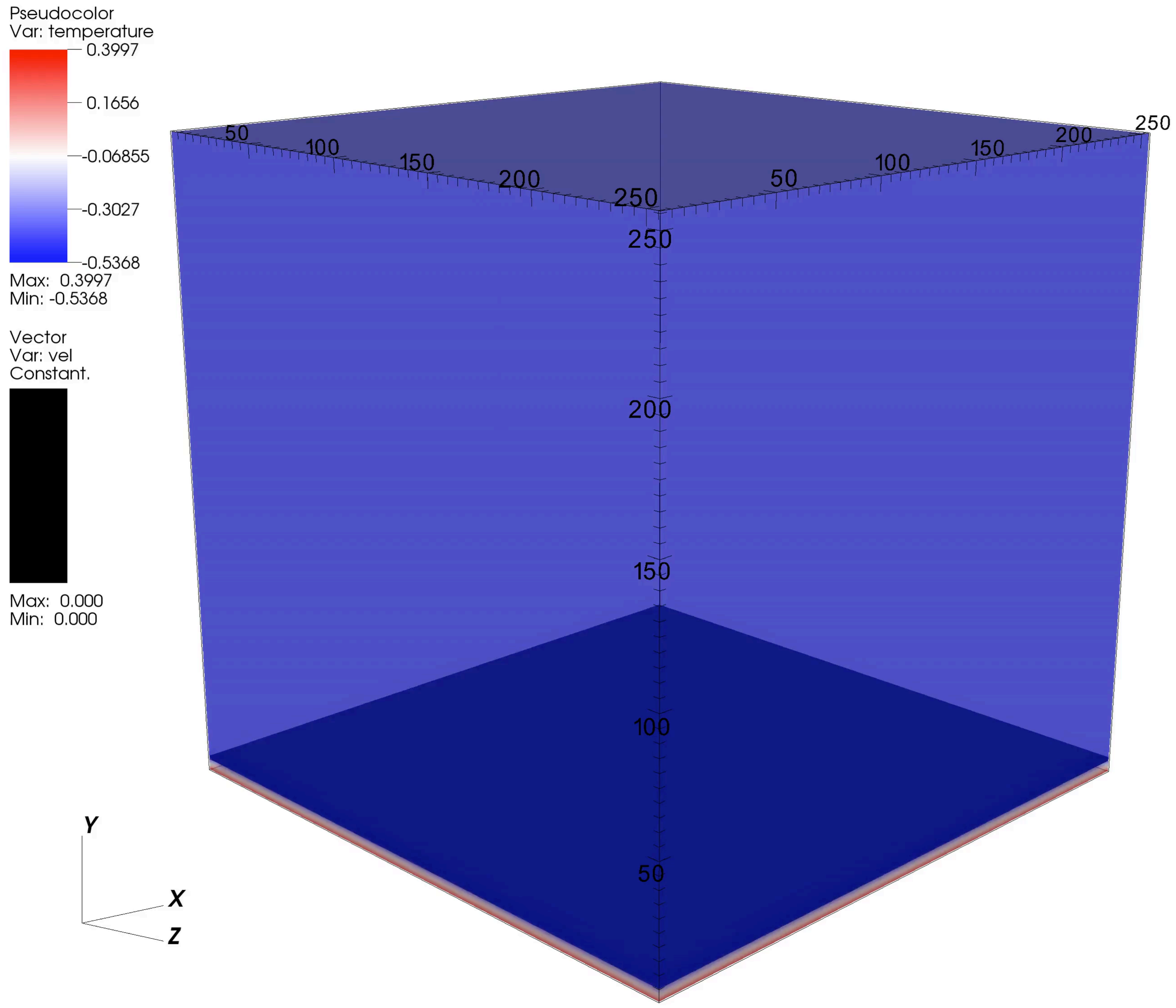


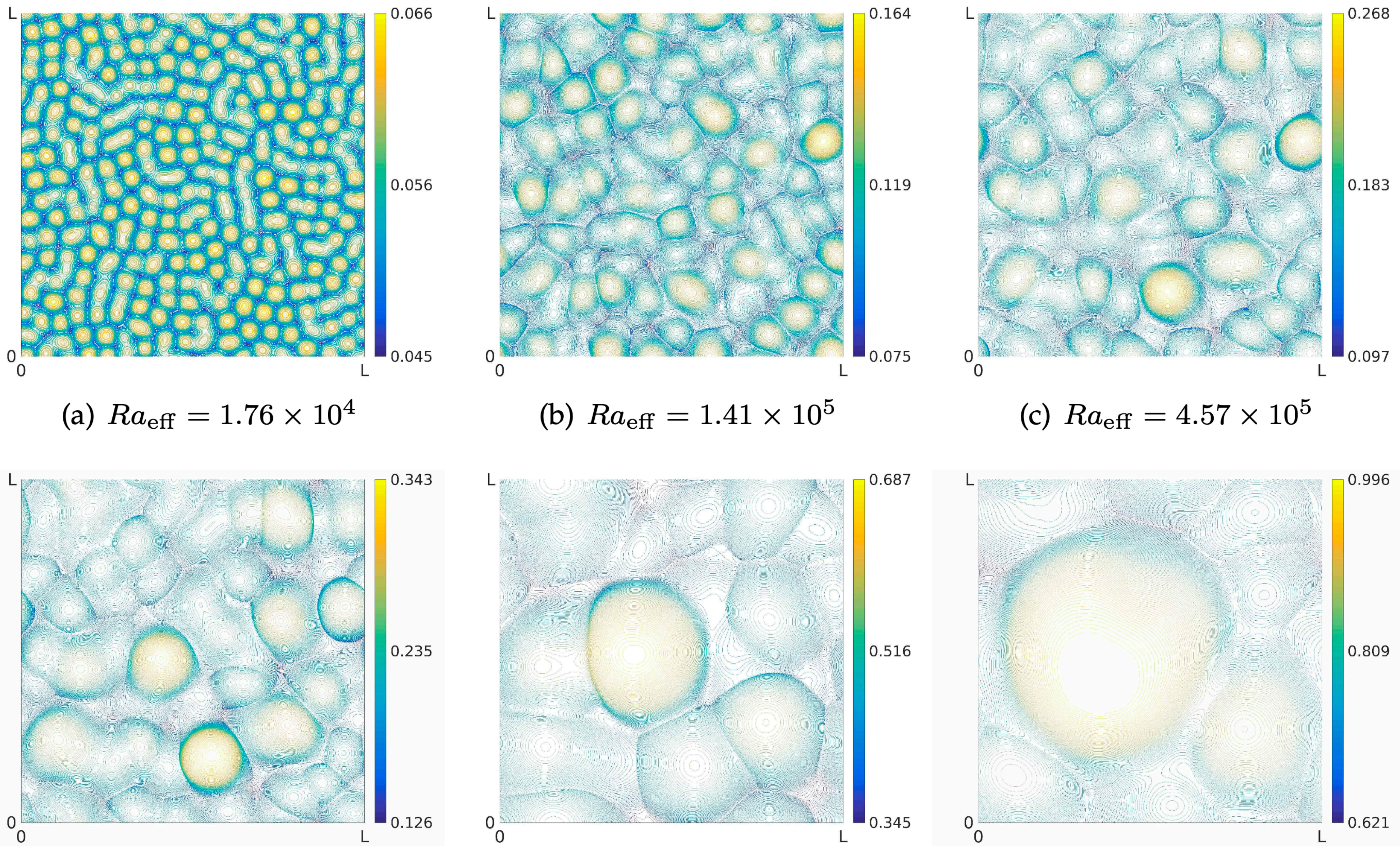
Effect of Stefan number



- Delayed onset compared to RB
- Nu_{eff} melting >= Nu RB up to Ra~$10^{7\text{-}8}$
- Nu_{eff} melting ~= Nu RB asymptotically in Ra

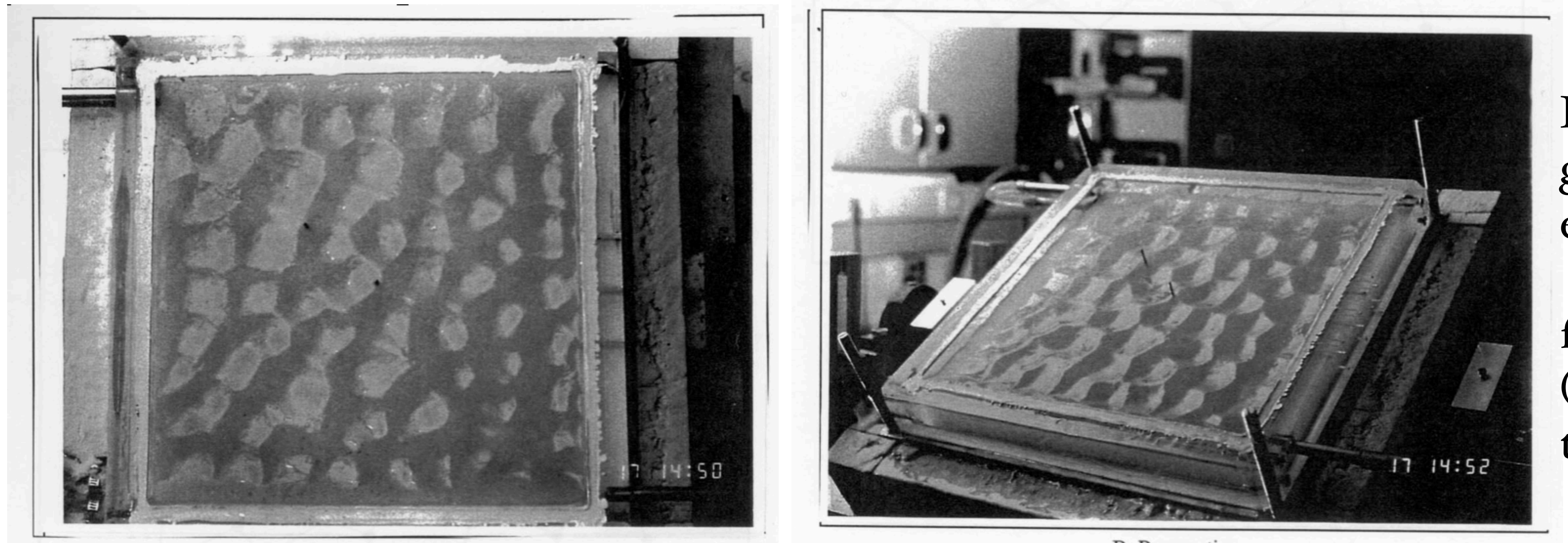
Convective melting 3D





(a) $Ra_{\text{eff}} = 1.76 \times 10^4$ (b) $Ra_{\text{eff}} = 1.41 \times 10^5$ (c) $Ra_{\text{eff}} = 4.57 \times 10^5$

(d) $Ra_{\text{eff}} = 1.04 \times 10^6$ (e) $Ra_{\text{eff}} = 1.15 \times 10^7$ (f) $Ra_{\text{eff}} = 5.54 \times 10^7$



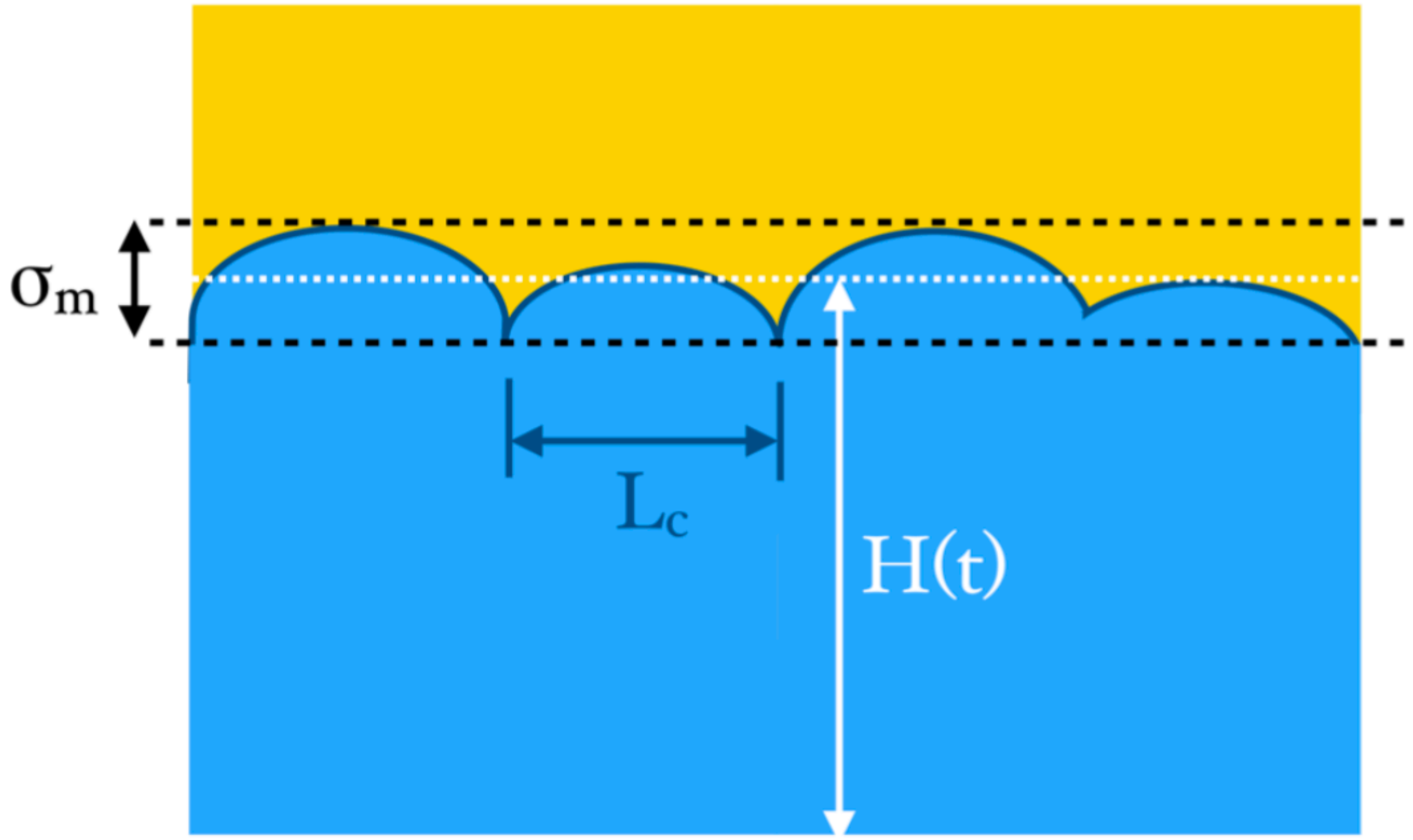
A. Vue en plan B. Perspective

Hill & Jaupart glycerol experiment

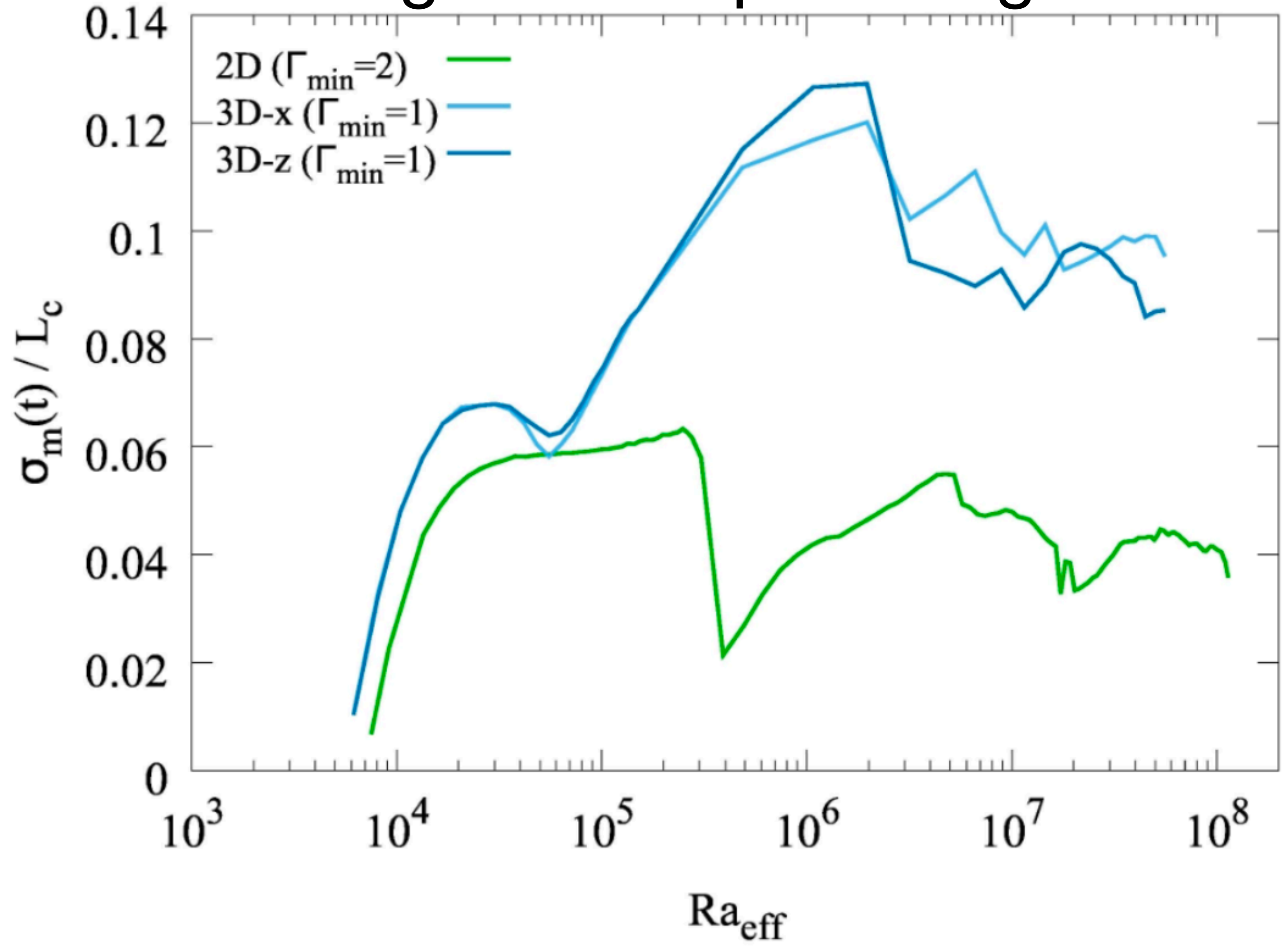
from X.Hill (1996) PhD thesis

Basal melting driven by turbulent thermal convection,
B. Rabbanipour Esfahani et al. **Phys. Rev. Fluids** 3, 053501 (2018)

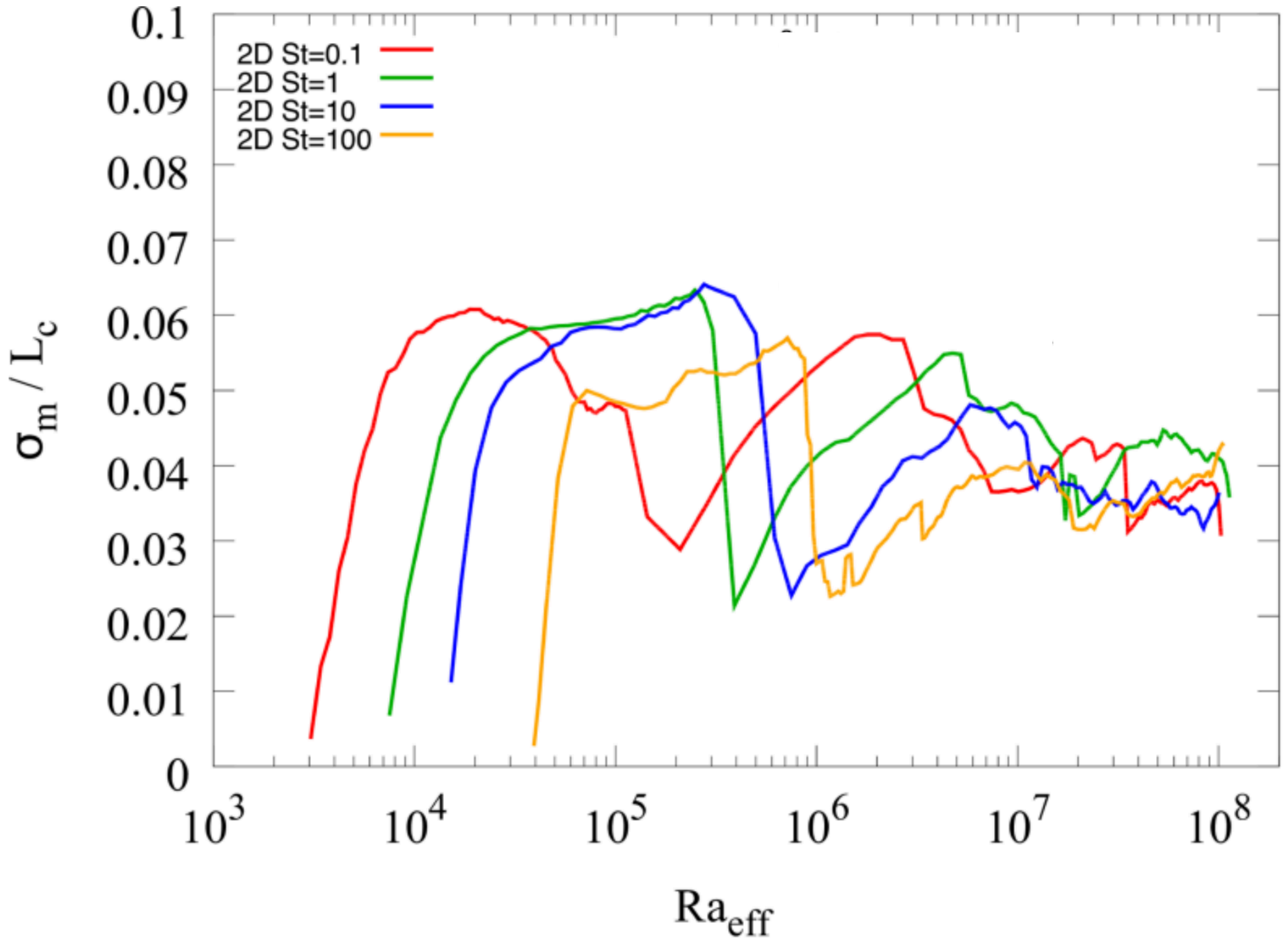
Topography



Roughness depth / lenght



Stefan effect?



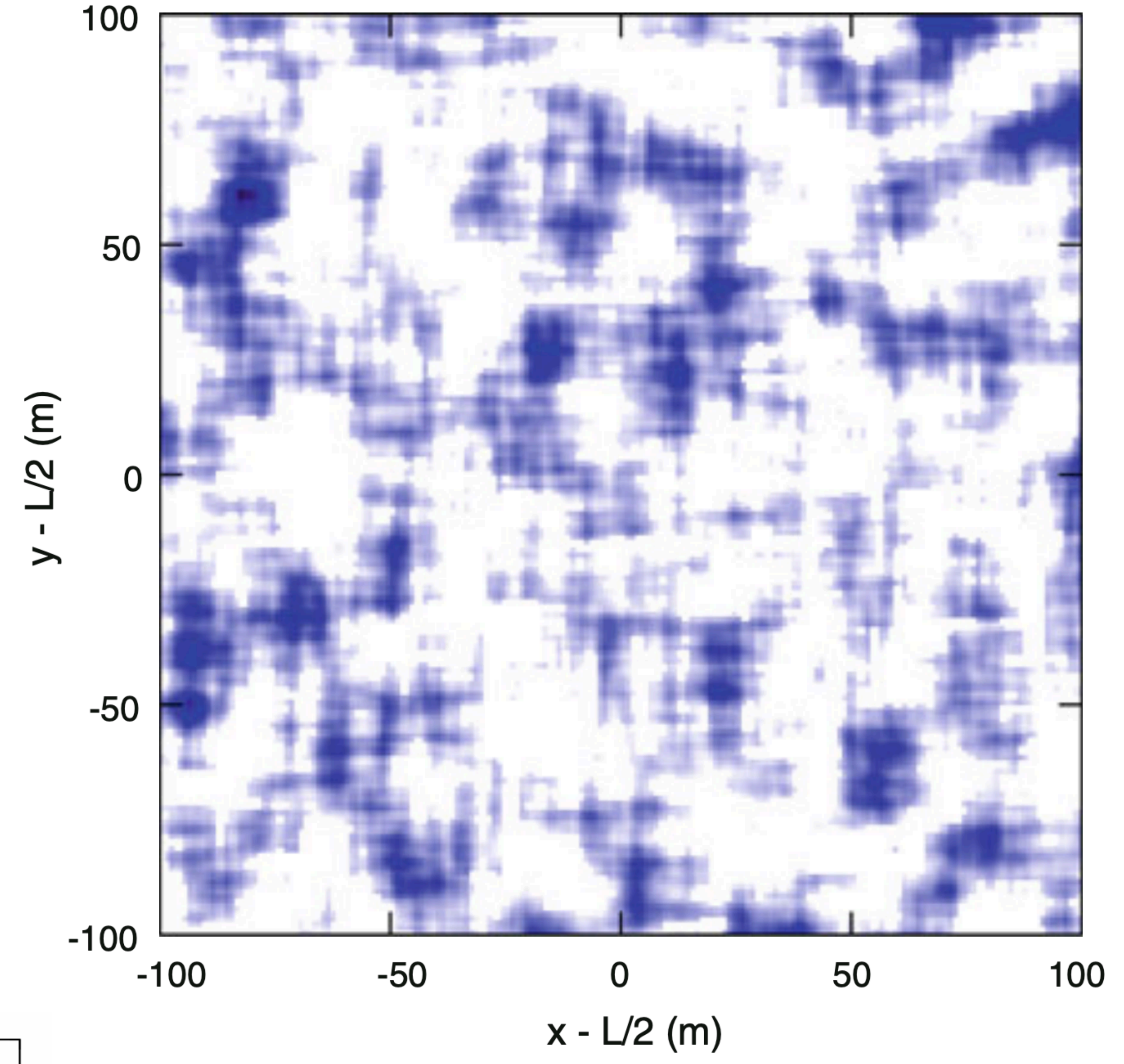
Multiple melt ponds

Sensitivity to small scale heat transfer mechanisms

ice thickness $\partial_t h = -\dot{f}$ ← Melting rate

water thickness (=pond depth) $\partial_t w = -\nabla \cdot (\mathbf{u} w) + \frac{\rho_i}{\rho_w} \dot{f} - s$

Meltwater flux ($\mathbf{u}w$), Seepage rate (s)



$$\dot{f} = \dot{m}_i + \dot{m}_p \left(\frac{w}{w_{\max}}\right)^{\alpha}, \quad \text{with } \alpha = \begin{cases} 1 & \text{if } w \in [0, w_{\max}] \\ 0 & \text{if } w > w_{\max}, \end{cases}$$

