Statistical properties of spheroids in turbulent flows: the effects of size and shape

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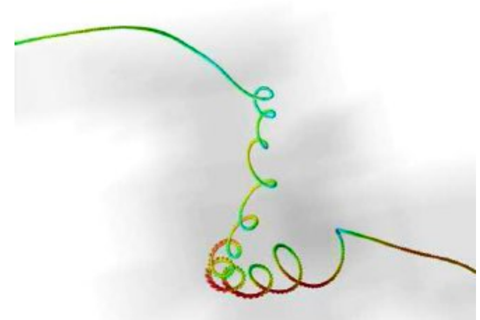
Talk's outline

Exploring turbulence through particles: an addendum to studies started @ENSLyon

- Recap on models, simulations and experiments
- Interesting questions
- A new numerical study on large neutral (a)spherical particles in turbulence
- Acceleration statistics
- Rotation statistics
- Conclusions

A brief recap:
Models for the motion of a particle in a flow (1)

- Fluid tracers, *or the Lagrangian point of view*

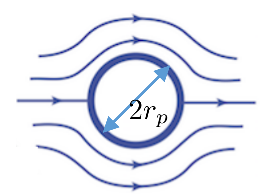


$$\frac{d\mathbf{x_f}}{dt} = \mathbf{u}(\mathbf{x_f}(t), t) \qquad \mathbf{a}_f \equiv \frac{D\mathbf{u}}{Dt} = -\frac{1}{\rho_f}\nabla p + \nu\Delta\mathbf{u} + \mathbf{f}$$

- Small inertial particles, *an equation with many authors and many names*

$$\frac{d\mathbf{x_p}}{dt} = \mathbf{v}(t)$$

$$m_p\frac{d\mathbf{v}}{dt} = \underbrace{6\pi\mu r_p(\mathbf{u}-\mathbf{v})}_{\text{drag}} + \underbrace{m_f\frac{D\mathbf{u}}{Dt}}_{\text{fluid acceleration}} + \underbrace{\frac{m_f}{2}\frac{d(\mathbf{u}-\mathbf{v})}{dt}}_{\text{added mass}} + \underbrace{6r_p^2\sqrt{\pi\rho_f\mu}\int_0^t \frac{d(\mathbf{u}-\mathbf{v})}{d\tau}\frac{d\tau}{\sqrt{t-\tau}}}_{\text{history}} + \underbrace{(m_p-m_f)\mathbf{g}}_{\text{buoyancy}}$$



$$Re_p = \frac{||\mathbf{u}-\mathbf{v}||(2r_p)}{\nu} \ll 1$$

$$Re_s = \frac{||\nabla\mathbf{u}||(2r_p)^2}{\nu} \ll 1$$

Stokes (1851)
Boussinesq (1885), Basset (1888), Oseen (1927) -> BBO eq.
Tchen (1947), Corrsin-Lumley(1956)
Maxey-Riley (1983), Gatignol (1983) -> MRG eq. *extension to non-uniform creeping flows*

A brief recap: Models for the motion of a particle in a flow (2)

- Corrections to the point-particle equation

Finite-size effect (Faxén laws 1922)

$$\mathbf{u} \to [\mathbf{u}]_S \simeq \mathbf{u} + \frac{r_p^2}{6}\Delta\mathbf{u}$$

in drag , history

$$\mathbf{u} \to [\mathbf{u}]_V \simeq \mathbf{u} + \frac{r_p^2}{10}\Delta\mathbf{u}$$

in fluid acceleration , added mass

Finite-Reynolds effect (Shiller-Neumann 1935) wake drag empirical correlation

$$6\pi\mu r_p \to 6\pi\mu r_p(0.15\ Re_p^{0.687}) \qquad Re_p \lesssim 10^3$$

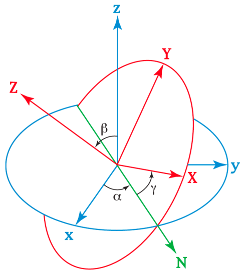
Lift force for light particles (Auton JFM 1987, inviscid calculation)

$$\mathbf{f}_{lift} = \frac{m_f}{2}(\mathbf{u}-\mathbf{v}) \times (\nabla \times \mathbf{u})$$

- Rigid body Newton-Euler equations

$$m_p \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \mathbf{F} \qquad \mathbf{F} = \oint_{S_p} \boldsymbol{\sigma}\cdot\mathbf{n}\, dS$$

$$\frac{\mathrm{d}\boldsymbol{\mathcal{I}\Omega}}{\mathrm{d}t} = \mathbf{T} \qquad \mathbf{T} = \oint_{S_p} (\mathbf{x}-\mathbf{r}) \times (\boldsymbol{\sigma}\cdot\mathbf{n})\, dS$$



A brief recap:
Acceleration