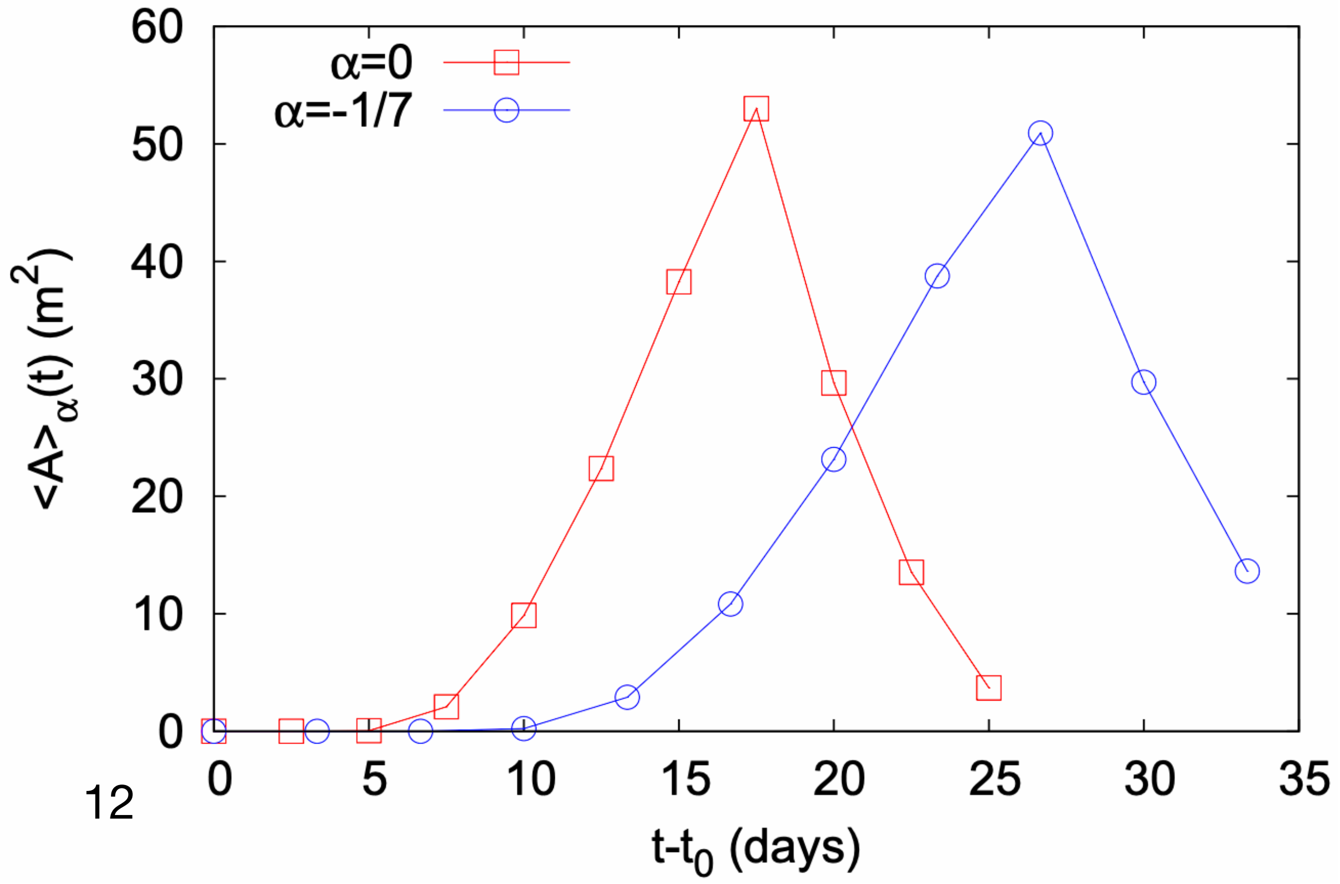
$w_{\max} \approx 0.1\text{m}$ radiation-conduction

convection

$$Nu_{eff} \sim Ra_{eff}^{c}$$

$c = \frac{1}{3}$ classical

$c = \frac{2}{7}$ anomalous



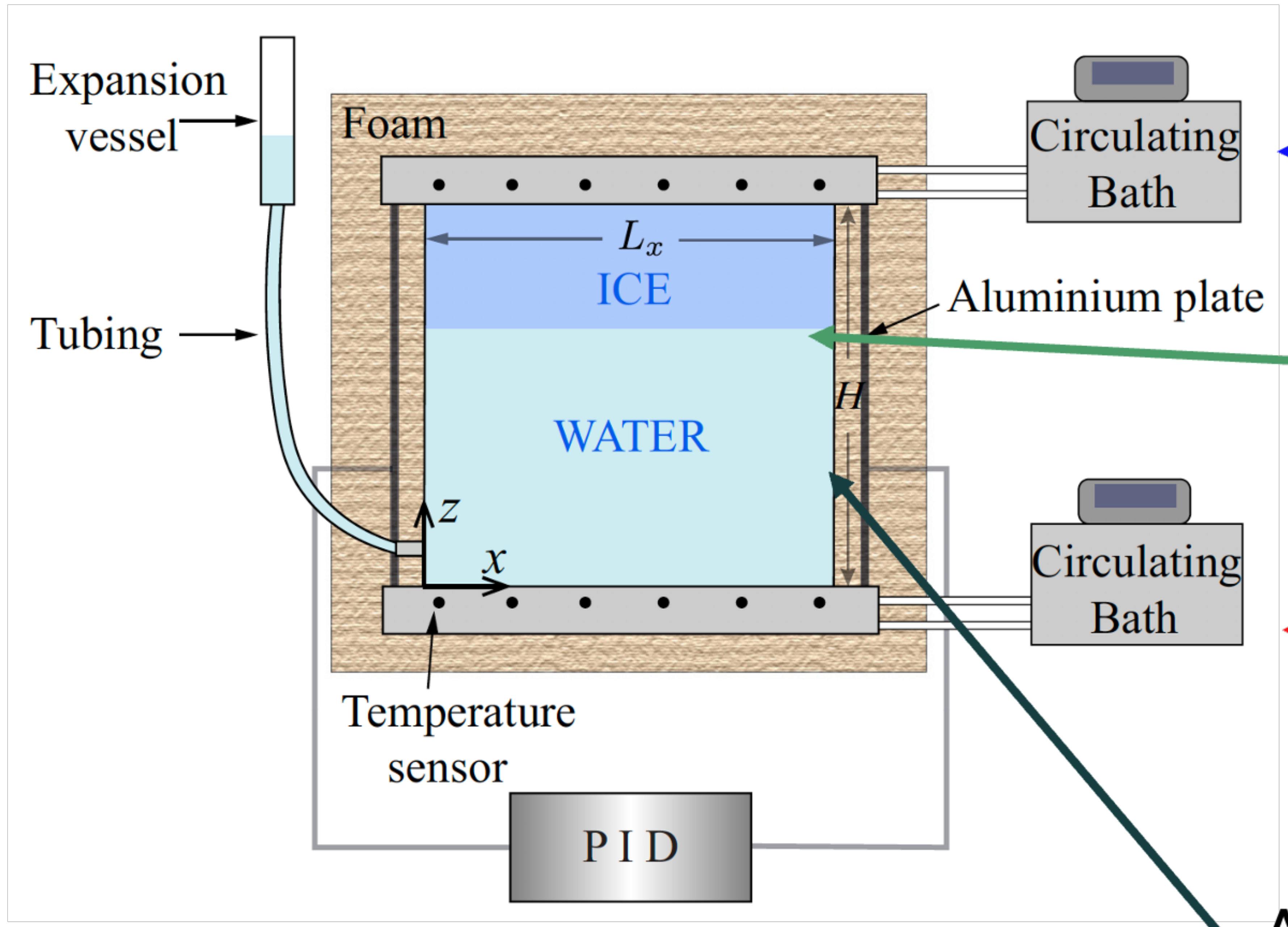
Modelling the evolution of melt ponds: sensitivity to microscale heat transfer mechanisms, A. Scagliarini et al. **Springer INdAM Series**, vol 38. Springer, Cham (2020)

Convection and freezing

A step towards experiments

Experimentalists prefer freezing

Experiments quasi-2D system

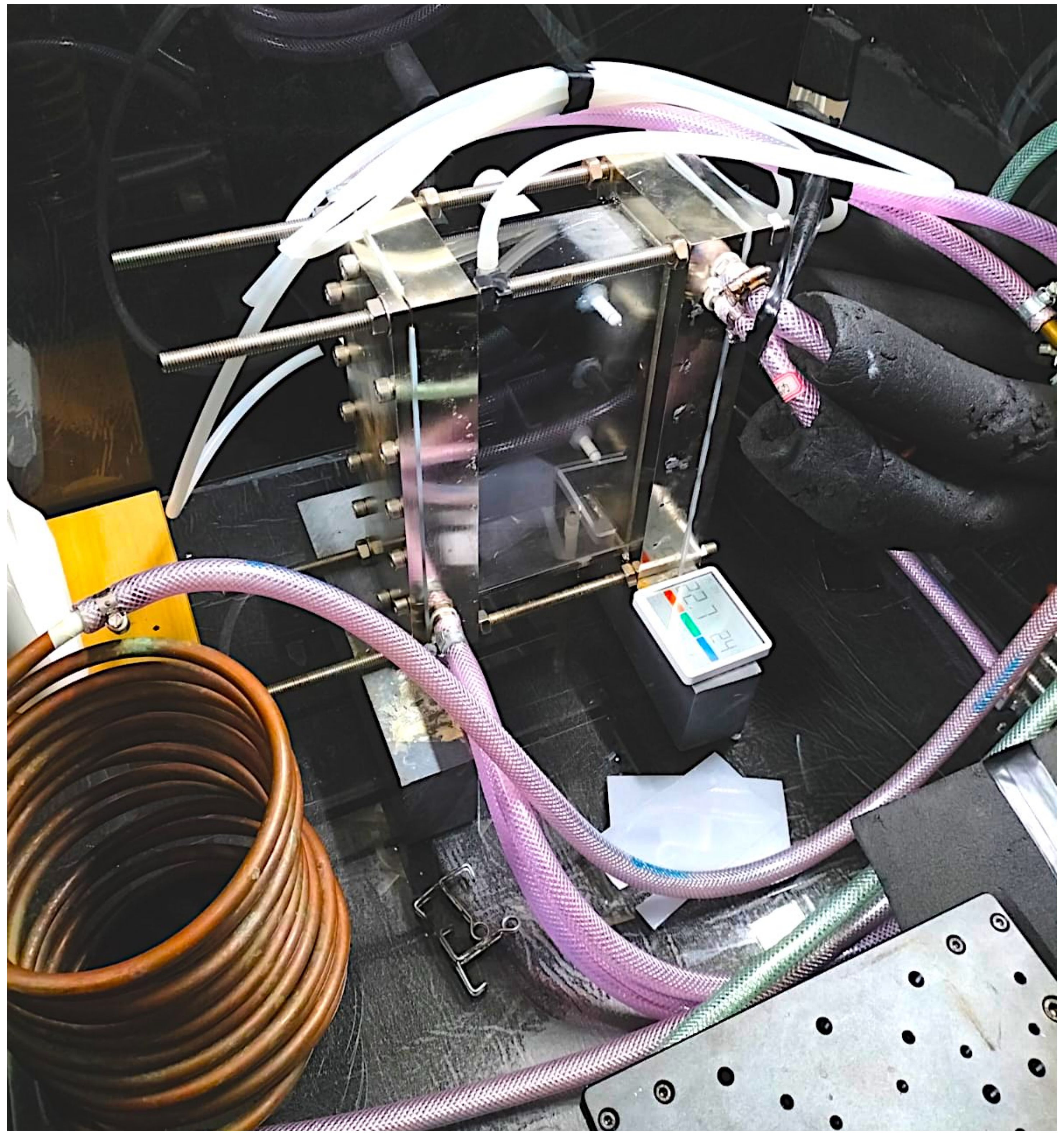


Constant Temperature
T < 0 °C

Ice water interface
T = 0 °C

Constant Temperature
T > 0 °C

Adiabatic sidewall



RB-VC freezing cell @Chao Sun LAB

Measured quantities: ice height spatial and temporal evolution

Water is special

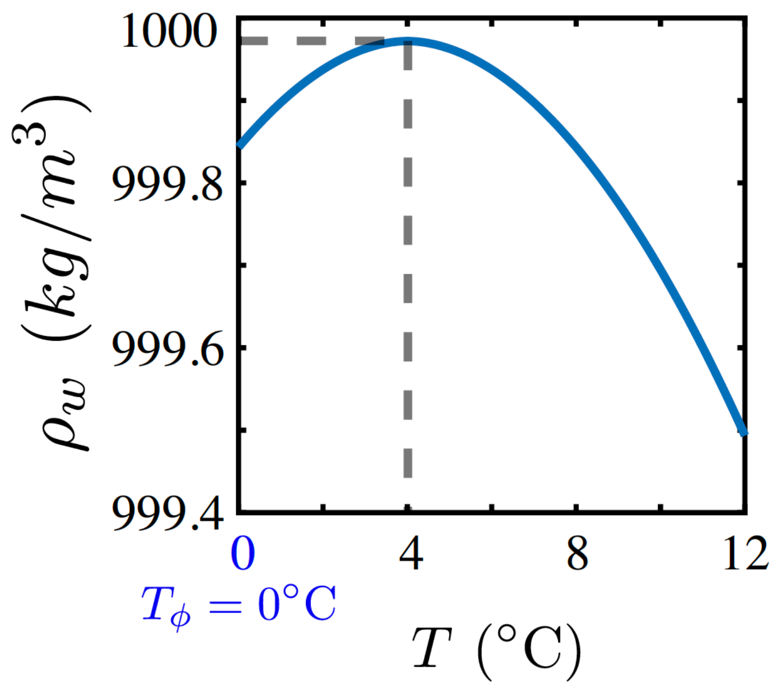
Adapting the simulations to freezing water

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u} = -\frac{\boldsymbol{\nabla} p}{\rho_0} + \nu_w \nabla^2 \mathbf{u} + \rho_w(T)\mathbf{g}$$

$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial}{\partial t}(\rho c_p T) + \boldsymbol{\nabla}\cdot(\mathbf{u}\ \rho c_p T) = \boldsymbol{\nabla}\cdot(\lambda \boldsymbol{\nabla} T)$$

1) Consider water equation of state



$$\rho_w = \rho_0(1 - \alpha^* |T_b - T_c|^q),$$

$$\rho_0 = 999.972 kg/m^3$$

$$\alpha^* = 9.30 \times 10^{-6}(K^{-q}),\ q = 1.895$$

$$T_c = 4^o C$$

Water/Ice interface condition

$$\mathcal{L}\ \rho_{ice}\ \dot{\mathbf{x}}_{\phi_w}(t) = \hat{\mathbf{n}}\cdot\lambda_{ice}\boldsymbol{\nabla} T|_{\mathbf{x}=\mathbf{x}_{\phi_w^+}} - \hat{\mathbf{n}}\cdot\lambda_w \boldsymbol{\nabla} T|_{\mathbf{x}=\mathbf{x}_{\phi_m^-}}$$

2) Local properties of ice & water

$$\rho(\mathbf{x}), c_p(\mathbf{x}), \lambda(\mathbf{x})$$

$$\rho_{ice}, c_{p(ice)}, \lambda_{ice} \quad \rho_w, c_{p(w)}, \lambda_w$$

Conjugate heat transfer

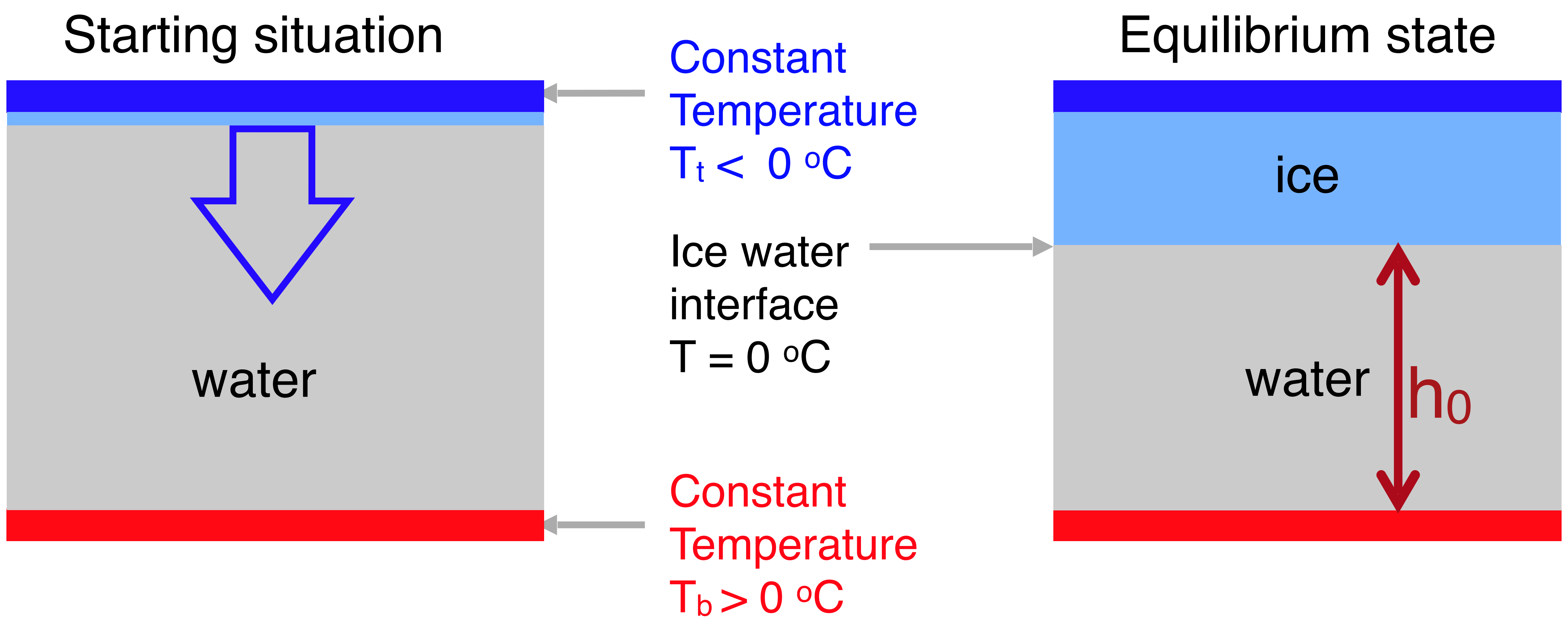
Using enthalpy h to separate two phases

$$\mathrm{h} = \begin{cases} L\phi_w + \mathrm{C_{pI}}T, & when\ T < T_\phi, \\ L\phi_w + \mathrm{C_{pI}}T_\phi, & when\ T = T_\phi, \\ L\phi_w + \mathrm{C_{pI}}T_\phi + \mathrm{C_{pW}}(T - T_\phi), & when\ T > T_\phi. \end{cases}$$

3) Neglect: Volume change; Gibbs-Thomson effect (small interface curvature)

Wang, Calzavarini, Sun, Toschi, PNAS (2021)

Freezing study protocol

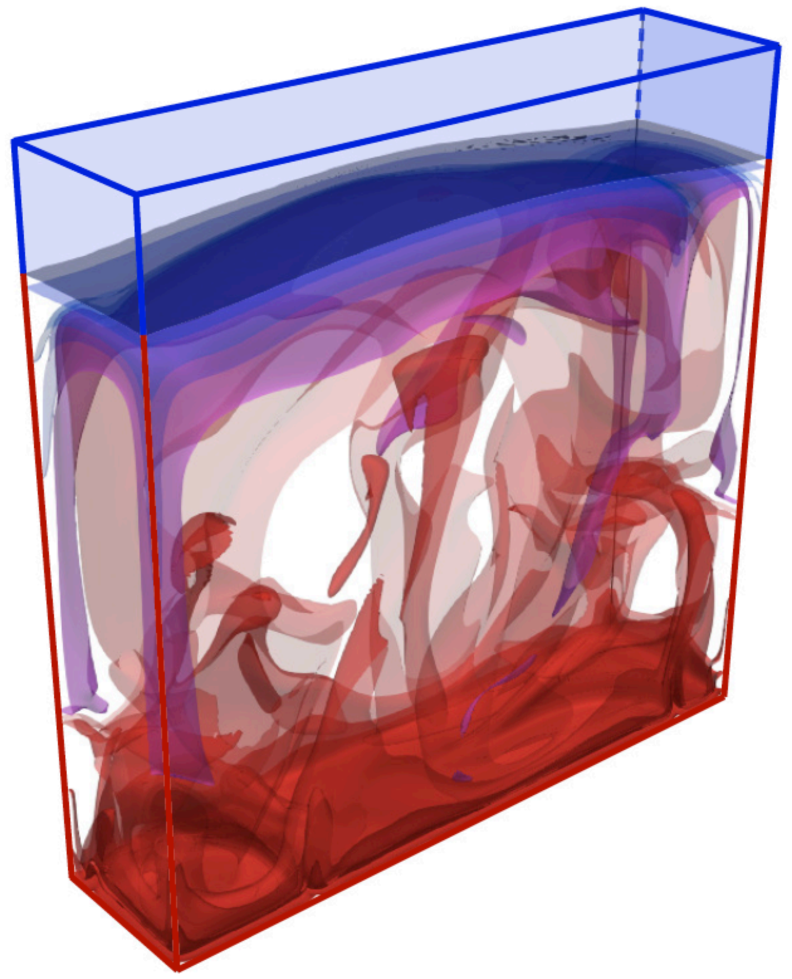


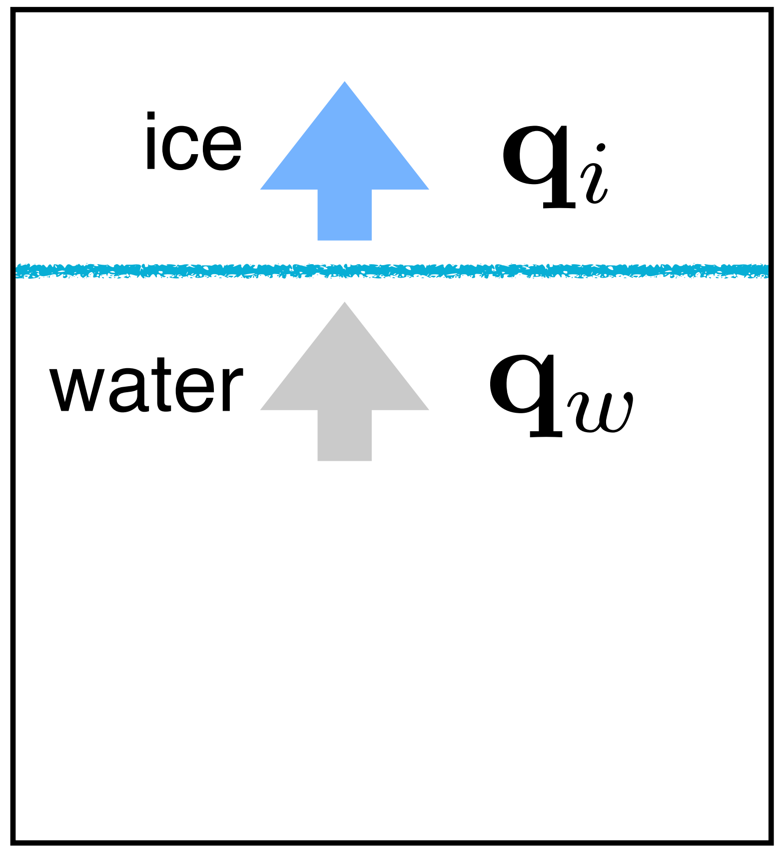
Here T_t = -10 °C

T_b = [0, 15] °C

$h_0 = \langle h(x,t) \rangle$ measured

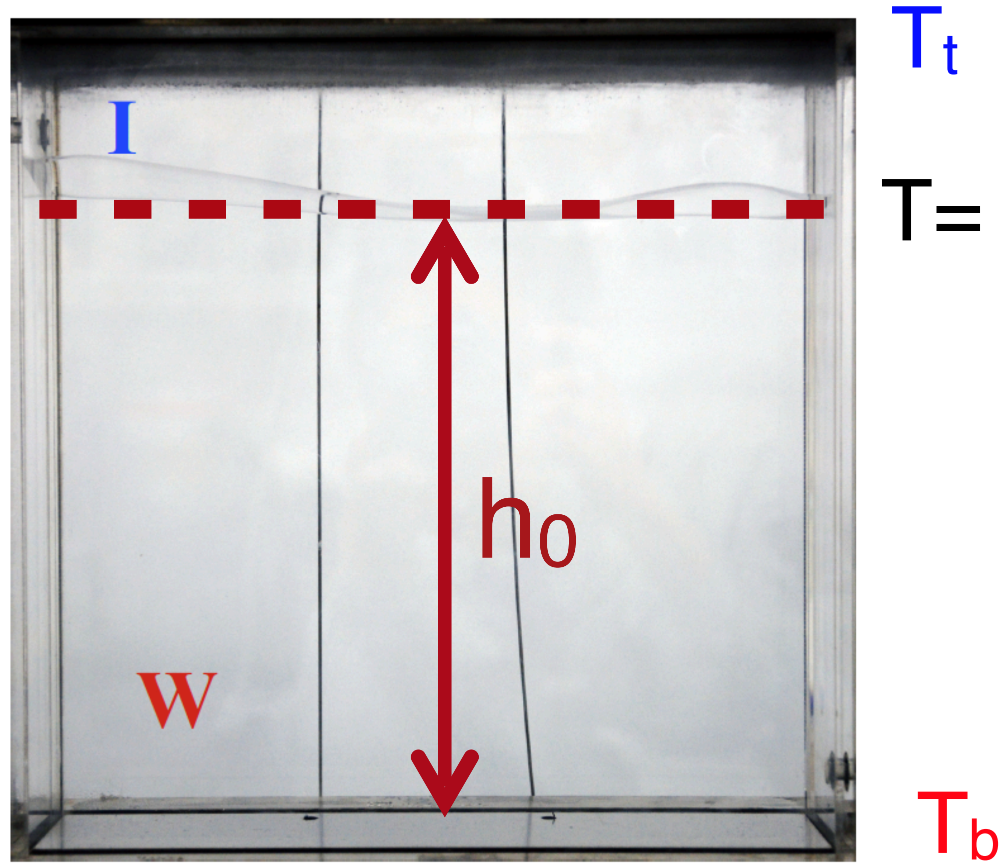
Experiments + Numerical Simulations (DNS) + 1D Modelling

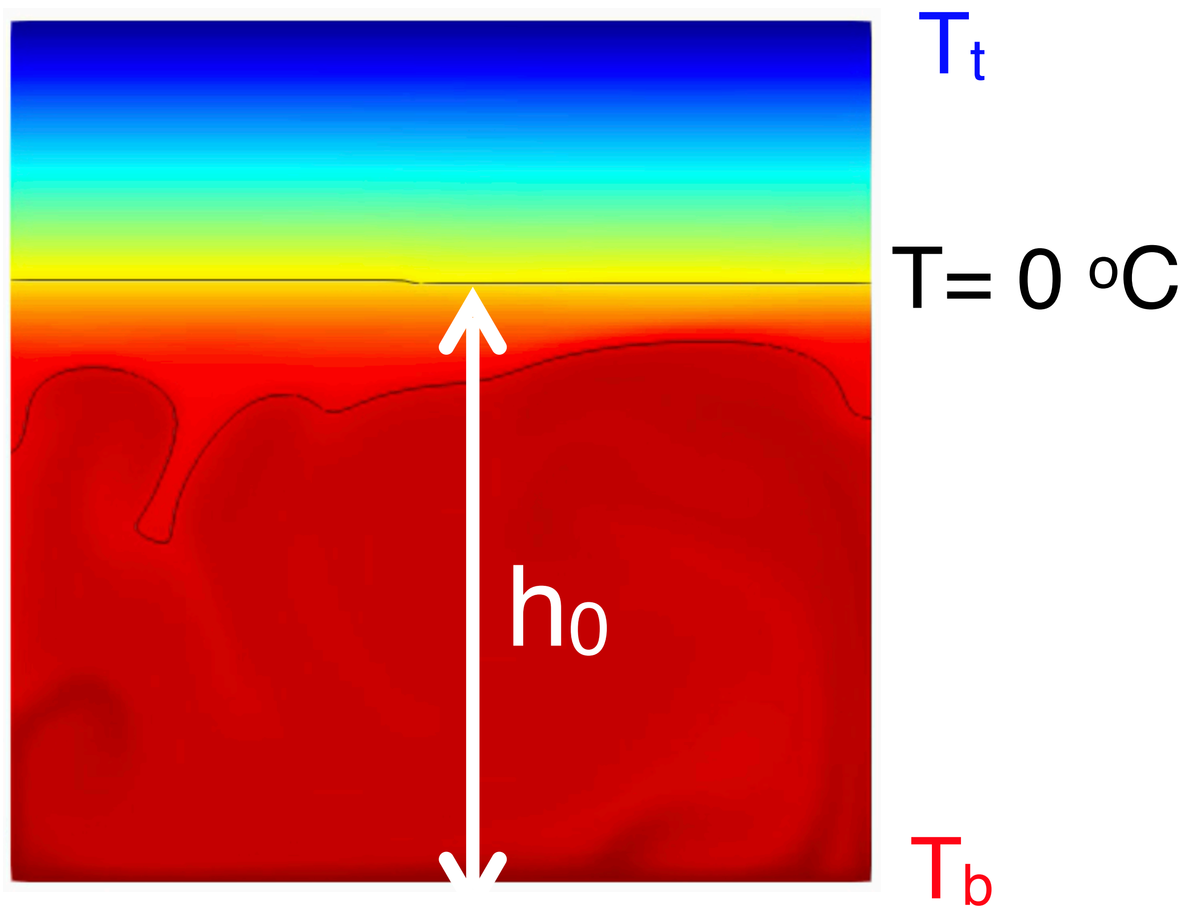


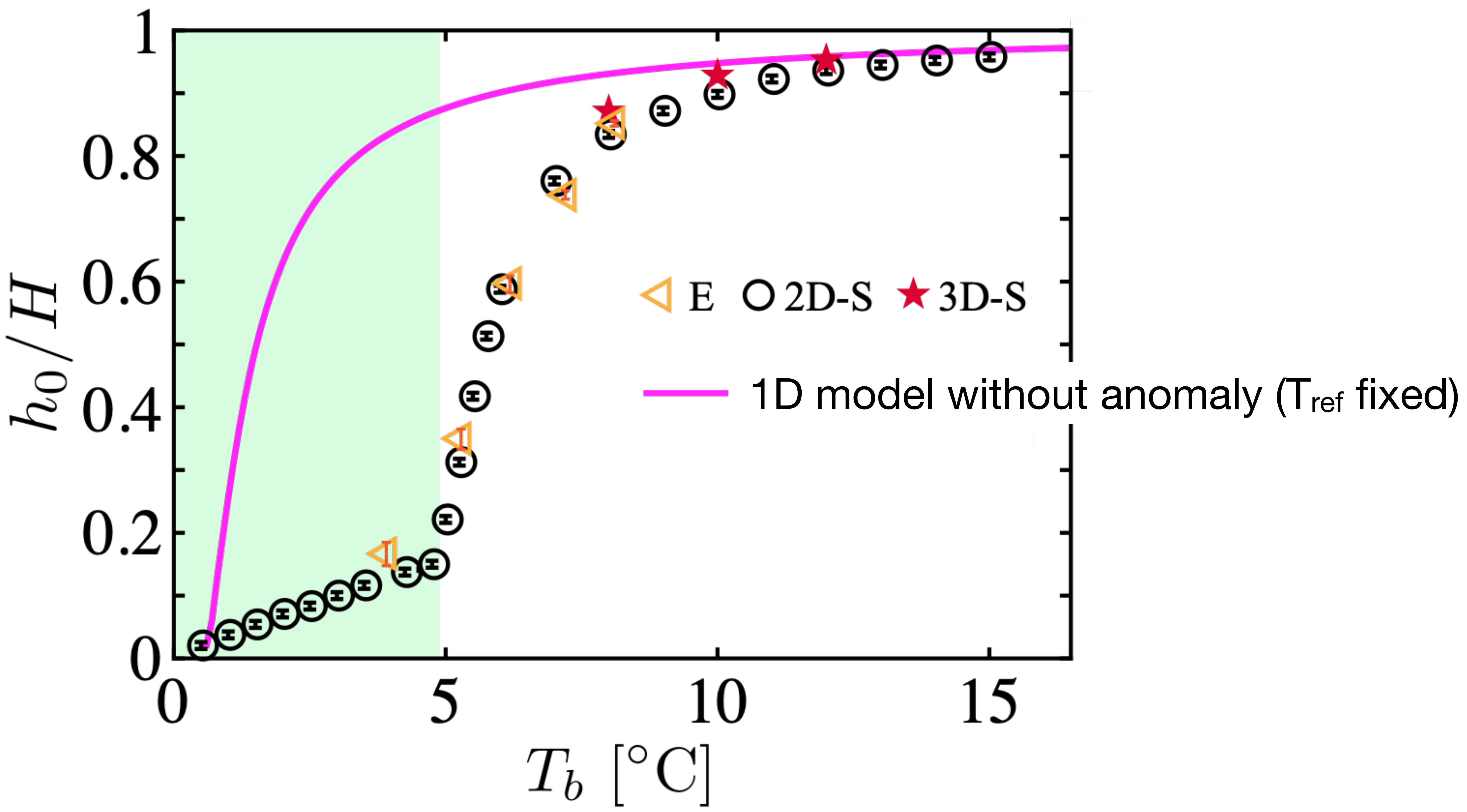


How the growth of ice depends on the fluid dynamics underneath,
Z. Wang, E. Calzavarini, C. Sun, F. Toschi, **PNAS** 118 (10) e2012870118
(2021)

Averaged ice thickness vs. T_{bottom}



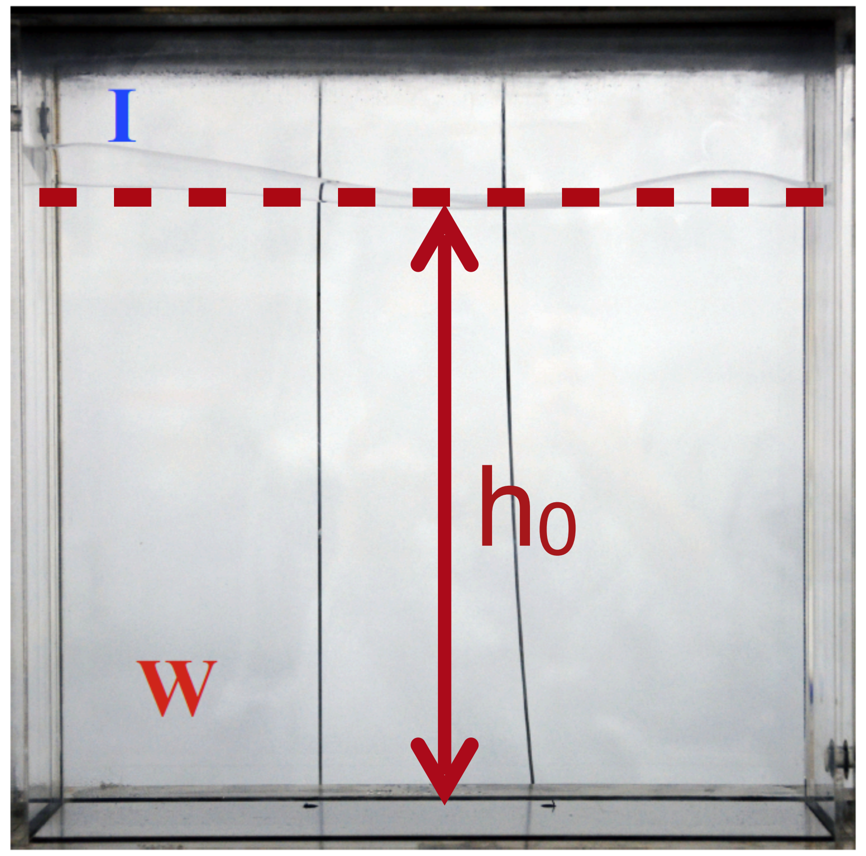


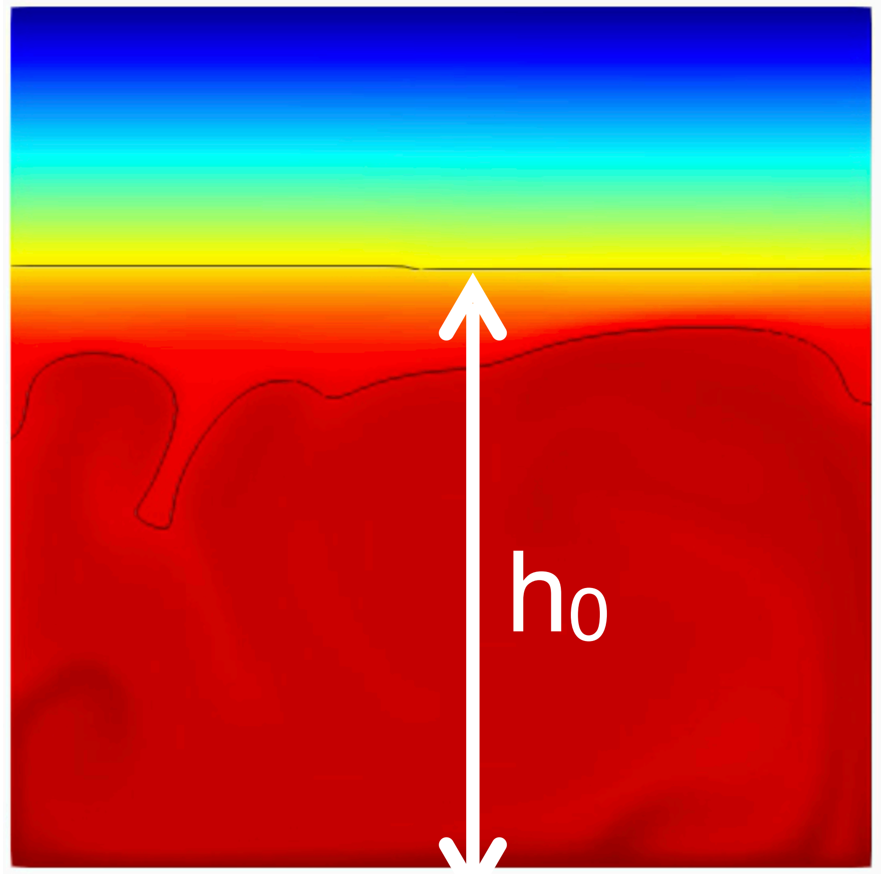


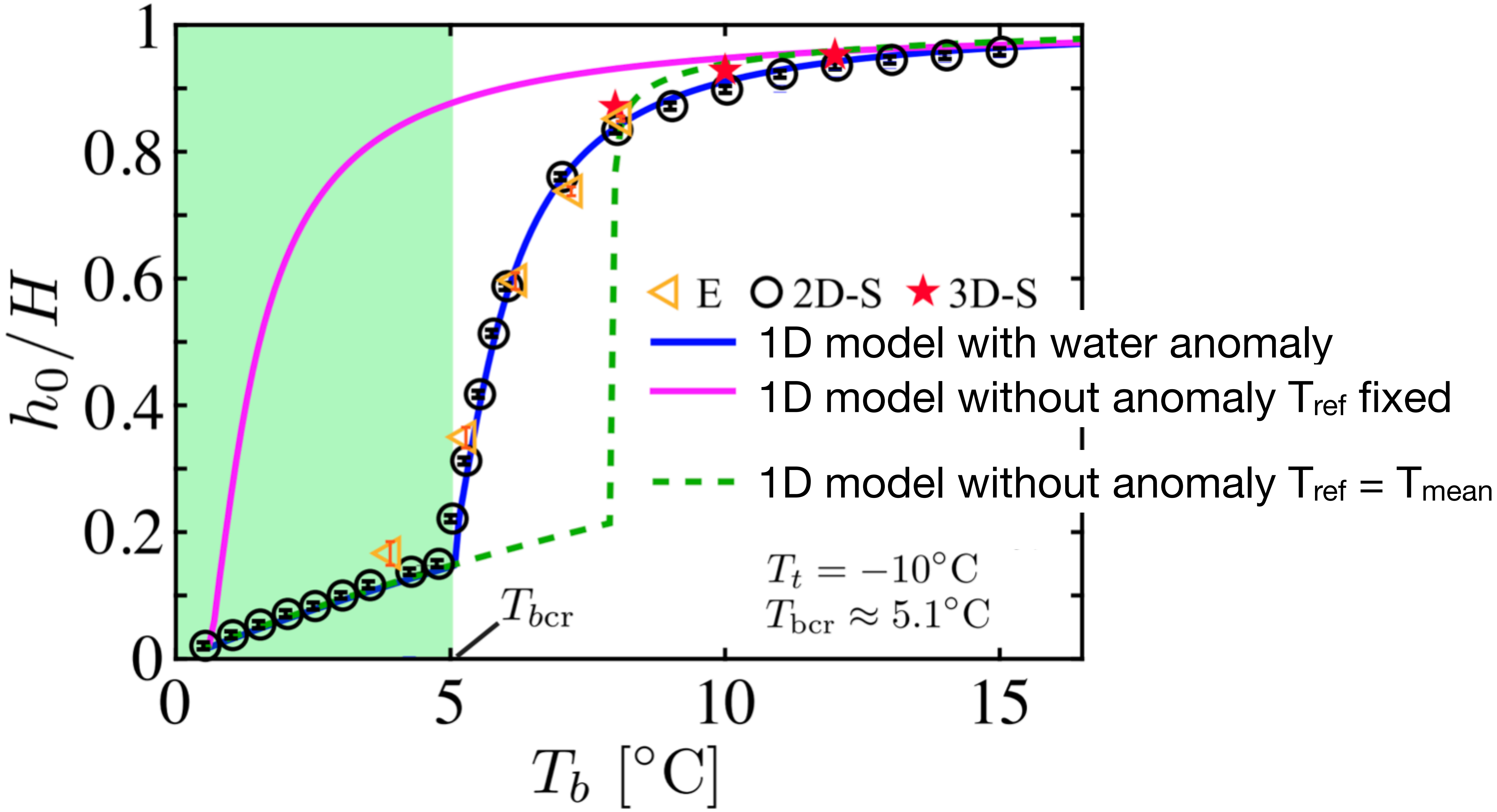
- Ice thickness decreases with increasing T_b
- Experiments, 3D, and 2D simulations agree well

- 1D model without density anomaly not working

Averaged ice thickness vs. T_{bottom}



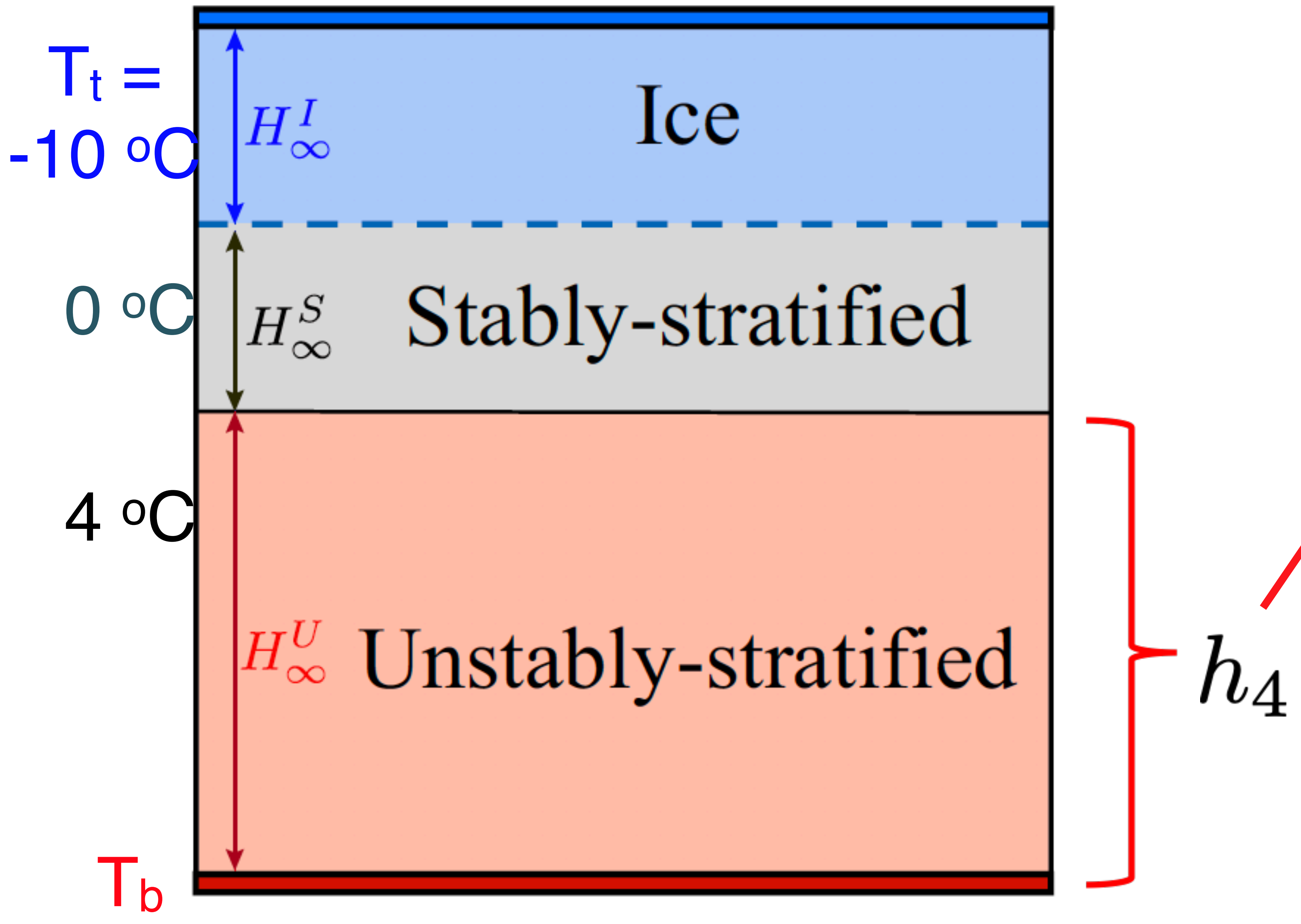




- Ice thickness decreases with increasing T_b
- Experiments, 3D, and 2D simulations agree well
  - 1D model with density anomaly agrees with EXP and DNS

It is crucial to consider the density anomaly
What happens in the system?

Coupled heat transfer dynamics of ice and water layers

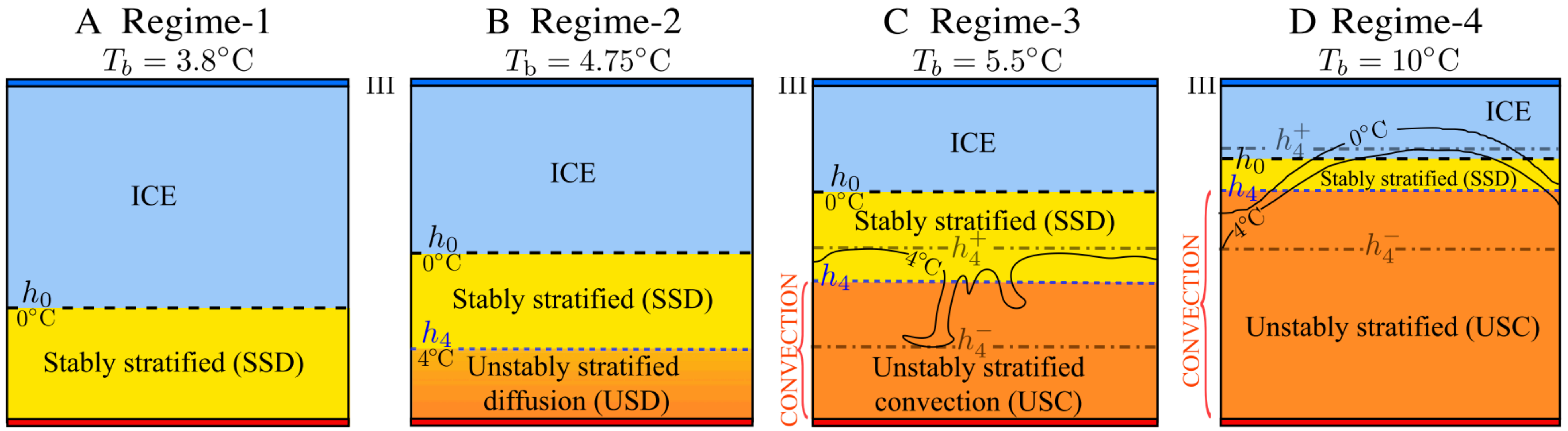


The unstably-stratified layer can be stable (diffusive) or unstable (convecting), depending on the effective Ra_e of the layer vs. the critical Ra_c for the onset of convection

$$\mathrm{Ra_e} = \frac{(\Delta\rho/\rho_0)g(h_4)^3}{\nu\kappa} \qquad Ra_{cr} \approx 1708$$

$Ra > Ra_{cr}$ Convection

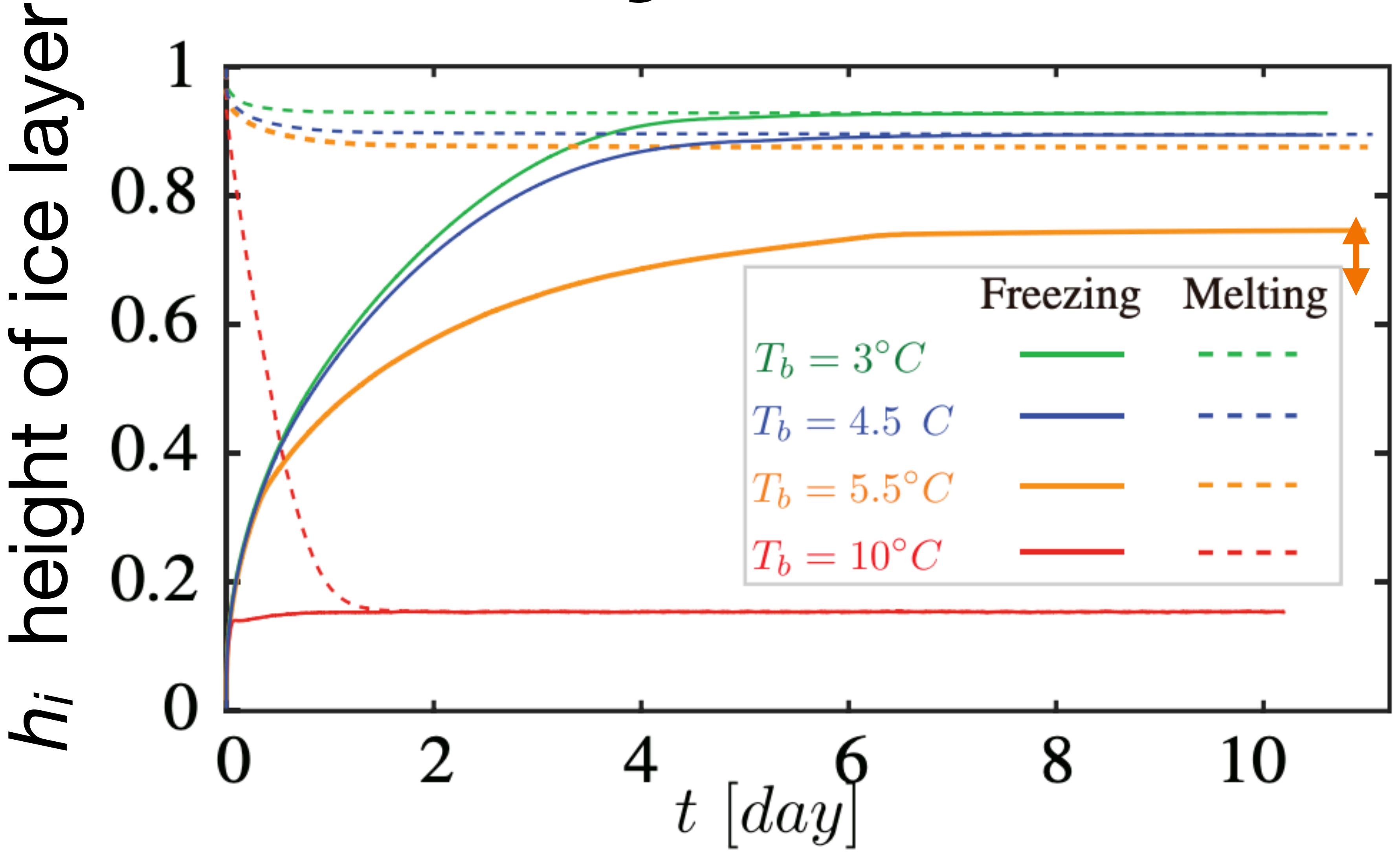
$Ra < Ra_{cr}$ Conduction

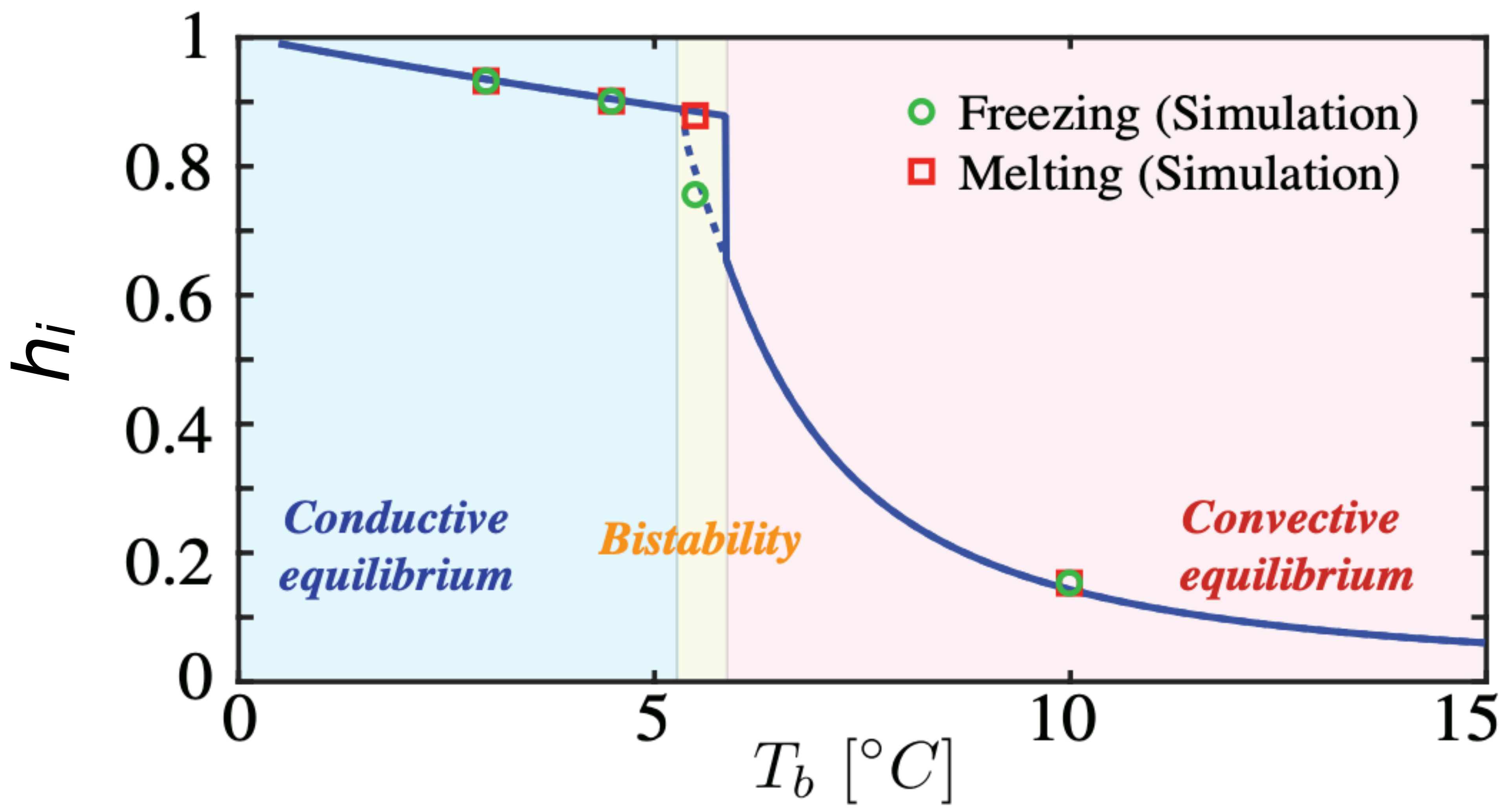


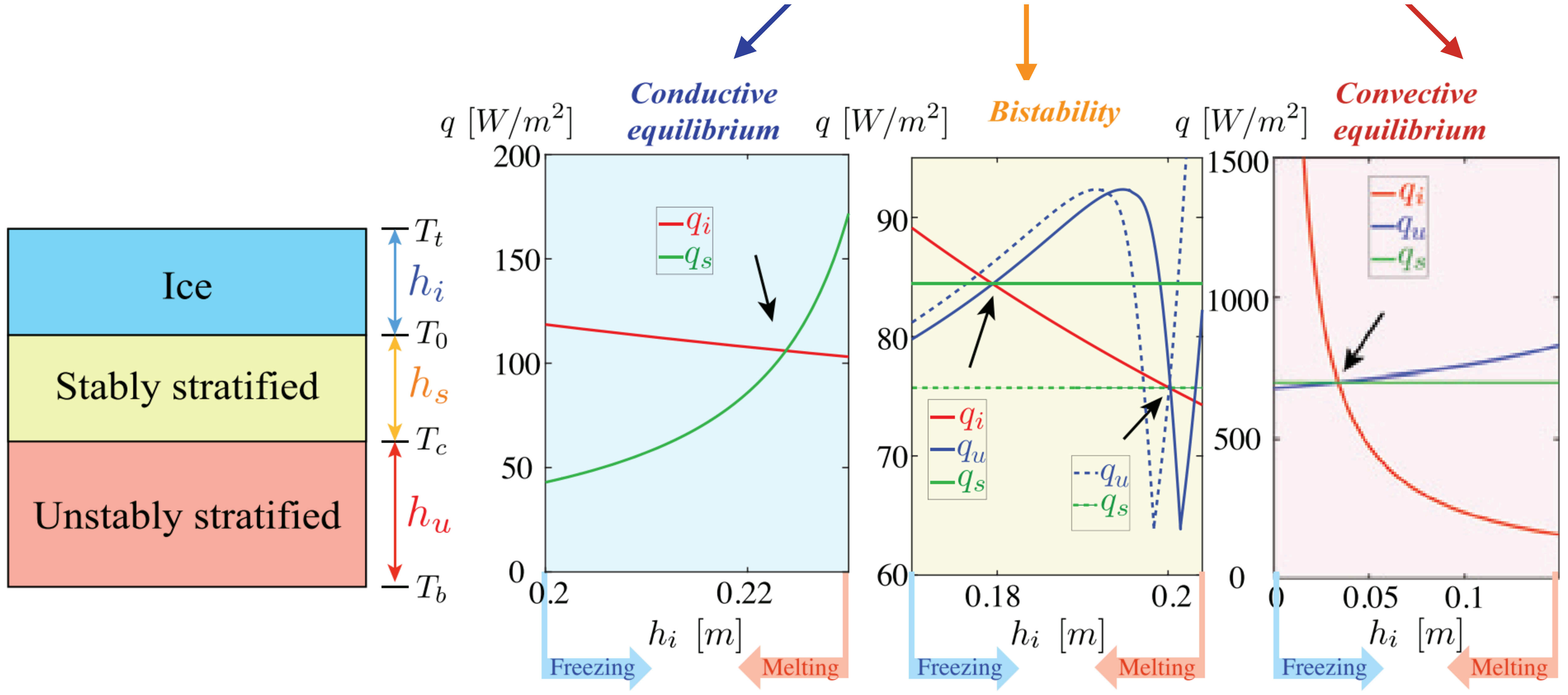
During the experiment the unstably-stratified layer even can switch from unstable to stable due to he growth of the ice and decrease of h_4

Freezing vs Melting

Bistability







On bistability

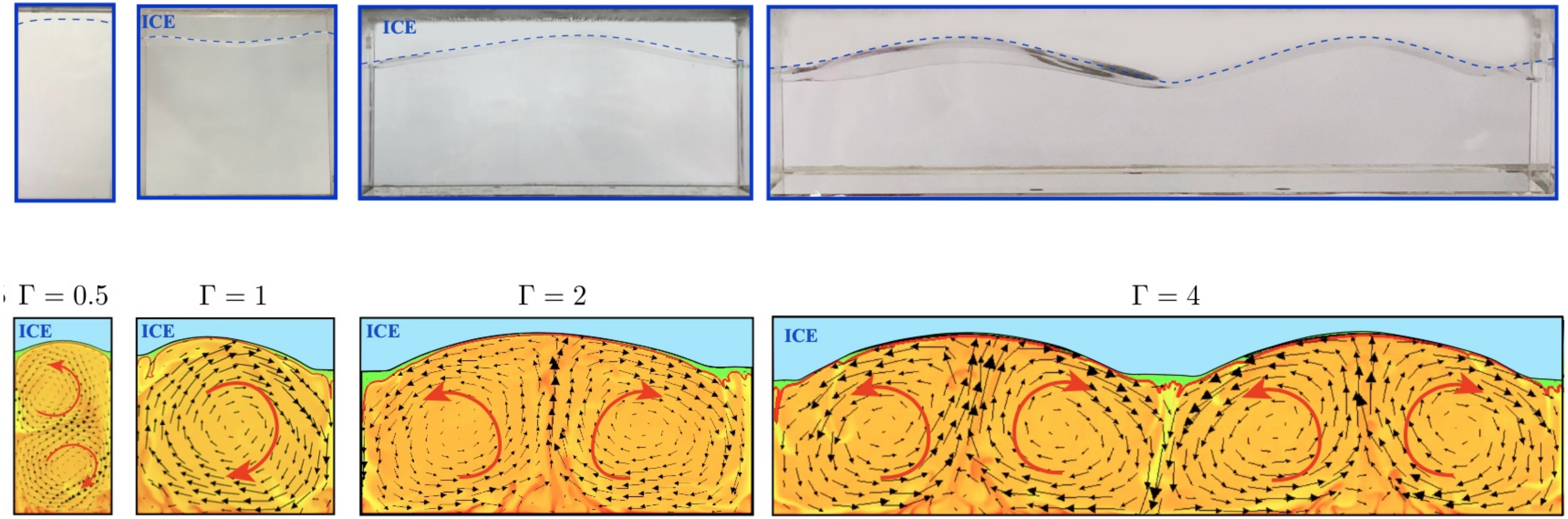
For a positive buoyancy fluid:
Bistability in Rayleigh-Bénard convection with a melting boundary, J. Purseed, B. Favier, L. Duchemin, E. W. Hester PRF (2020)

In water-ice
Equilibrium states of the ice-water front in a differentially heated rectangular cell, Z. Wang, E. Calzavarini, C. Sun, EPL (2021)

In melt-ponds
Bistability in Radiatively Heated Melt Ponds, R.Yang, C.J. Howland, H. Liu, R. Verzicco, D. Lohse PRL (2023)

Boxes of all kinds

RB varying the aspect ratio Γ

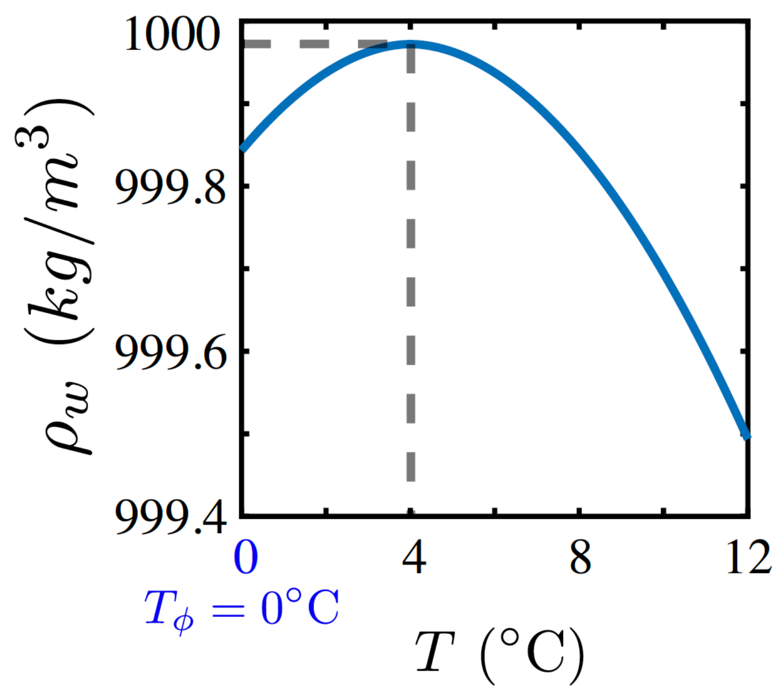


Observation: the Large Scale Circulation determines the topography

Equilibrium states of the ice-water front in a differentially heated rectangular cell, Z. Wang, E. Calzavarini, C. Sun, **Europhys. Lett.** , 135 (2021) 54001

Boxes of all kinds

VC and effect of density anomaly on ice front



$\mathbf{T_b}$

Observation: Inflexion point due to density anomaly

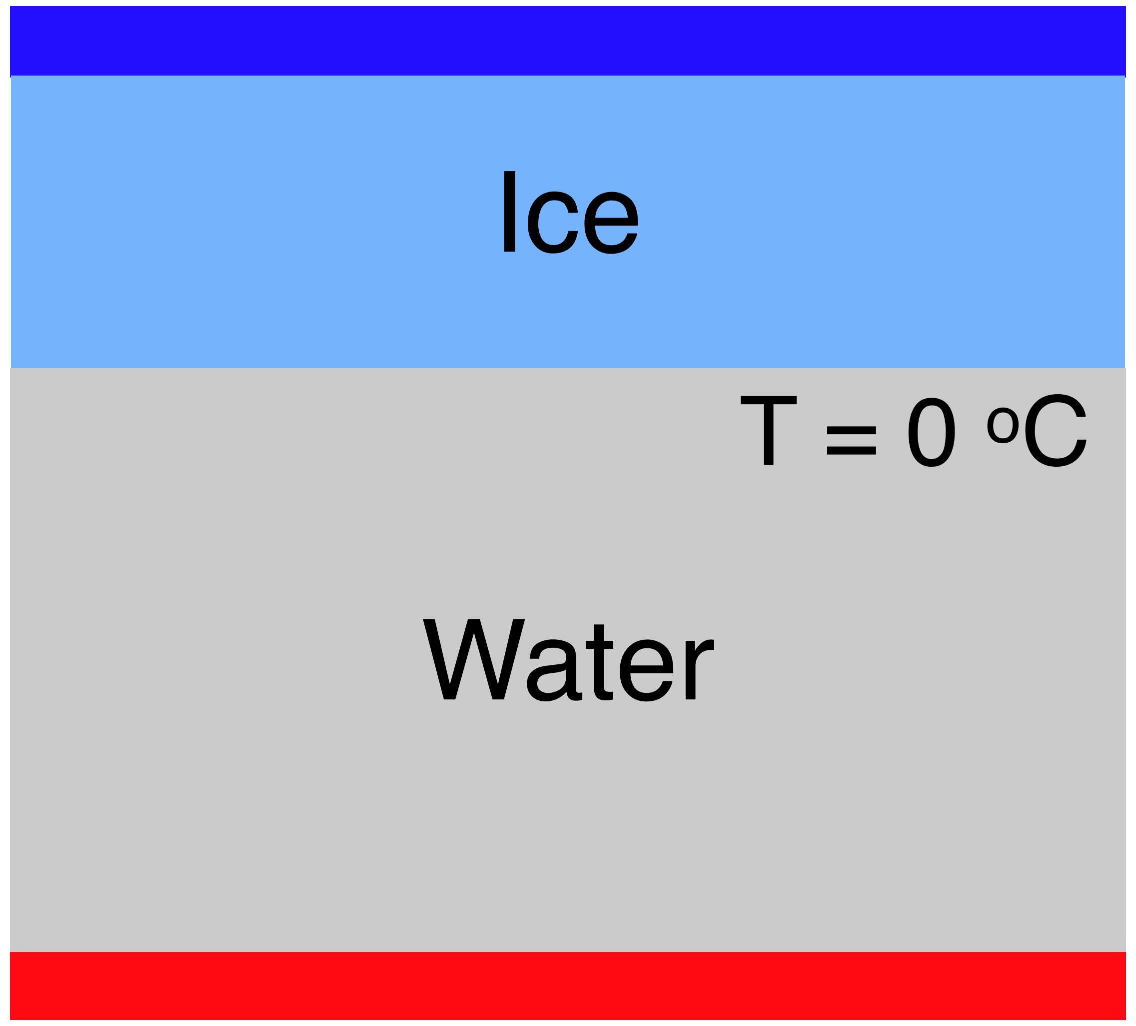
Linear buoyancy

Ice front shaping by upward convective current, Z. Wang, L. Jiang, Y. Du, C. Sun, E. Calzavarini **Phys. Rev. Fluids** 6, L091501(2021)

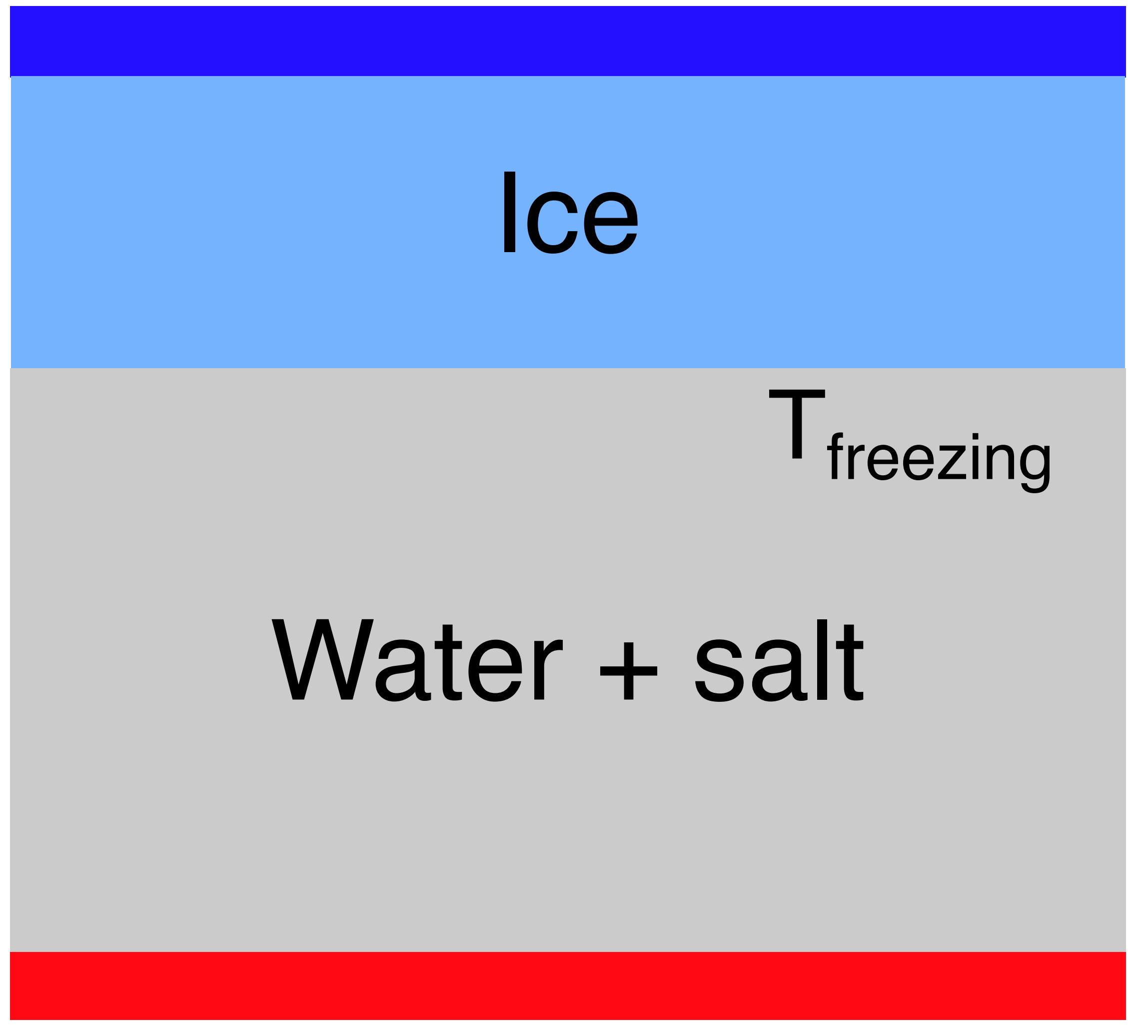
Salt comes in

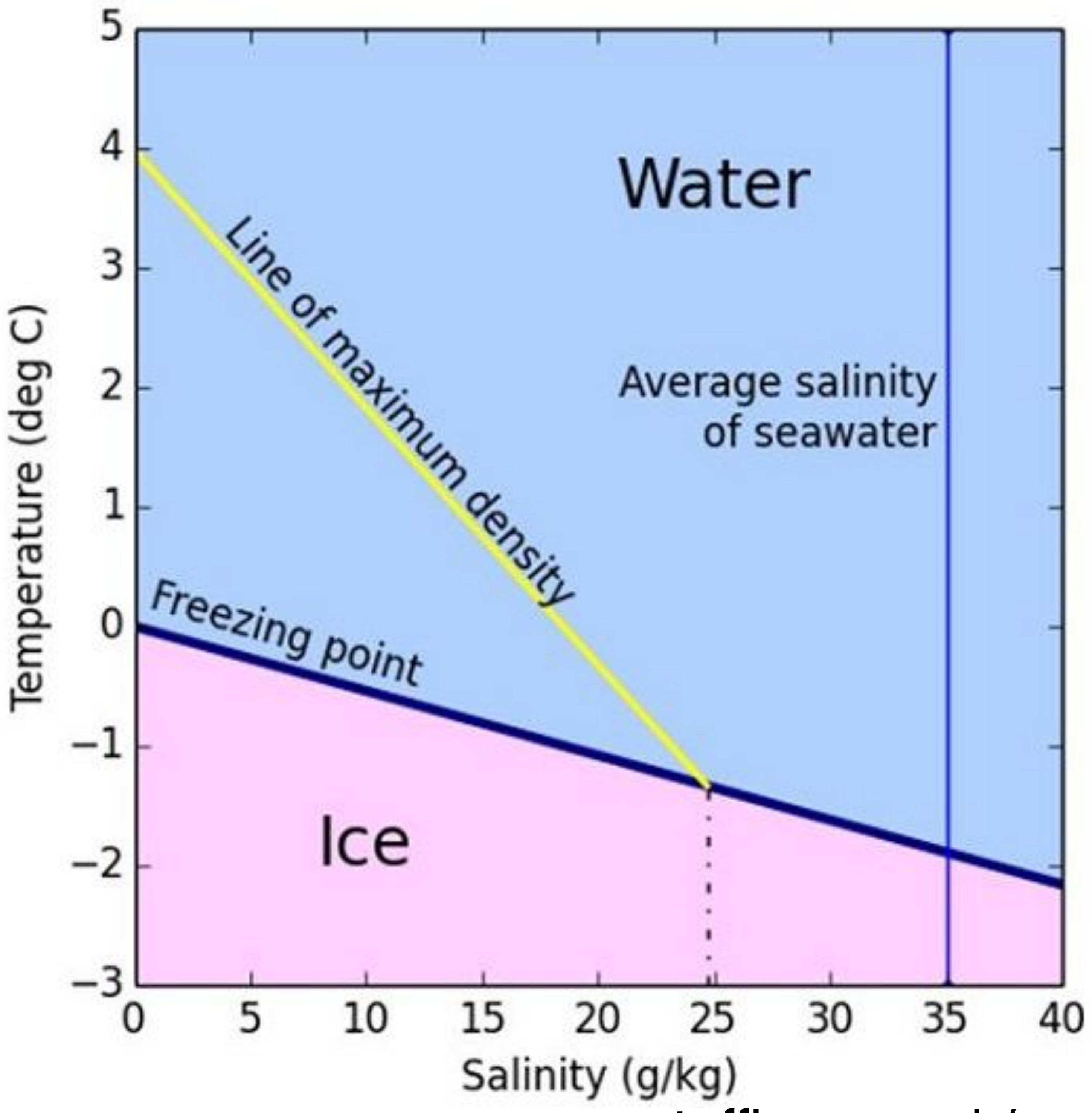
Equilibria not like before

Icing in pure water



Icing in salty water



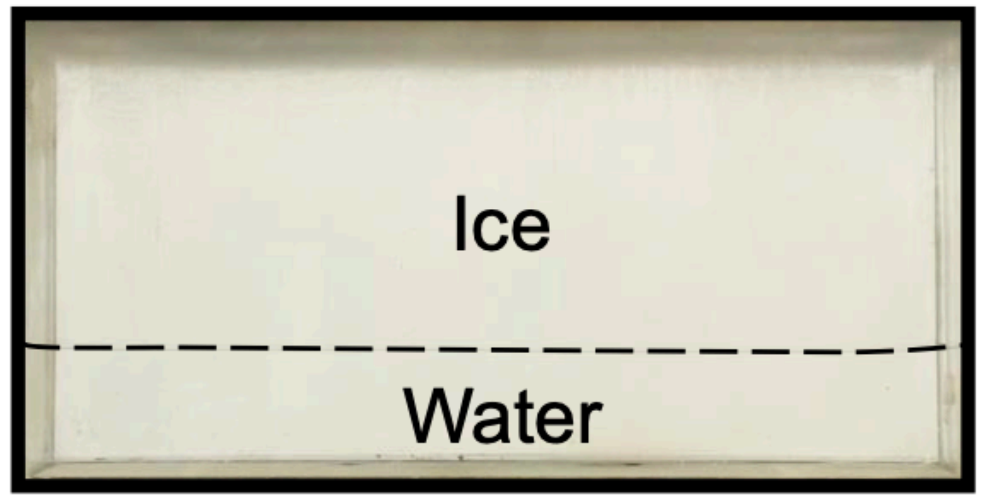


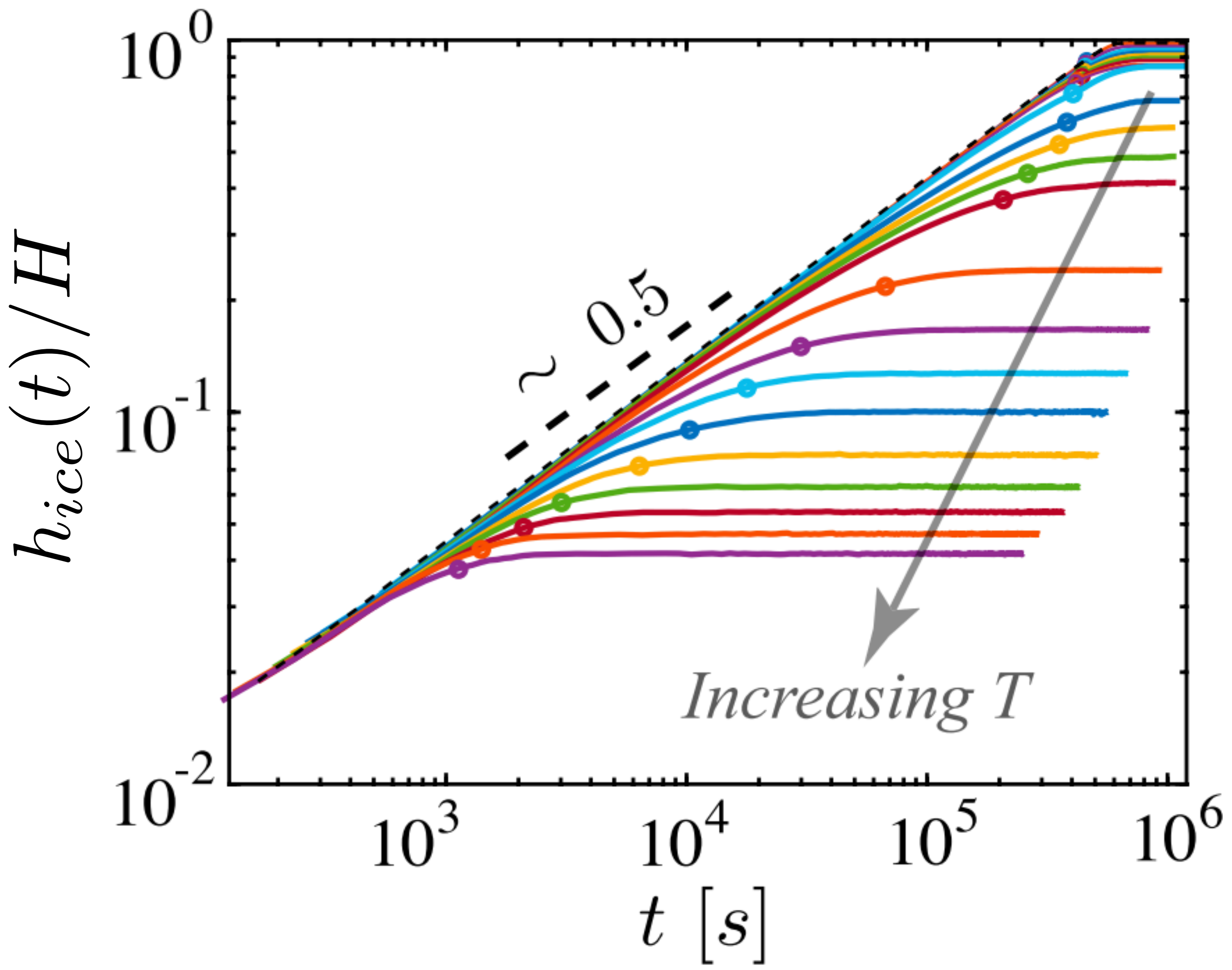
metoffice.gov.uk/

- Freezing point depends on salinity $T_m(S)$
- Density depends on temperature and salinity $\rho(T,S)$

Other differences?

Freezing of pure water vs. salty water



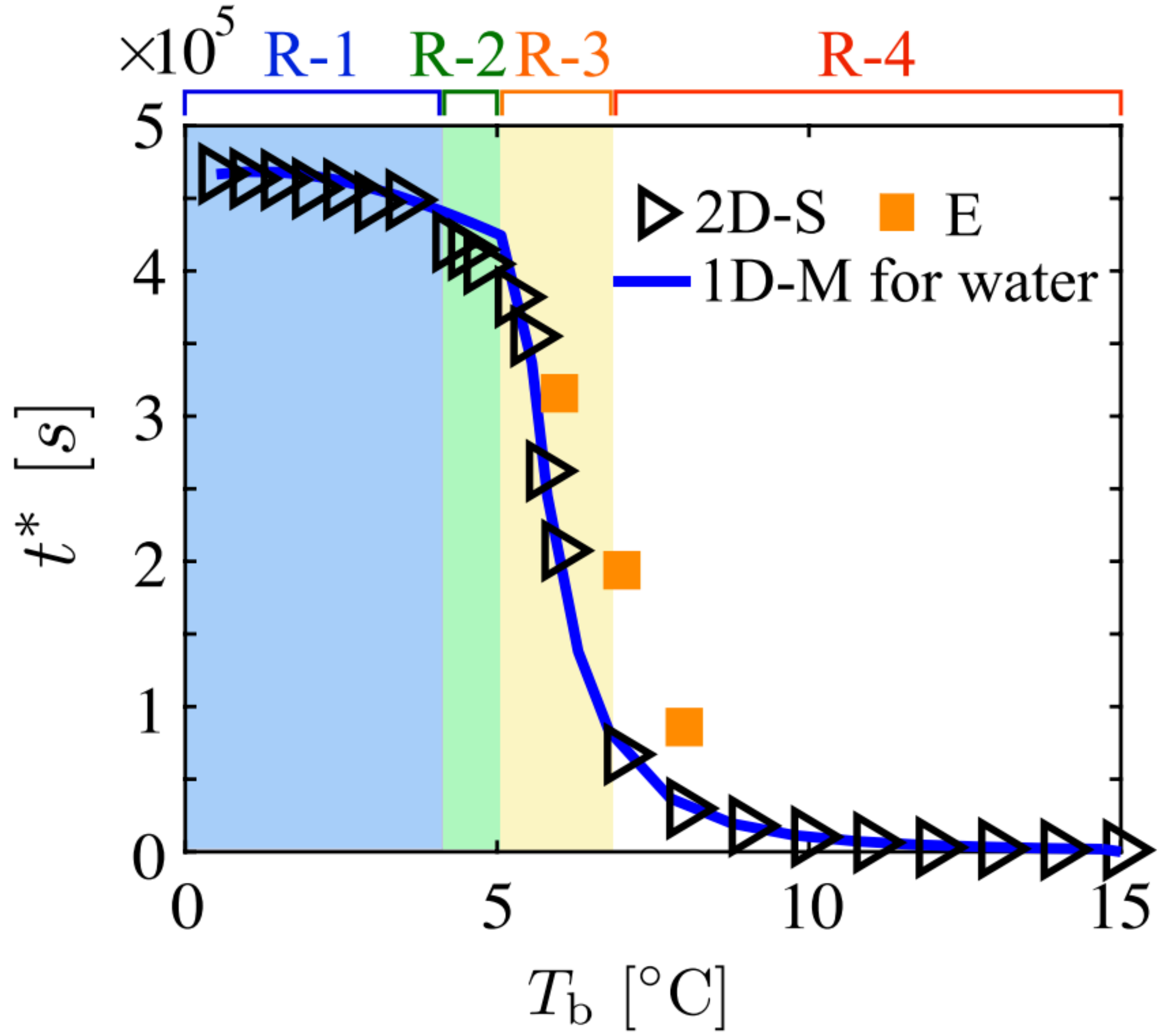


Ice layer growth:

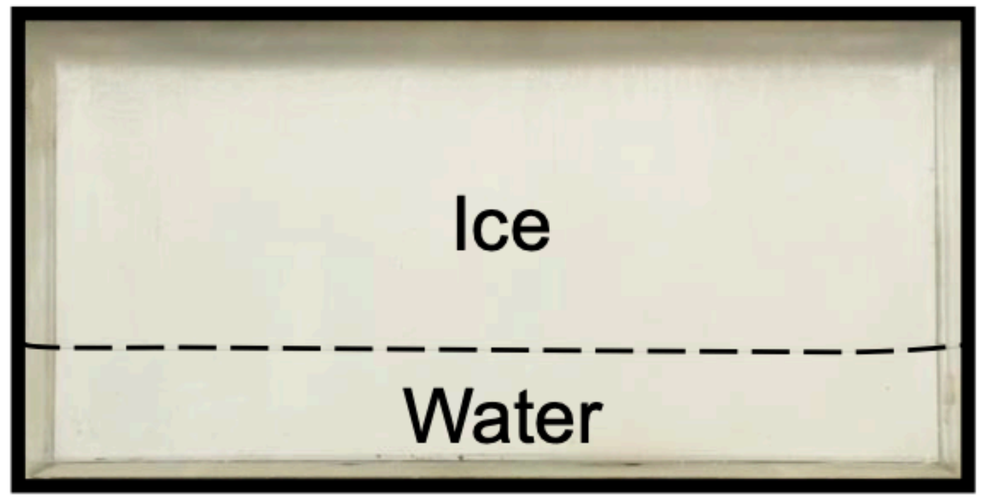
diffusion dominated at early stages

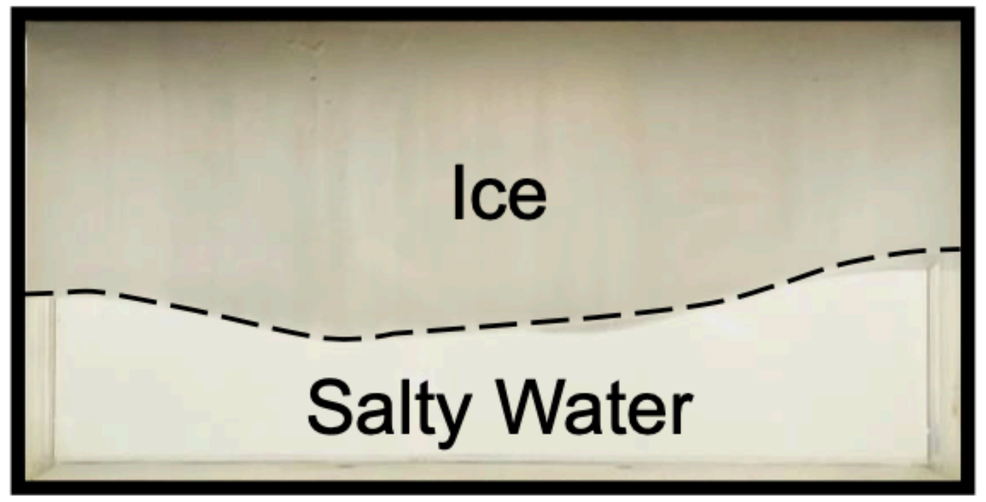
$$h_{ice}(t) \propto t^{0.5}$$

Saturates at later stages

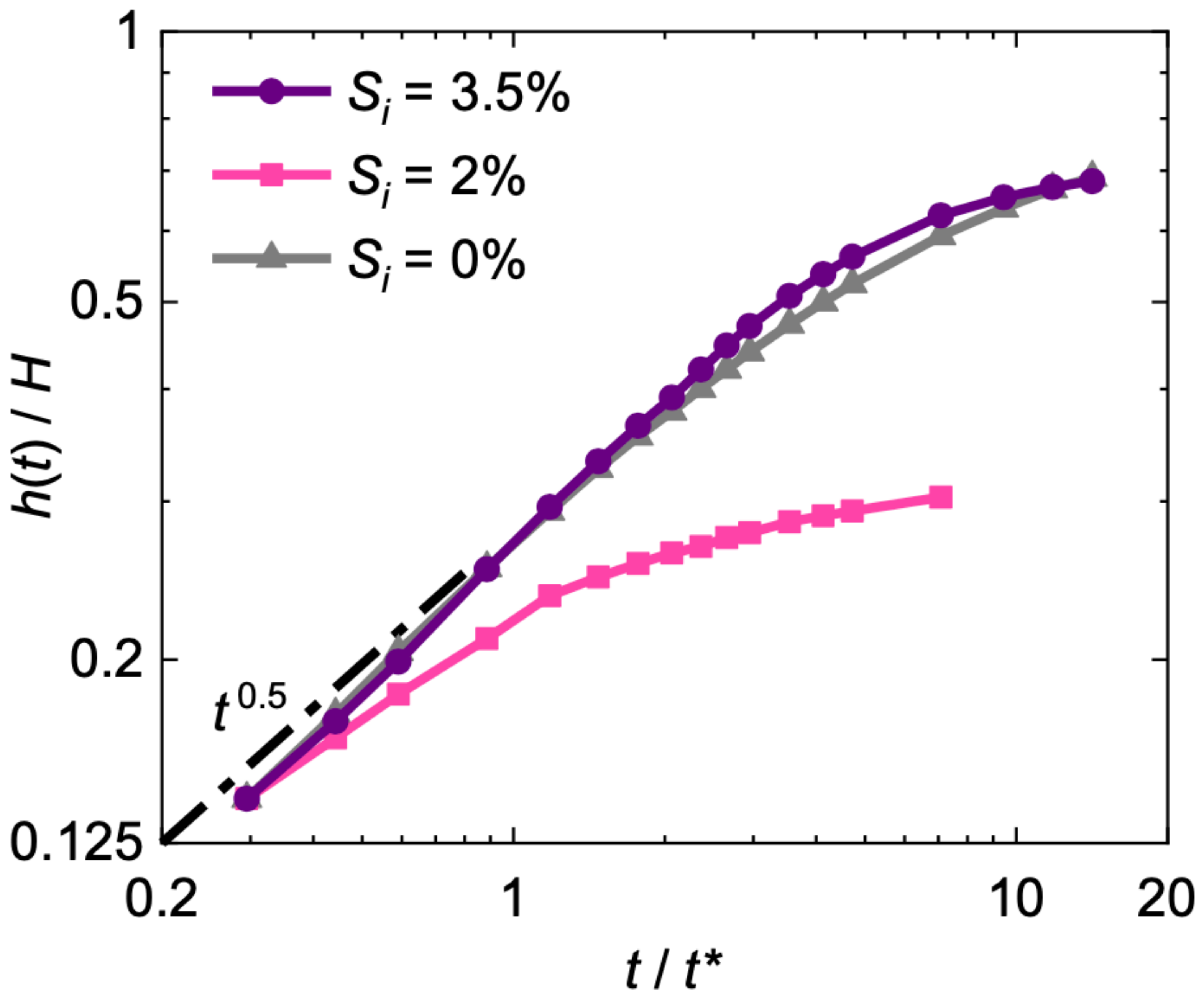


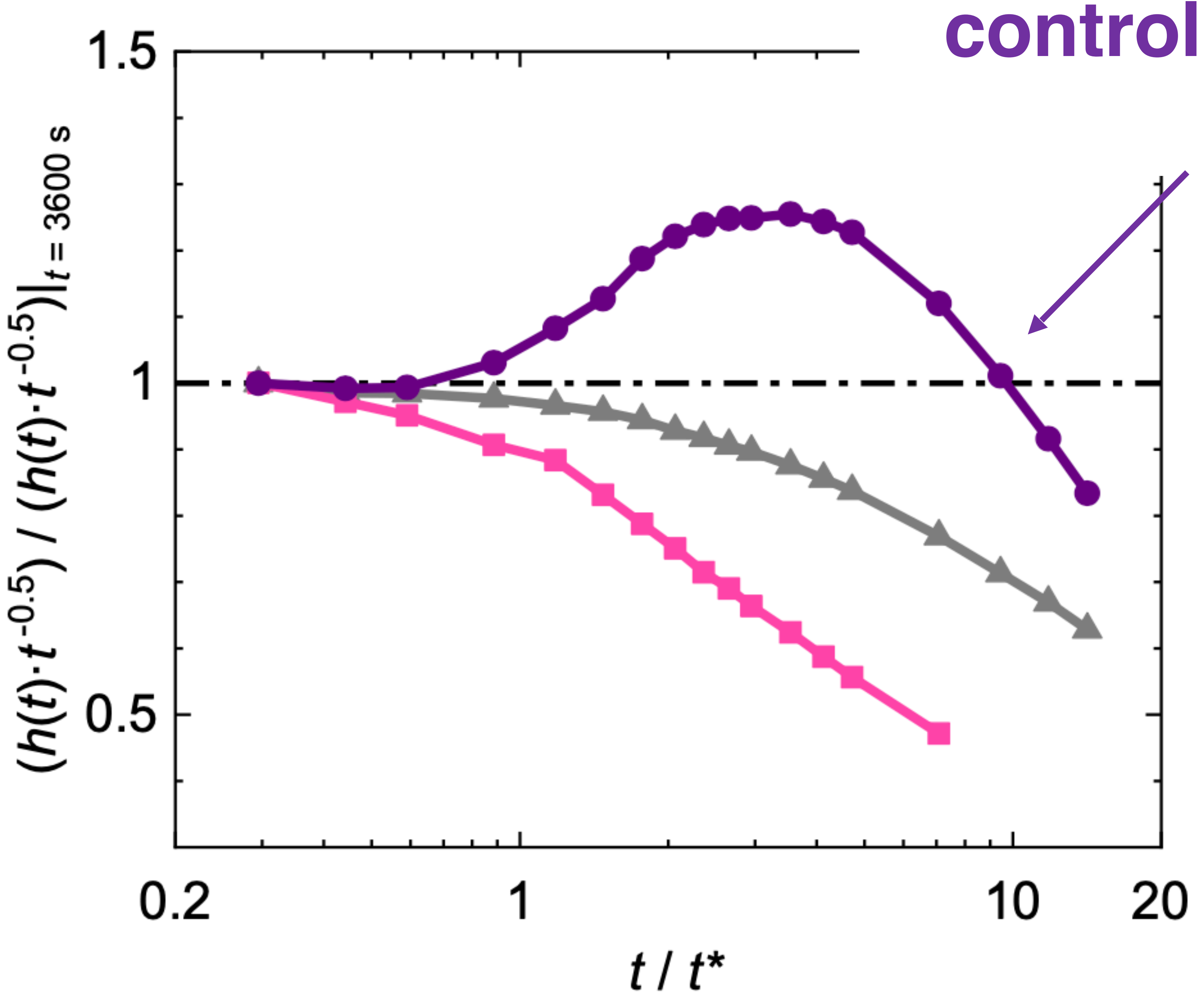
Freezing of pure water vs. salty water





Ice layer growth in salty water

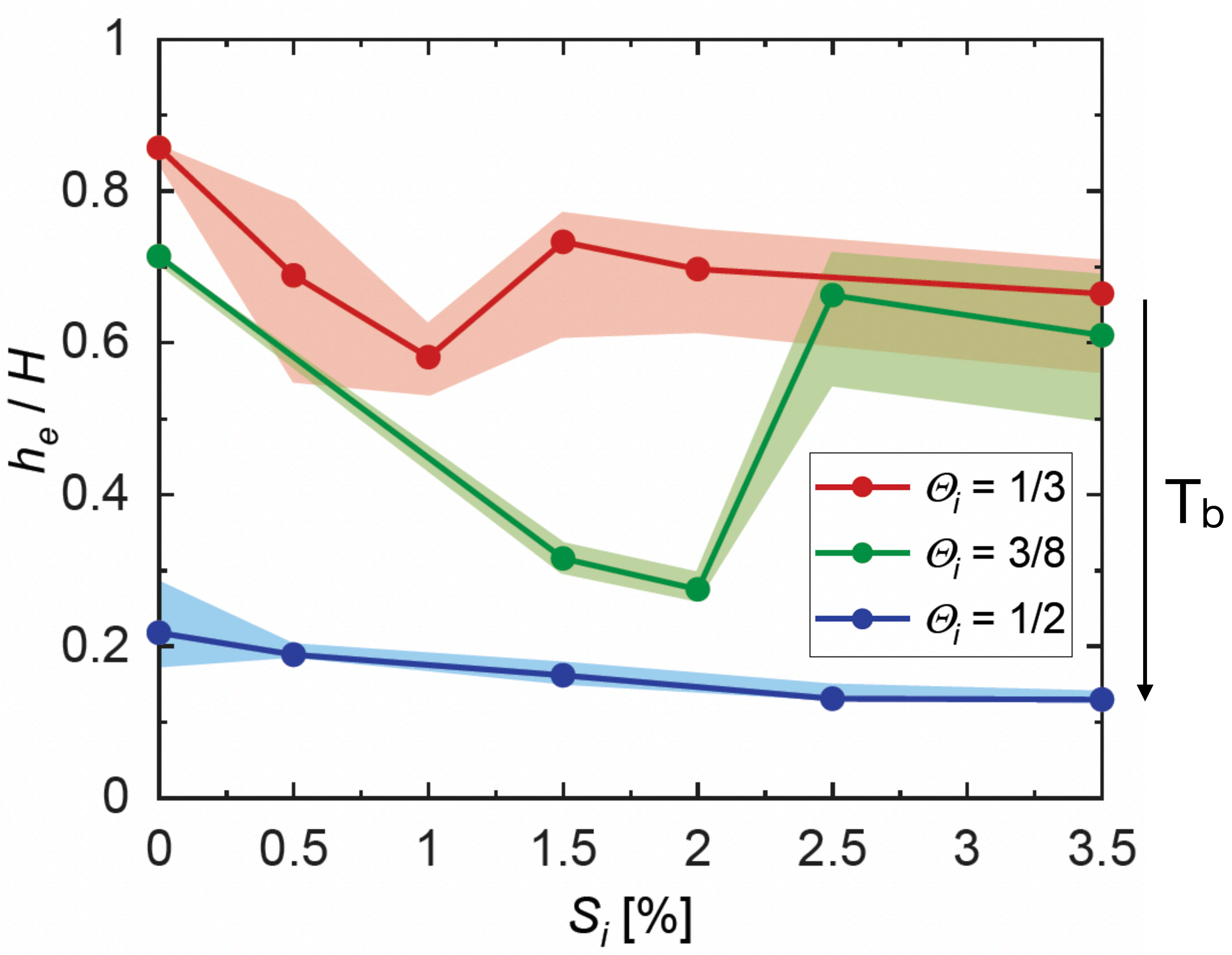




Exceeds the diffusion controlled growth rate
Why?

Equilibrium ice thickness

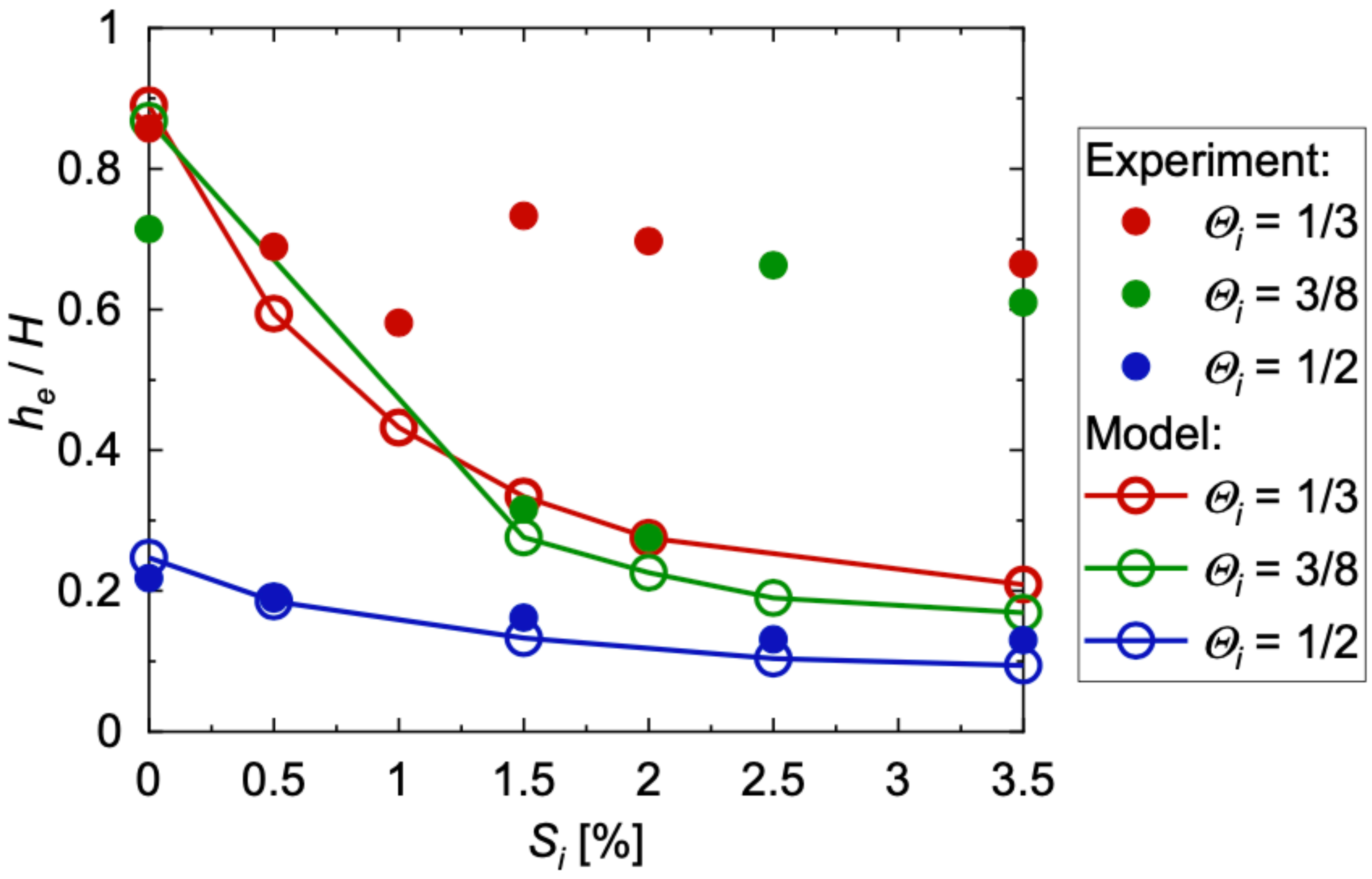
Measured average ice thickness in EXP



- **Salinity** varied from **0 to 3.5%** (fresh water, harbours at river mouths to sea water)
- Top plate temperature fixed at -10 K below freezing temperature at initial-water-salinity
- Bottom plate temperature varied (parameter Θ_i)

Equilibrium ice thickness

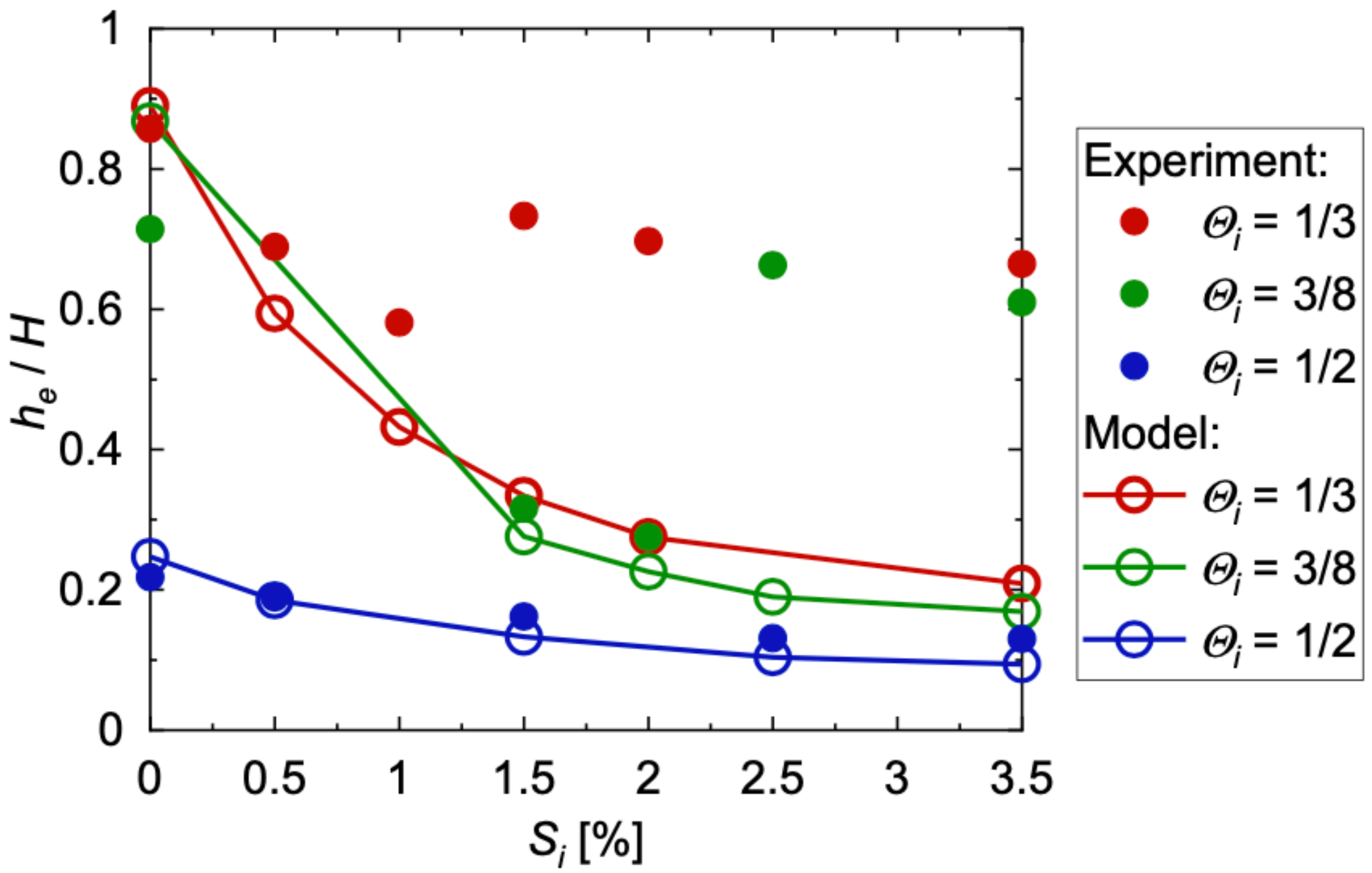
Measured average ice thickness



1D model considering the shift of the freezing point and salty-water equation of state

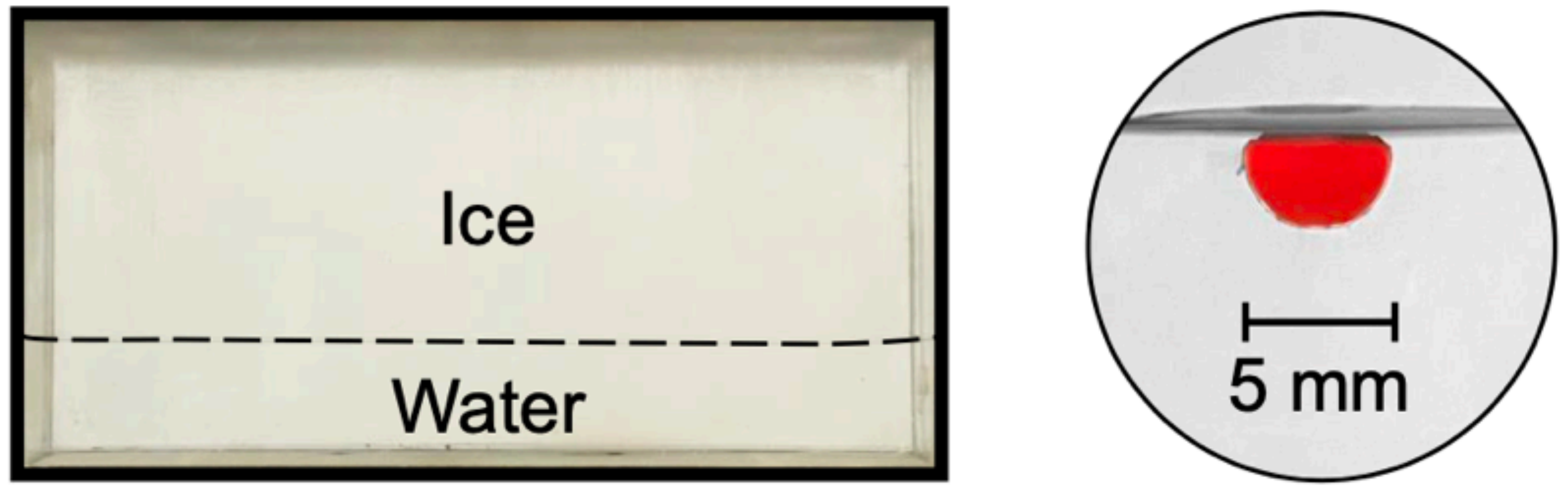
Equilibrium ice thickness

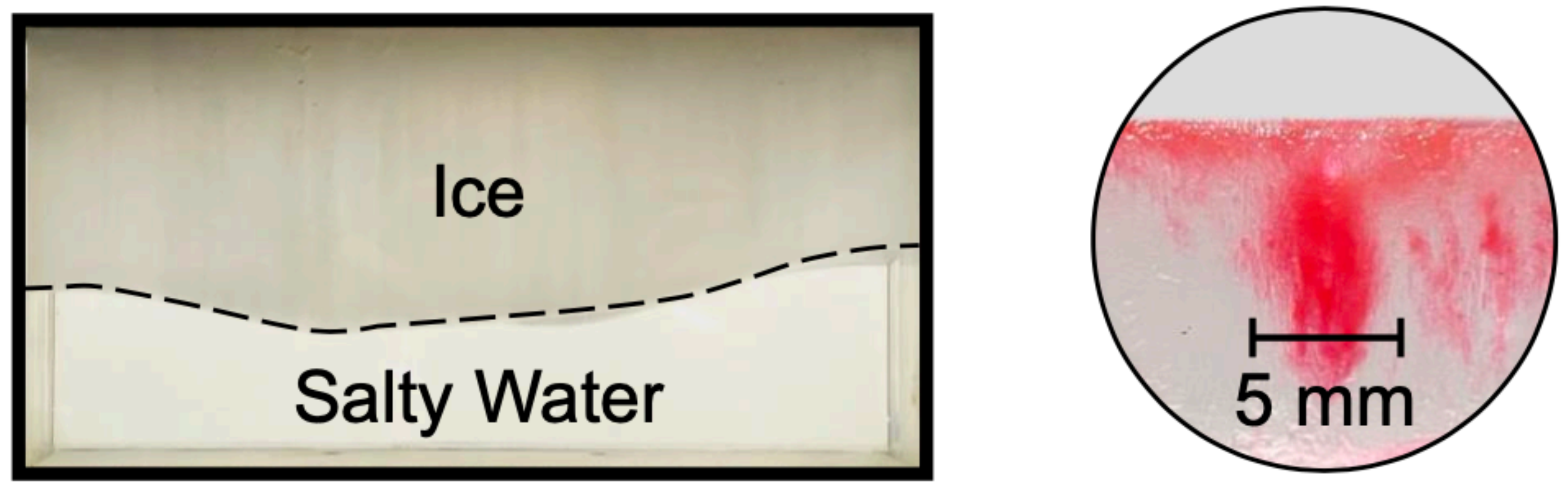
Measured average ice thickness



What is still missing?

A drop of ink in ice, after 5 minutes

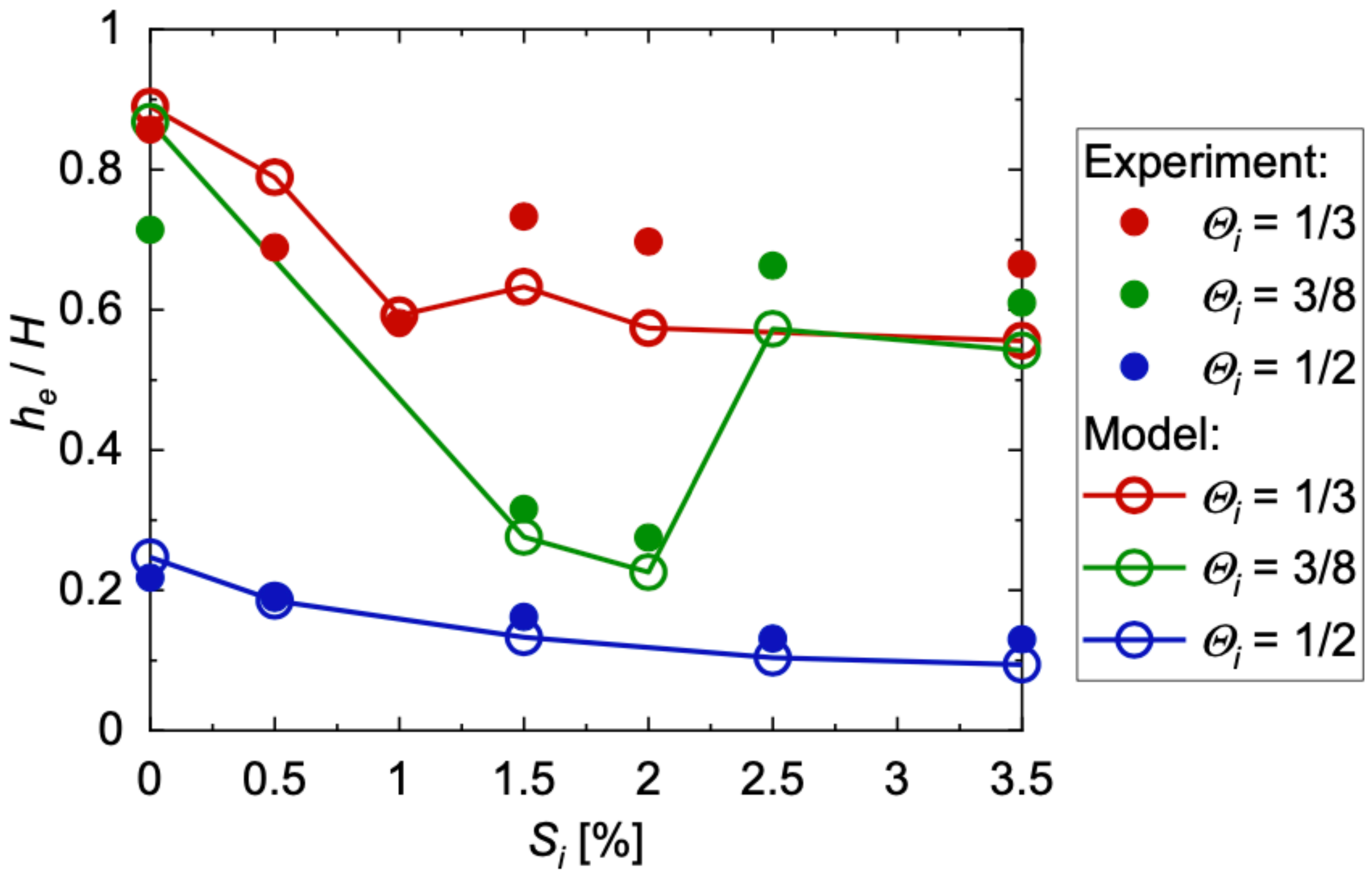




The ice is porous,
is a **mushy layer**

Equilibrium ice thickness

Measured average ice thickness

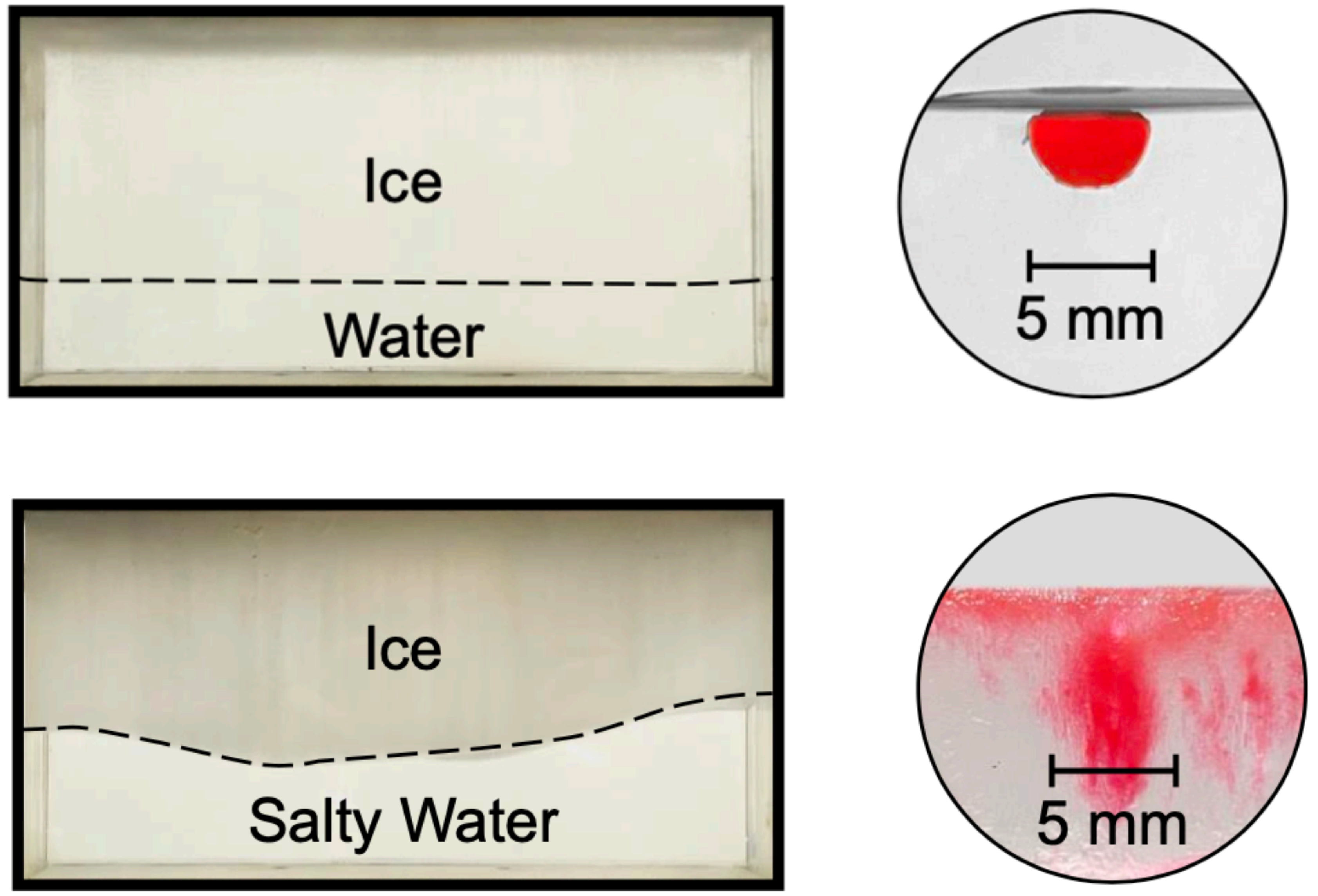


The model with convection in mushy ice

captures better the “jumps” in ice thickness

What is still missing?

A drop of ink in ice, after 5 minutes



The ice is porous,
is a **mushy layer**

Simulations of porous ice growth

A Unified Numerical Framework for Turbulent Convection and Phase-Change Dynamics in Coupled Fluid-Porous Systems
Rongfu Guo and Yantao Yang https://arxiv.org/abs/2510.22730

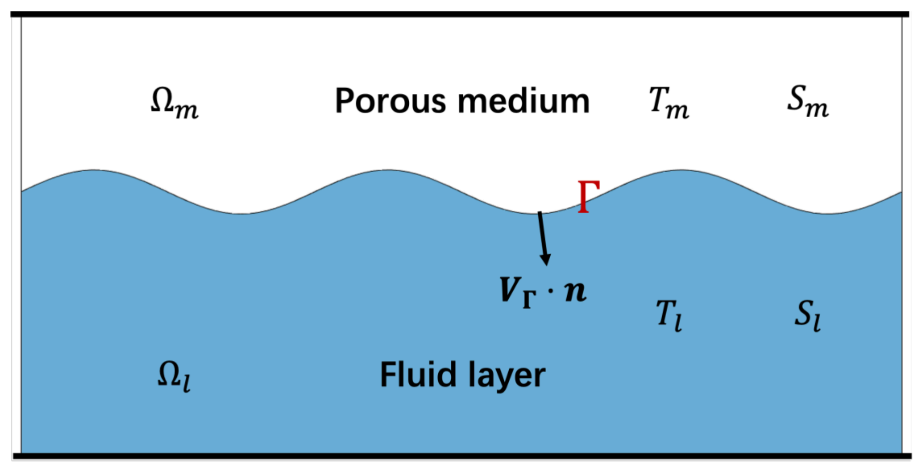
$$\partial_t \mathbf{U} = -\mathbf{U}\cdot\nabla\left(\frac{\mathbf{U}}{\phi}\right) - \frac{1}{\rho_{\mathrm{ref}}}[\nabla P_l] + \nu\left(\nabla^2\mathbf{U} - \frac{\phi}{\mathcal{K}}\mathbf{U}\right) - \frac{g\phi\rho'}{\rho_{\mathrm{ref}}}\mathbf{e}_z$$

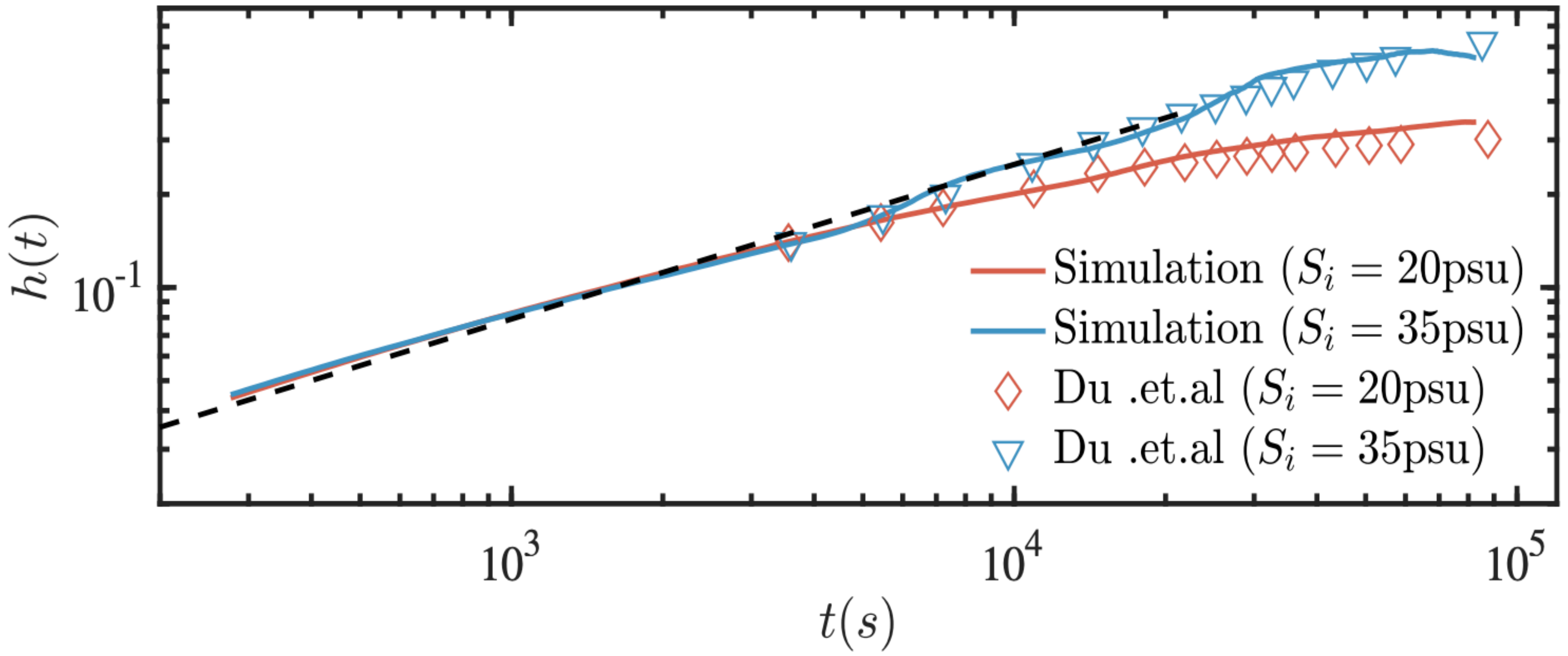
$$\partial_t (c_m T) = -c_f \mathbf{U}\cdot\nabla T + \nabla\cdot(k_e \nabla T) - \mathcal{L}\partial_t\phi$$

$$\partial_t (S\phi) = -\mathbf{U}\cdot\nabla S + \nabla\cdot(\kappa_S^e \nabla S)$$

$$\partial_t\phi + \mathbf{U}\cdot\nabla\phi = \kappa_\phi\nabla^2\phi + \mathcal{G}(T - T_\phi)$$

+ parametrizations





Effective porosity of mushy-ice ≠ phase field

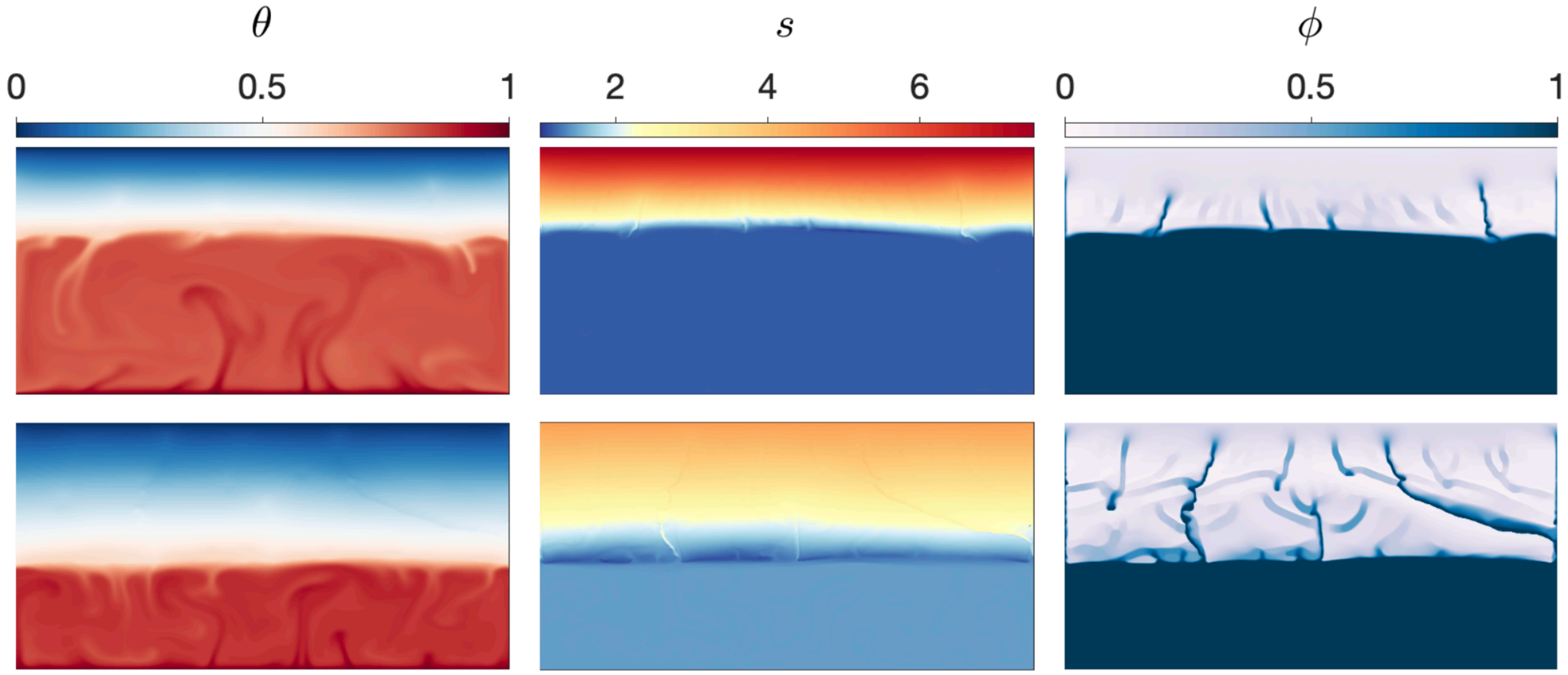
$$\phi_e = \begin{cases} 0 & \text{for} \quad \phi \le \phi_{cr}, \\ \alpha(\phi - \phi_{cr})^\beta & \text{for} \quad \phi_{cr} < \phi \le \phi_x, \\ \phi & \text{for} \quad \phi_x < \phi, \end{cases} \quad \text{with} \quad \phi_x = \frac{\phi_{cr}}{1-\beta}, \quad \alpha = \frac{1}{\beta}\left(\frac{\beta\phi_{cr}}{1-\beta}\right)^{1-\beta}$$

permeability $\mathcal{K}(\phi) = K_0 f(\phi)$ $\quad f(\phi_e) = \phi_e^3/(1-\phi_e)^2$

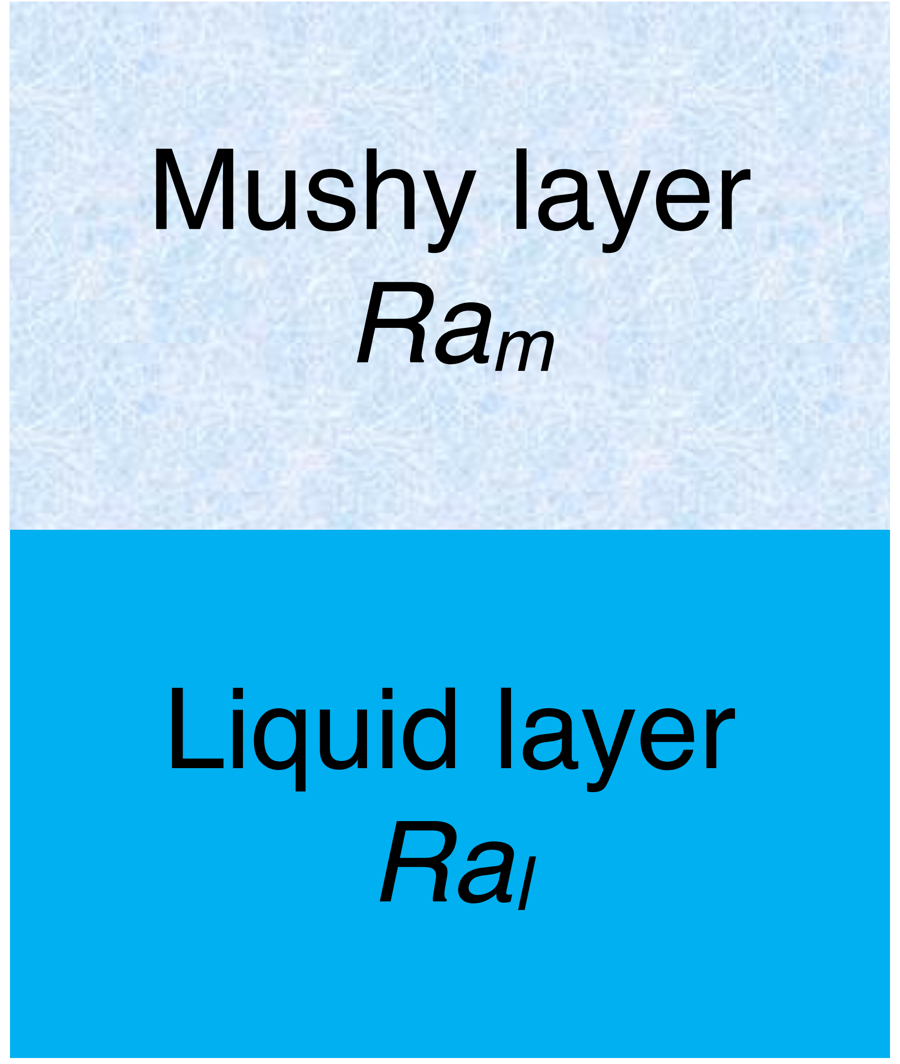
Thermal conductivity mushy-ice ≠ mixture conductivity

$$k_e = k_m + k_{dis} = \phi k_l + (1-\phi)k_s + \epsilon_{dis}(\phi) c_m |\mathbf{U}|\,.$$

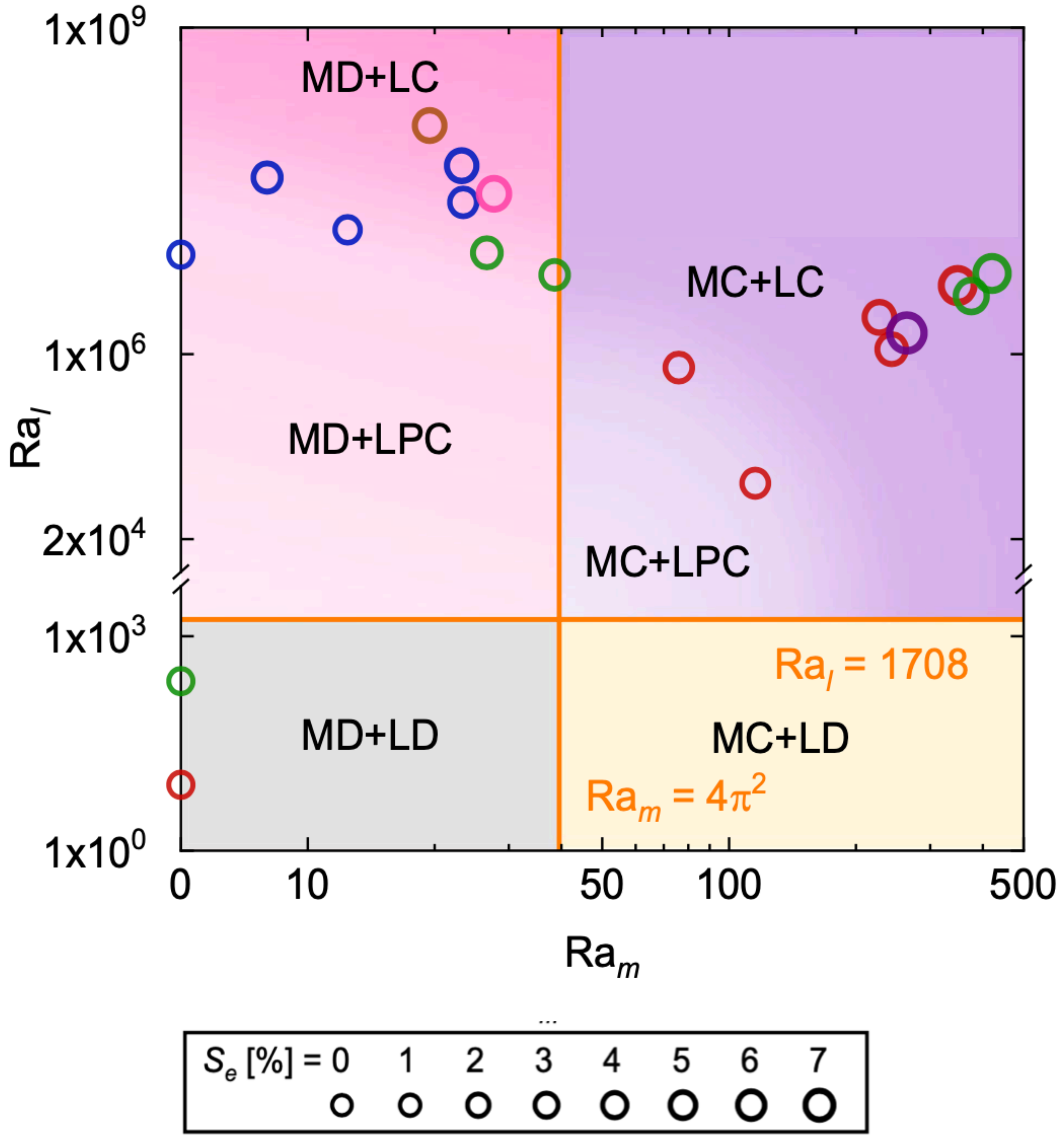
$$\epsilon_{dis}(\phi) = \begin{cases} \epsilon_{d0} & \text{for } 0 < \phi \le \phi_{bn} \\ \epsilon_{d0}\left(\frac{1-\phi}{1-\phi_{bn}}\right)^2 & \text{for } \phi_{bn} < \phi \le 1 \end{cases}$$



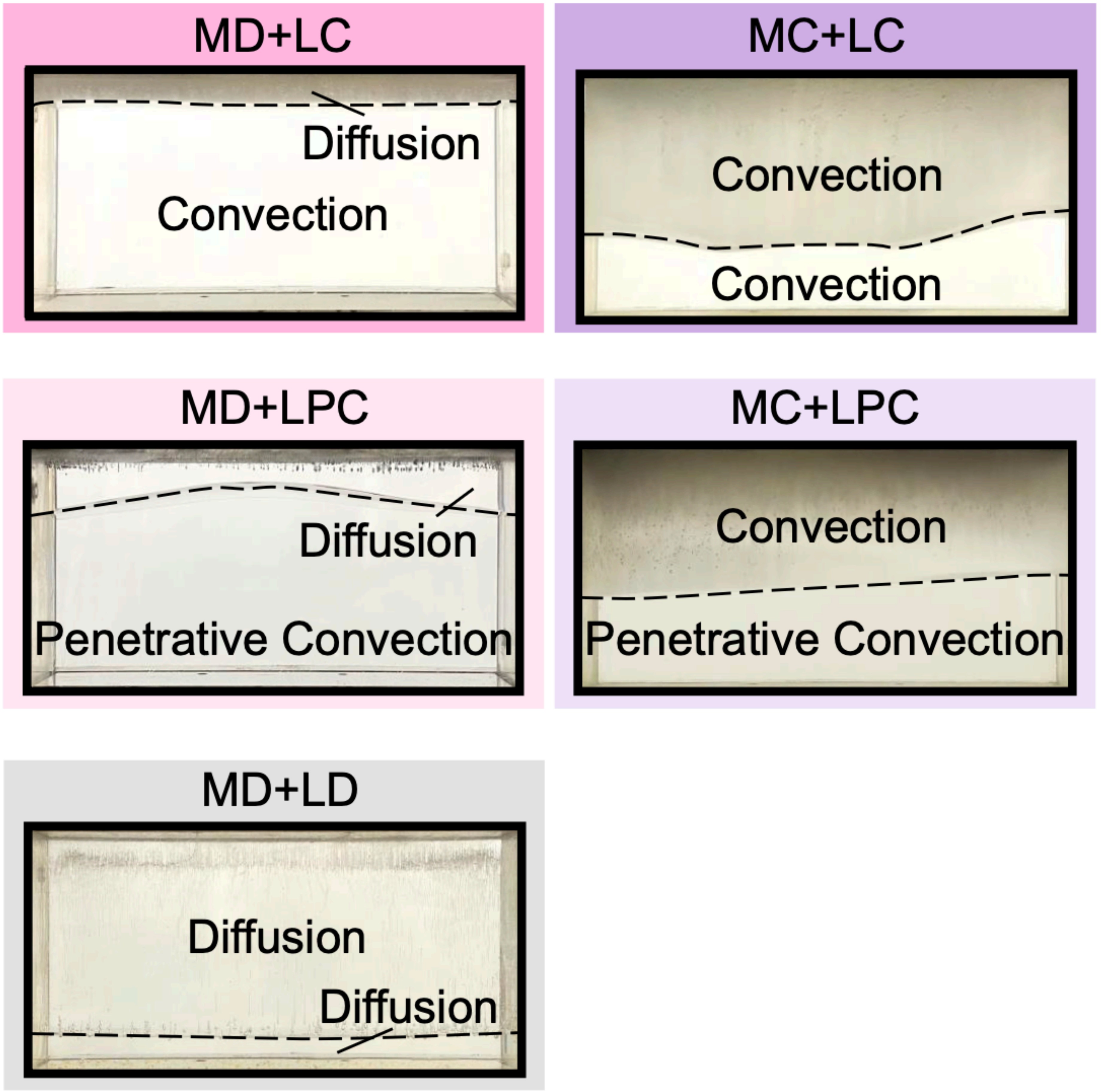
State modes for icing in salty water



Phase space diagram



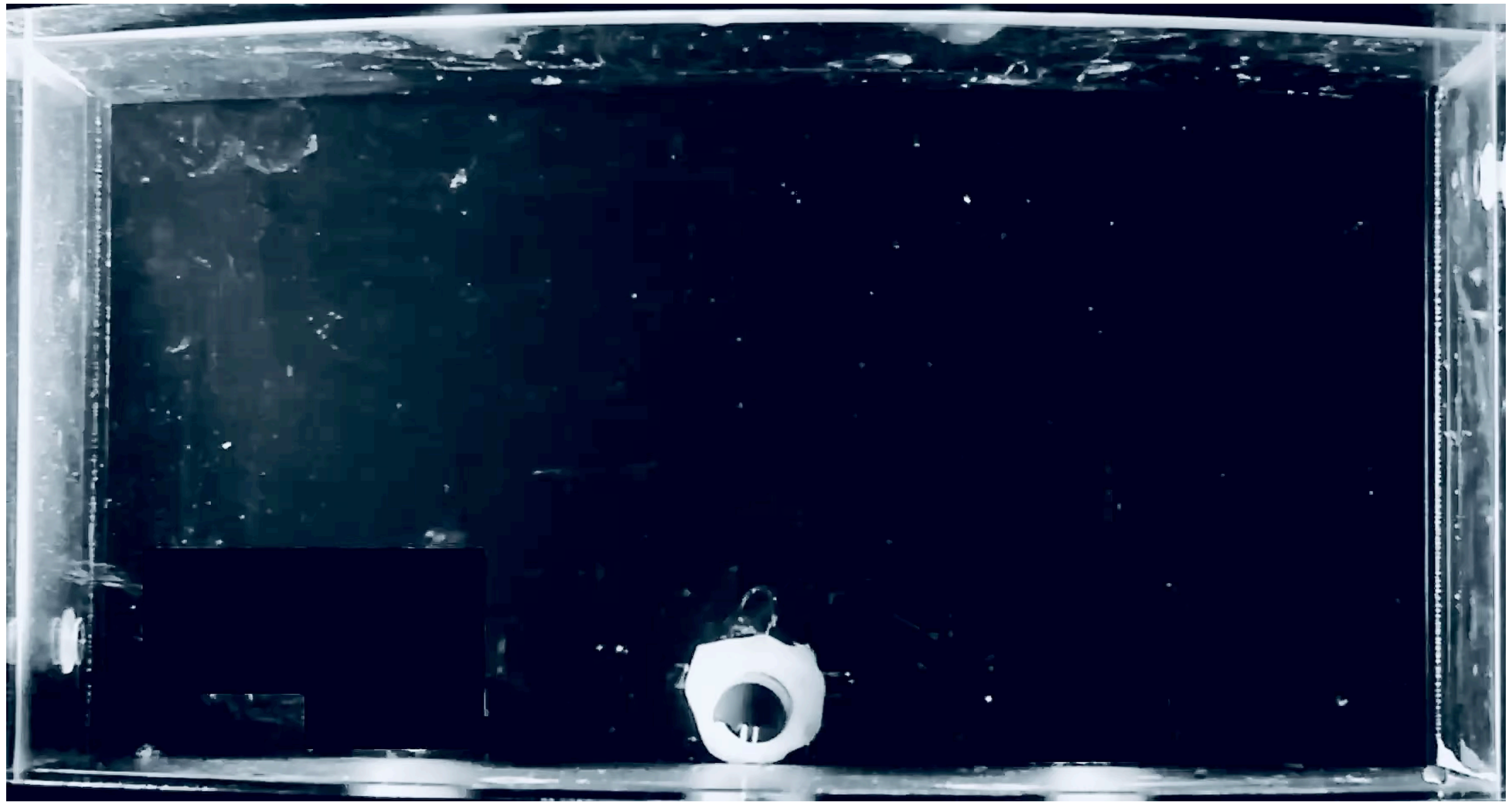
Identification of five out of six possible modes in the experiments



Du, Wang, Jiang, Calzavarini, Sun, JFM (2023)

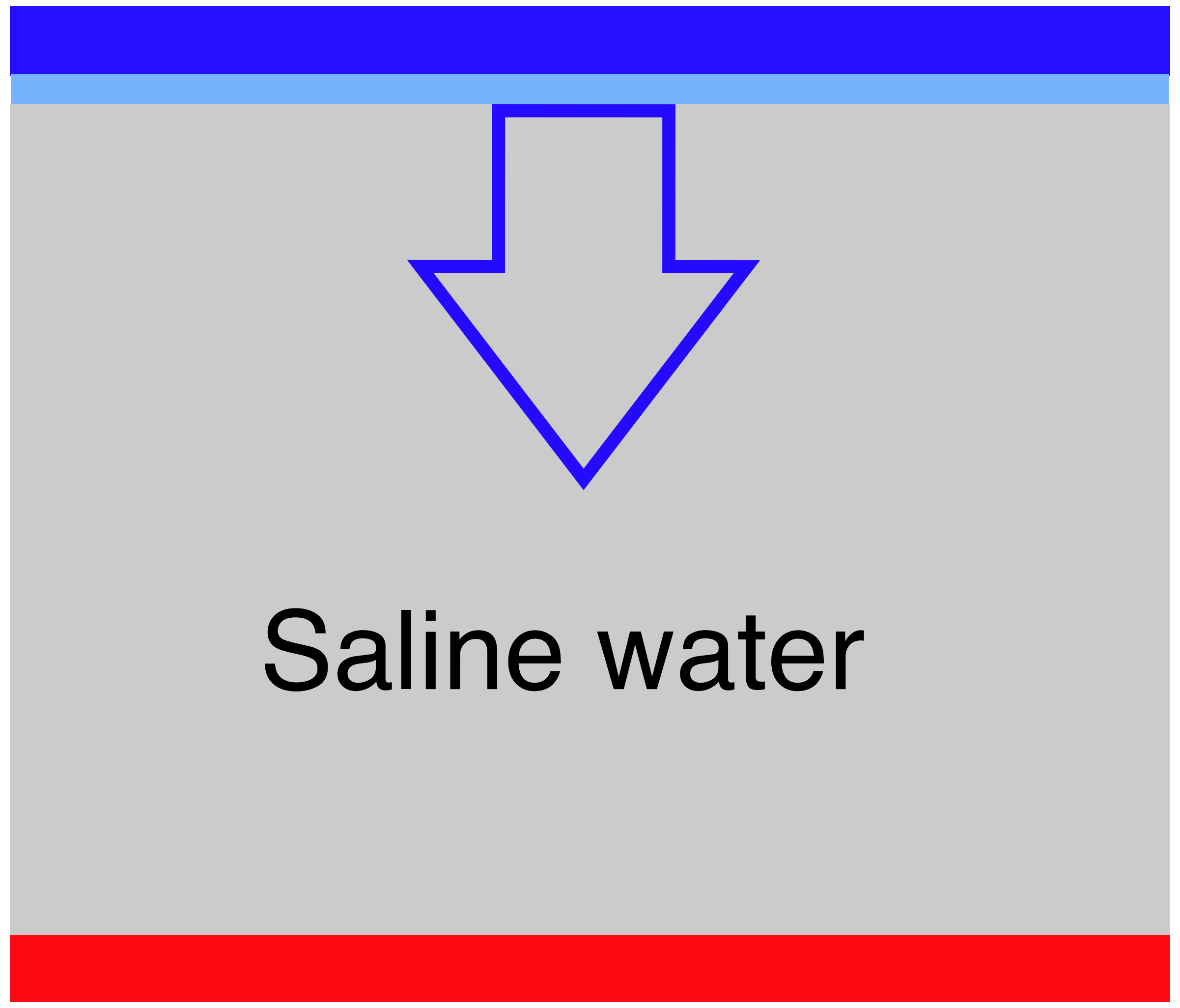
Ice aging

A very long time scale



Salinity probe

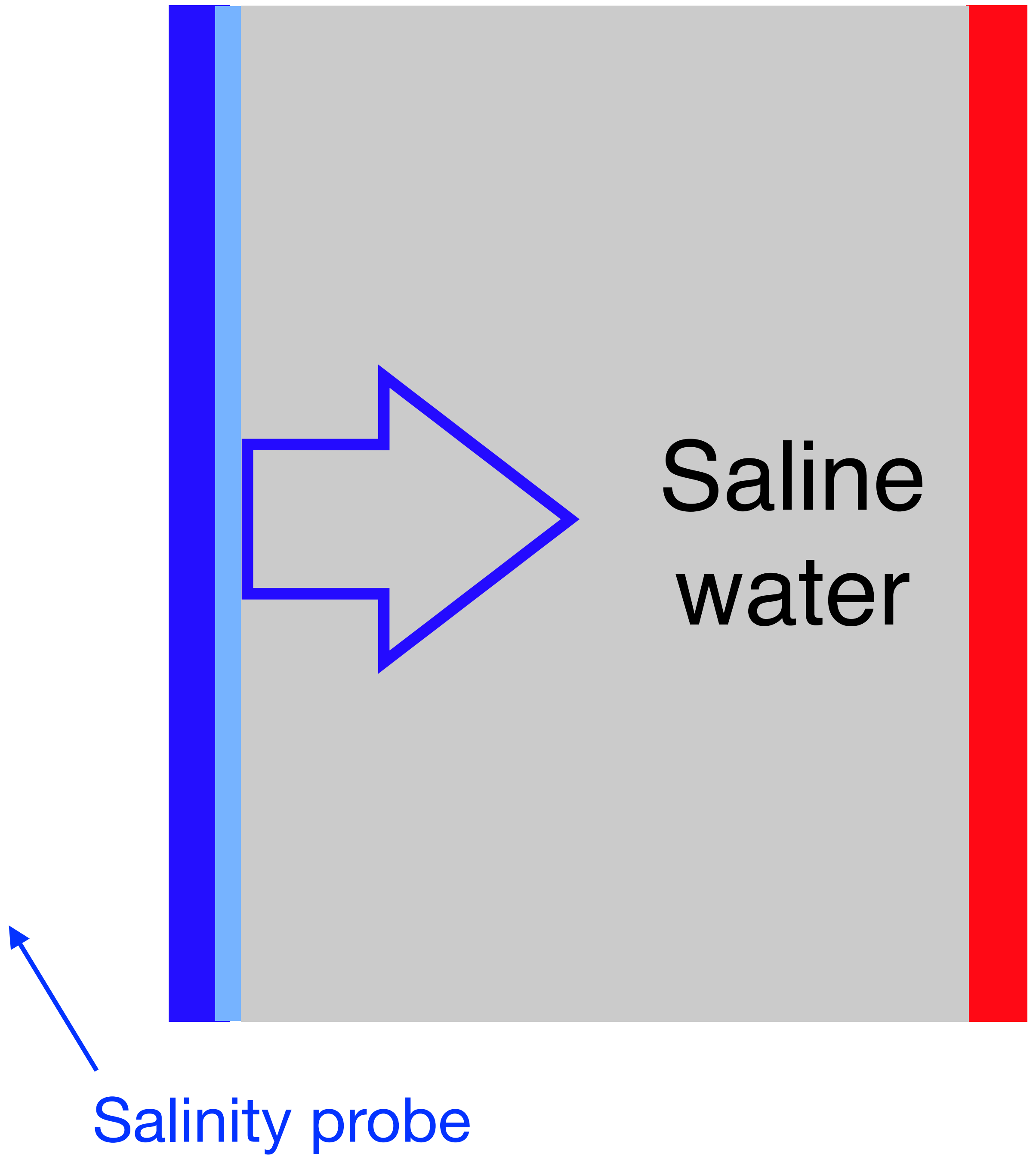
16 days experiment



Rapid formation of ice layer is followed by a slow decrease of the ice thickness

Ice aging

A very long time scale



Rapid formation of ice layer is followed by a slow decrease of the ice thickness

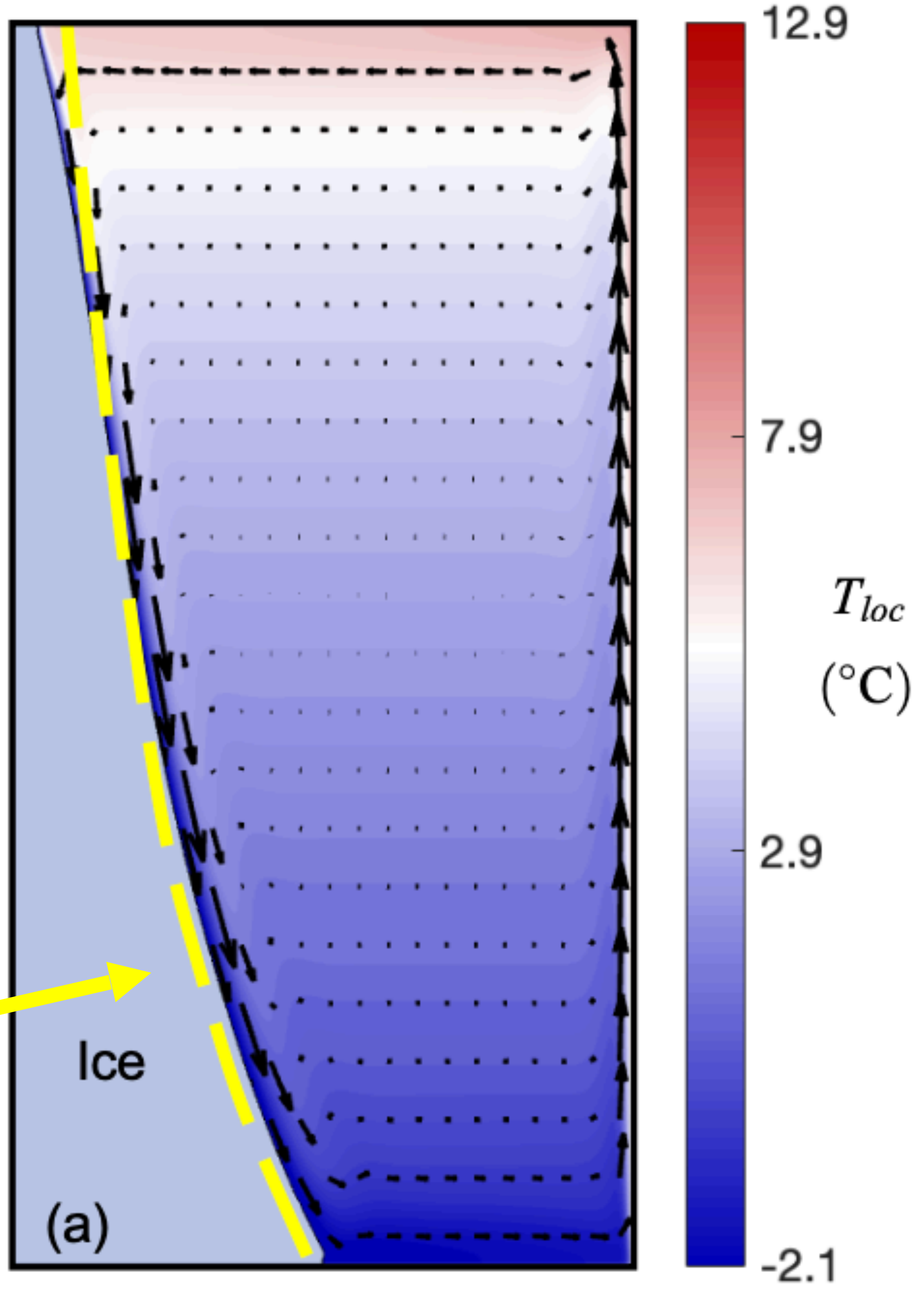
24 days experiment

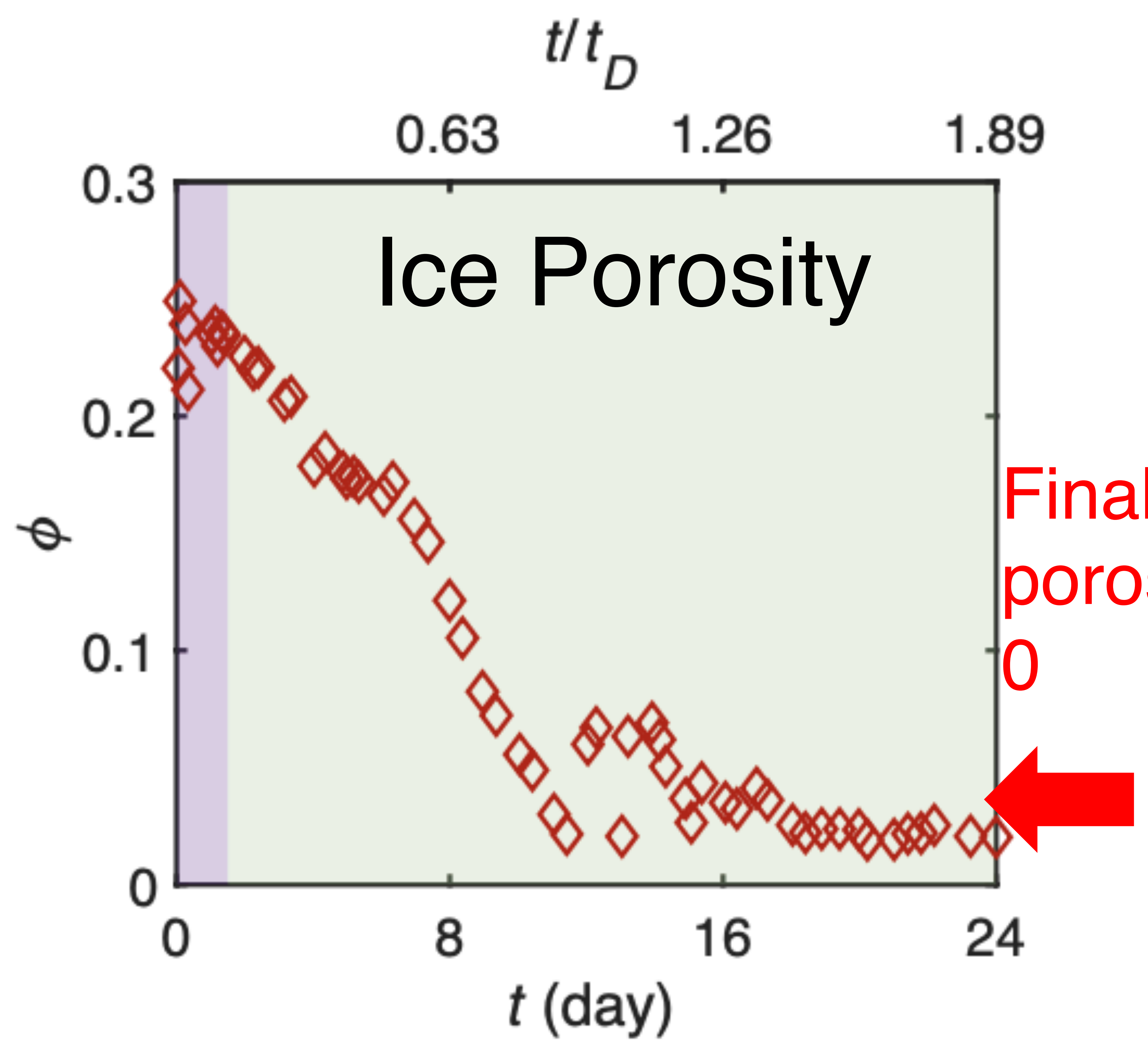
Long time evolution

Asymptotic equilibrium state of the system

DNS with buoyant pure liquid freezing under the same conditions

Yellow line: Final state of experiment





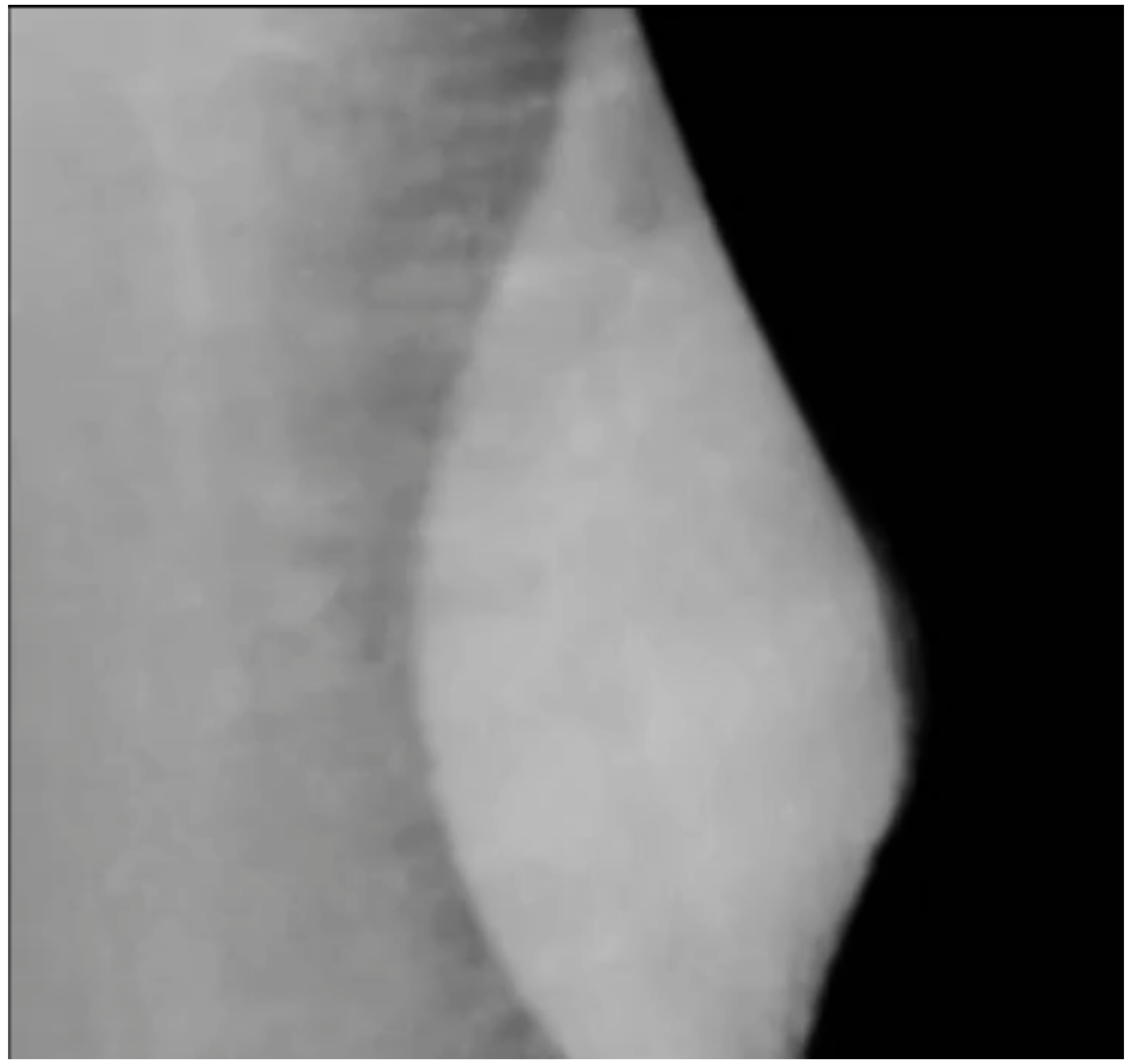
Final porosity ~ 0

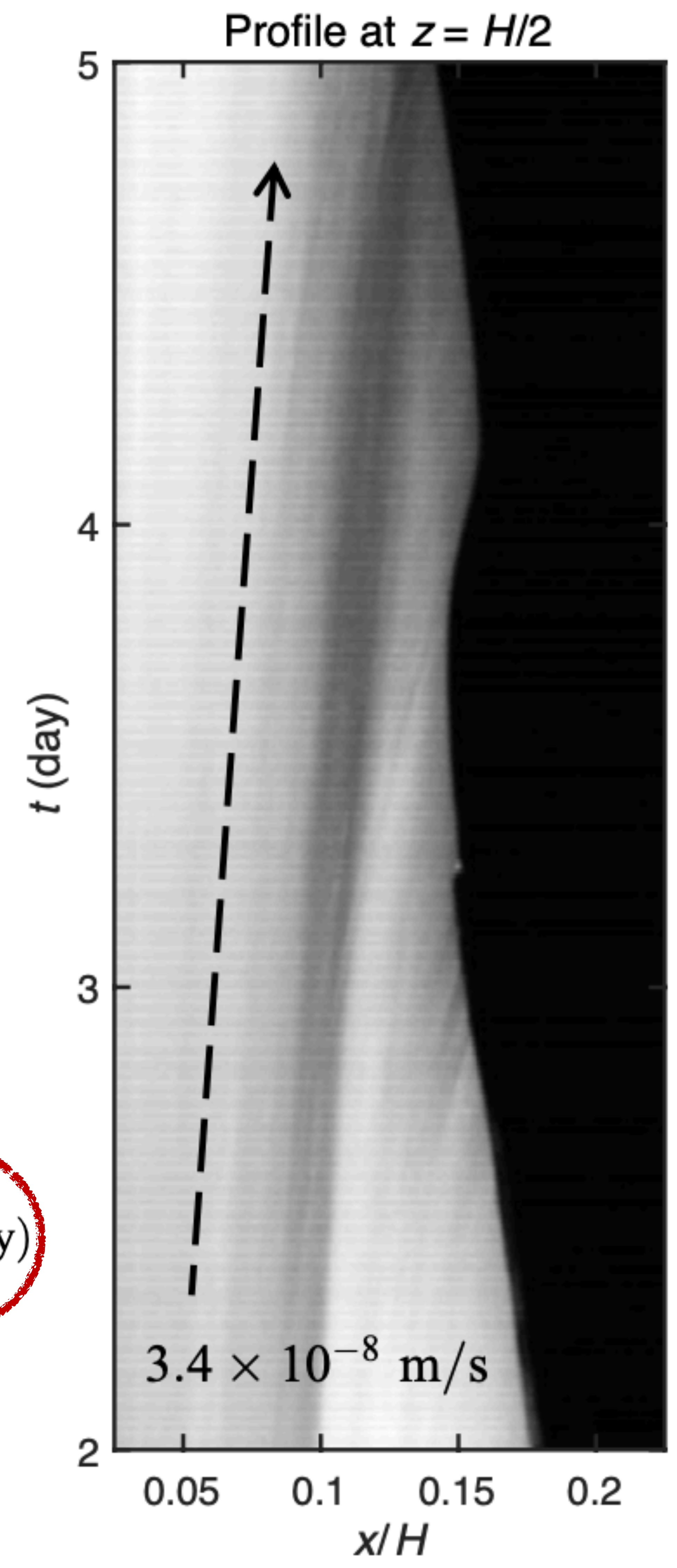
The system converges to a **dense freshwater ice layer** adjacent to a well-mixed highly saline water region.

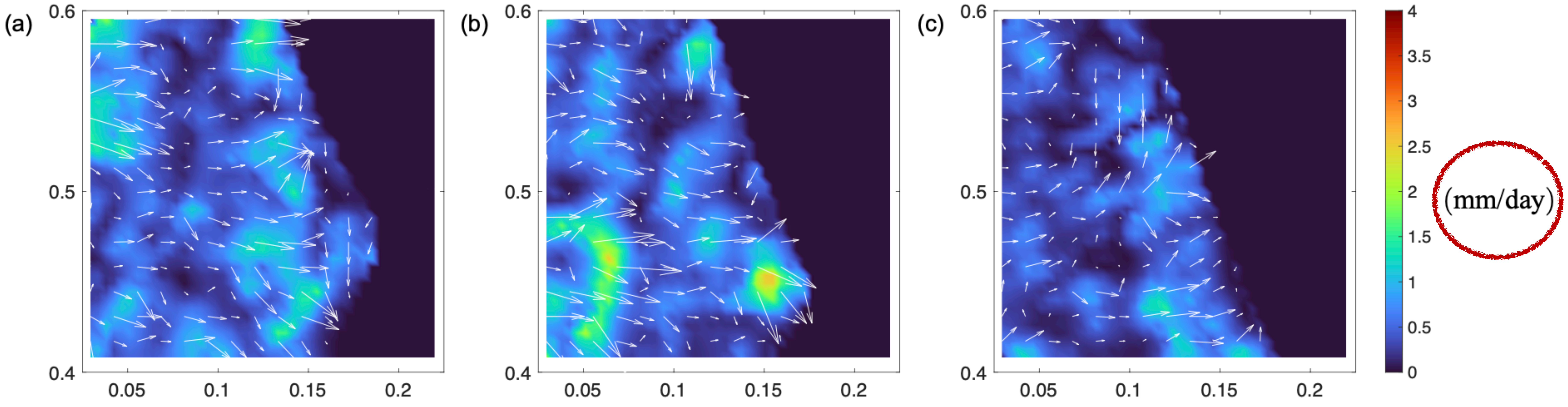
Dynamics of “salty” ice aging

Zoomed in

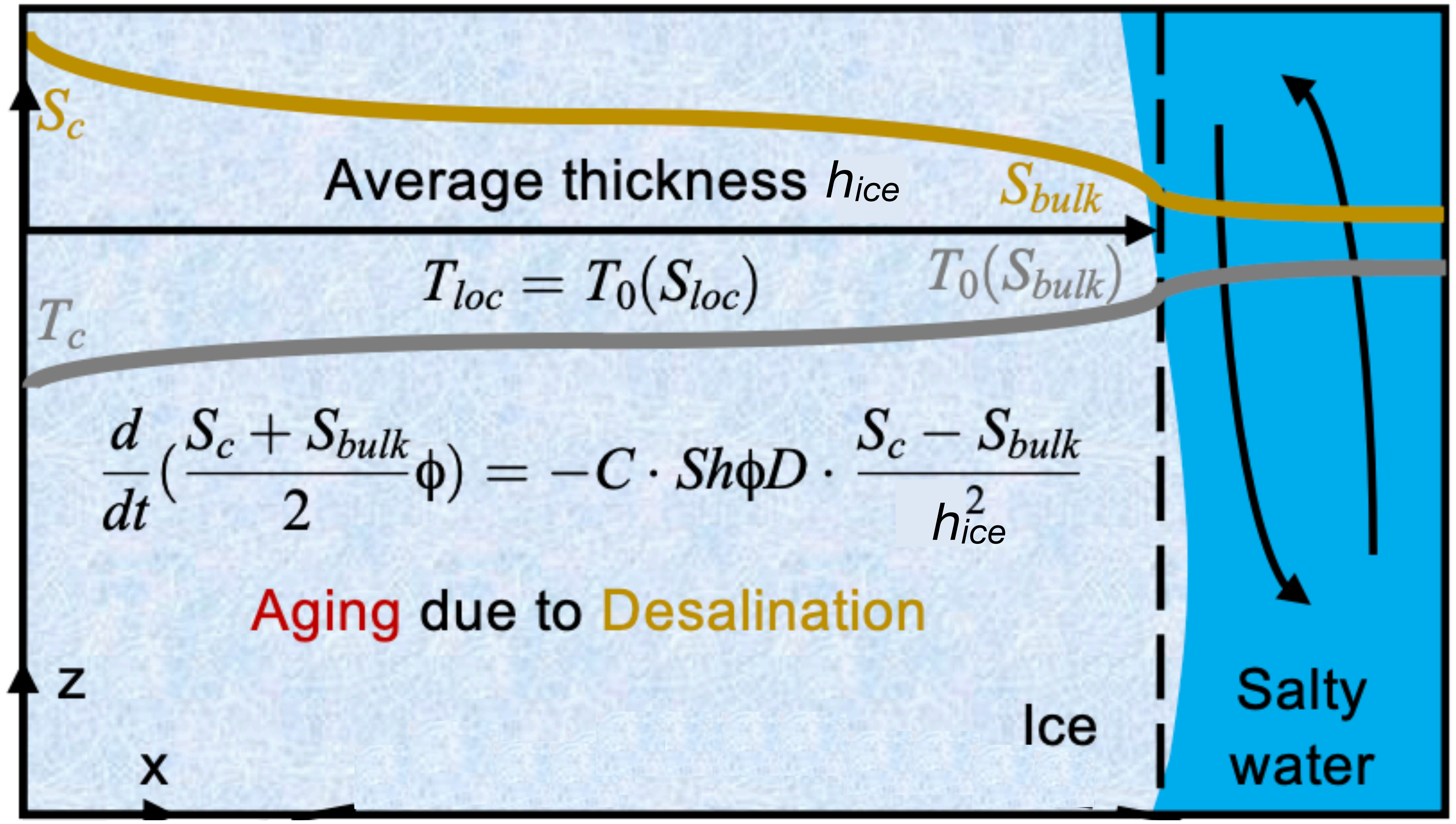
5 days record







Dynamics of “salty” ice aging



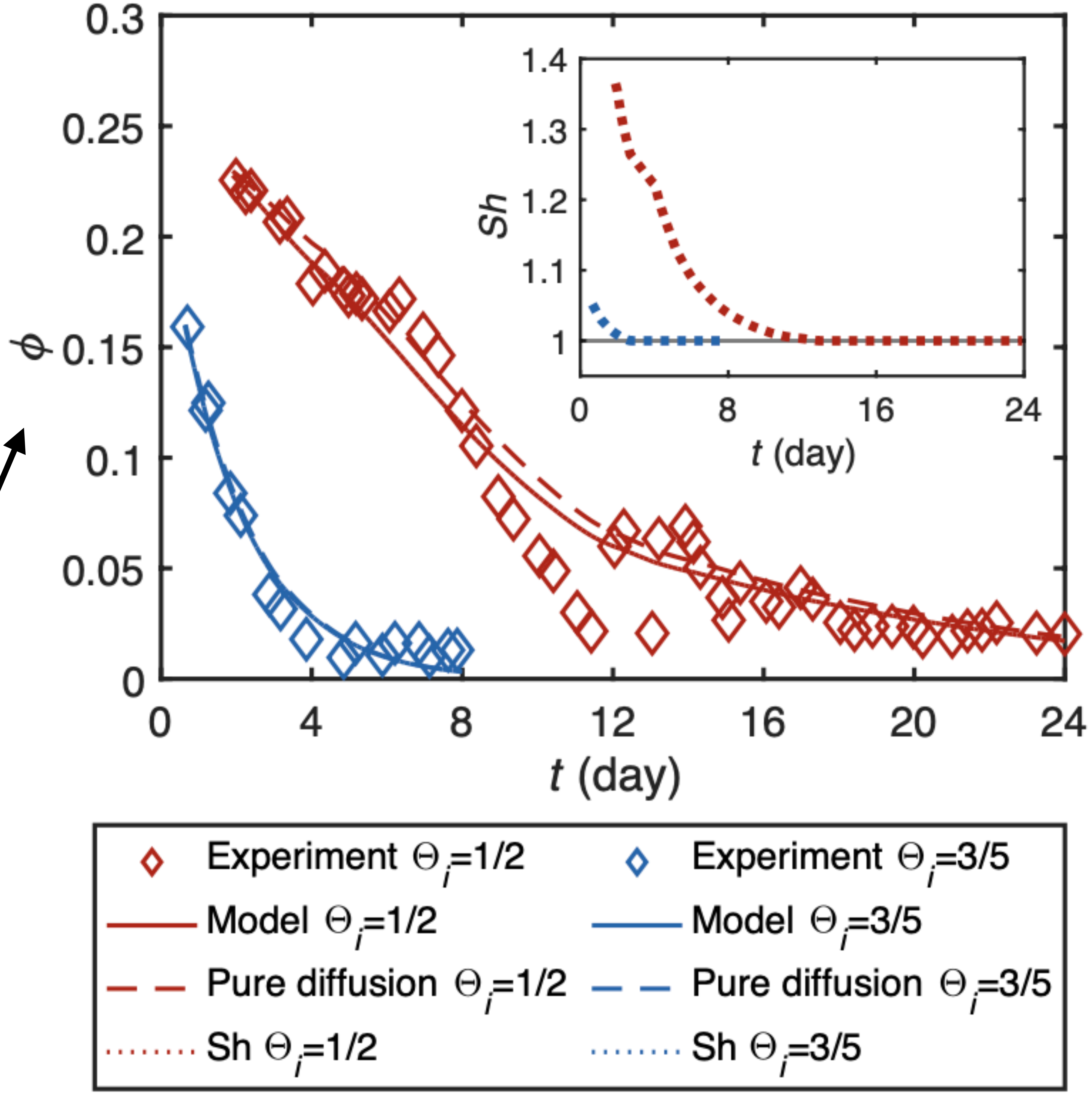
1D model

INPUT liquid salinity $S_{bulk}(t)$, ice layer $h_{ice}(t)$

OUTPUT porosity $\phi(t)$ [compared to the one measured in EXP]

⇒ Aging of mushy ice is driven primarily by **diffusive desalination** (Sherwood number ~ 1)

Two different experiments with different ΔT



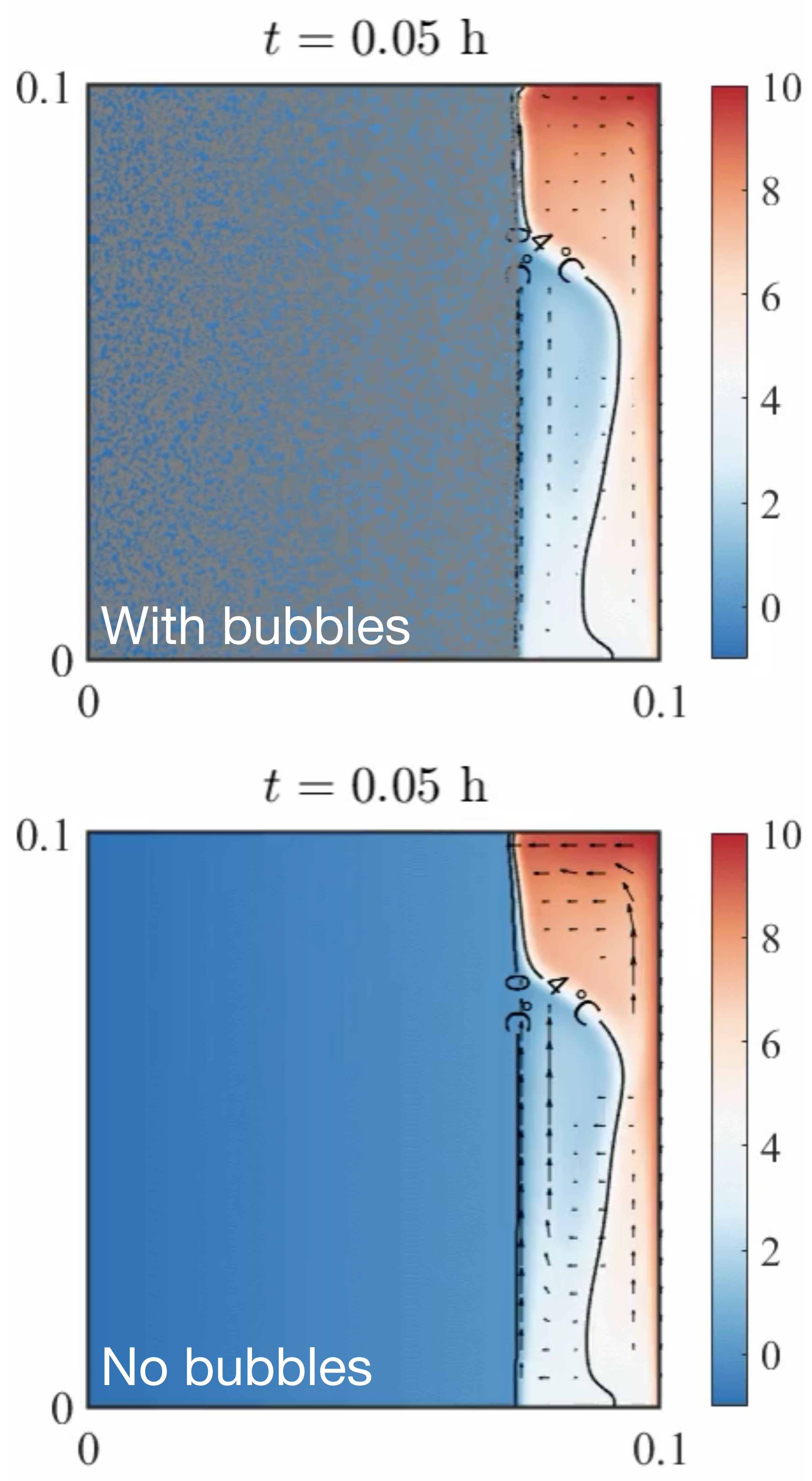
Du, Wang, Calzavarini, Sun, PRL (2025)

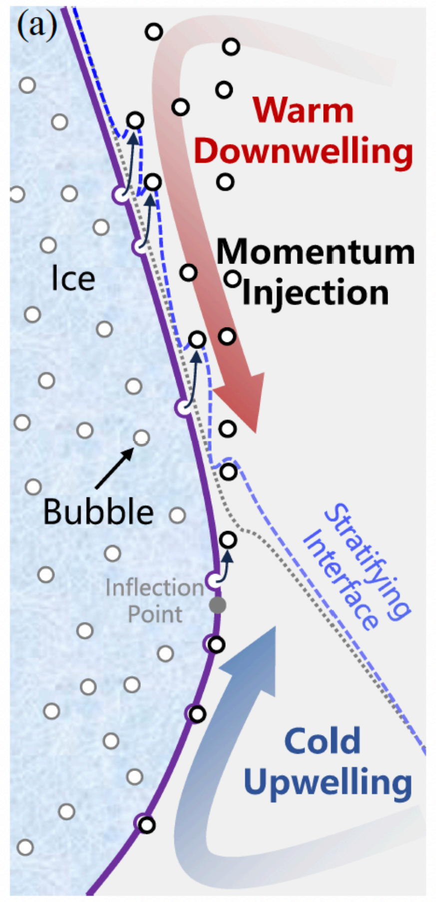
Natural ice has bubbles

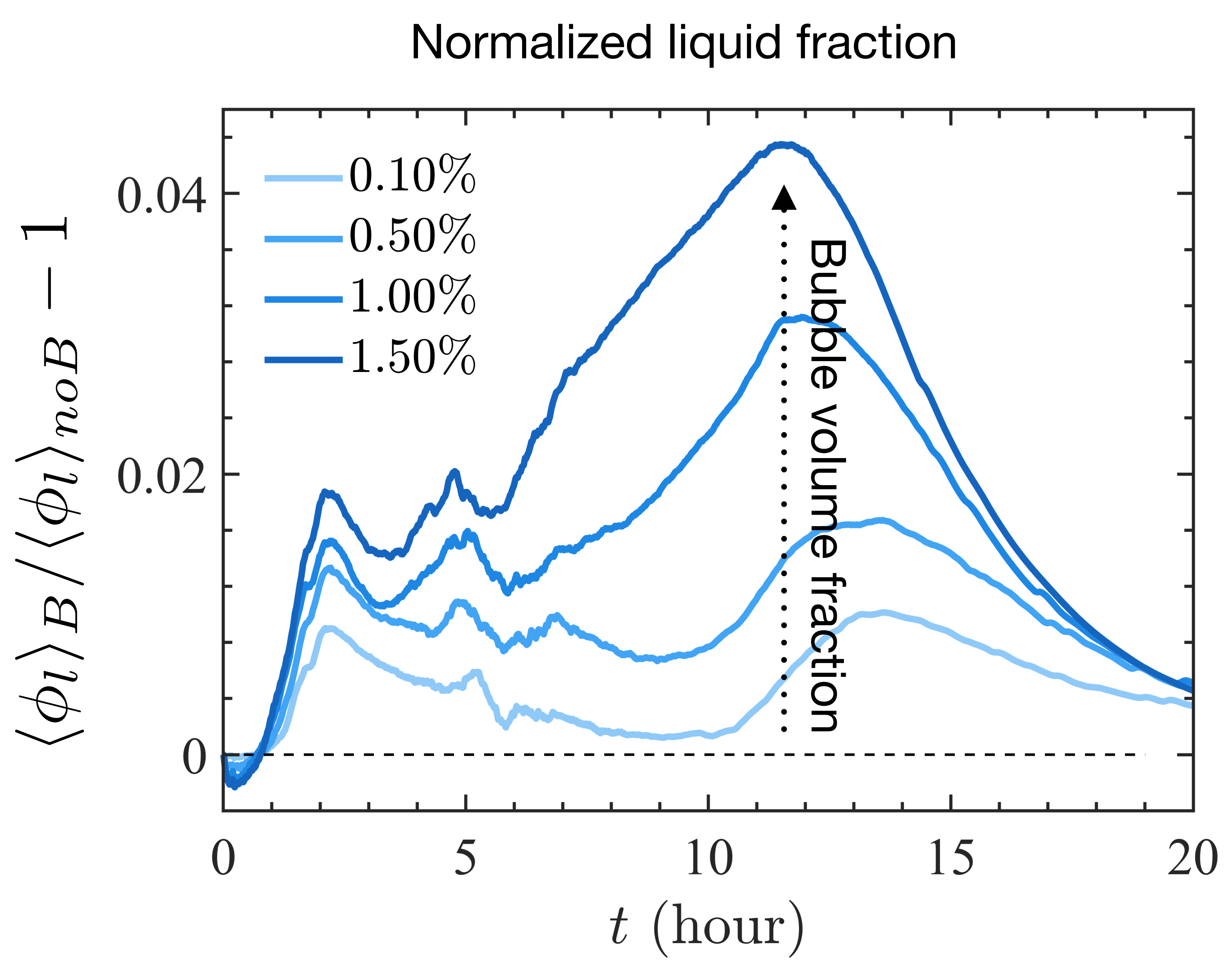
Ongoing Work!

The ice blocks cut from the glacier are full of bubbles.
(National Geographic —— By Douglas Fox)

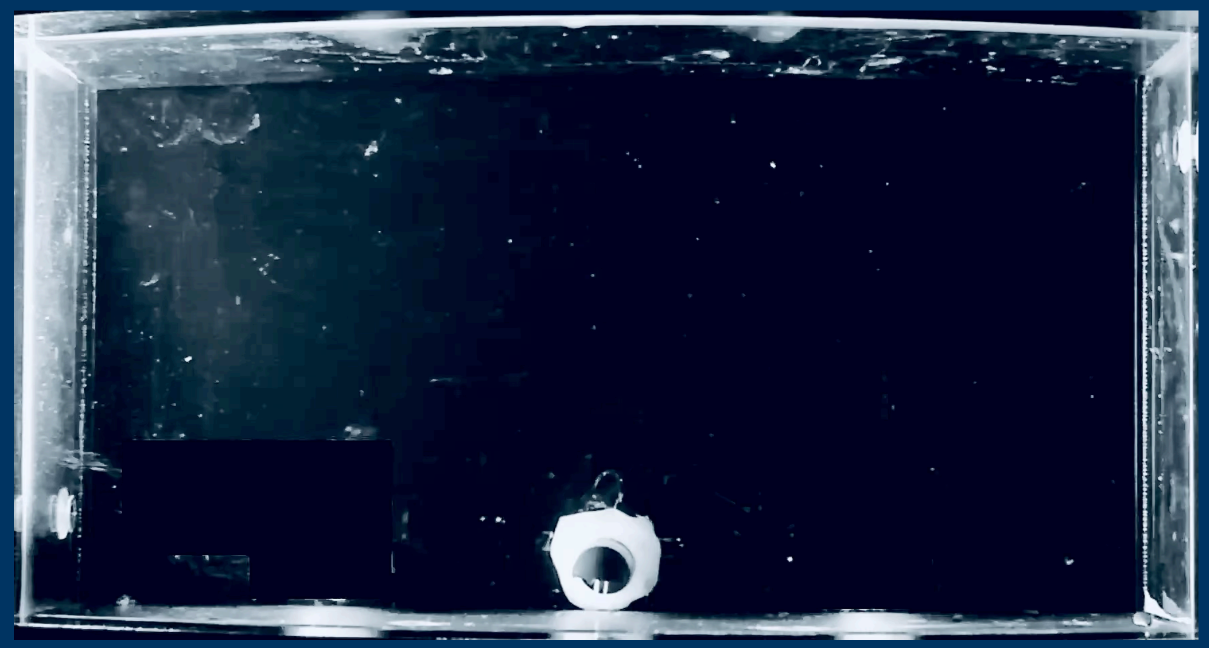
Can bubbles within ice significantly alter/ accelerate melting ?







Conclusions



Rich problem: closed system, simple geometry & bc → complex dynamics

Adding realism: water eq-of-state, salt, air inclusions → make it challenging

My works on melting/freezing (mostly) in a box

MELTING

•*Basal melting driven by turbulent thermal convection*, B. Rabbanipour Esfahani, S. C. Hirata, S. Berti and E. Calzavarini, **Phys. Rev. Fluids** 3, 053501 (2018)
•*Modelling the evolution of melt ponds: sensitivity to microscale heat transfer mechanisms,* A. Scagliarini, E. Calzavarini, D. Mansutti, F. Toschi, In Mathematical Approach to Climate Change and its Impacts. **Springer INdAM Series**, vol 38. Springer, Cham (2020)

FREEZING

•*How the growth of ice depends on the fluid dynamics underneath*, Z. Wang, E. Calzavarini, C. Sun, F. Toschi, **PNAS** 118 (10) e2012870118 (2021)
•*Ice front shaping by upward convective current*, Z. Wang, L. Jiang, Y. Du, C. Sun, E. Calzavarini **Phys. Rev. Fluids** 6, L091501(2021)
•*Equilibrium states of the ice-water front in a differentially heated rectangular cell*, Z. Wang, E. Calzavarini, C. Sun, **Europhys. Lett.** , 135 (2021) 54001

SALTY WATER

•*Sea Water Freezing Modes in a Natural Convection System*, Y. Du, Z. Wang, L. Jiang, E. Calzavarini, C. Sun, **J. Fluid Mech.** (2023), vol. 960, A35
•*Sea ice aging by diffusion-driven desalination*, Y. Du, F. Wang, E. Calzavarini, C. Sun, **Phys. Rev. Lett. 135**, 104201 (2025)
•*Freezing and ice aging dynamics in saline water under natural convection*, F. Wang, Y. Du, X. Xie, E. Calzavarini, C. Sun, **Phys. Rev. Fluids** 11, 013504 (2026)

BUBBLY ICE

•*In progress…*

REVIEW

•*The physics of freezing and melting in the presence of flows*, Y. Du, E. Calzavarini, C. Sun, ***Nature Reviews Physics***, 2024, ⟨10.1038/s42254-024-00766-5⟩

Thanks!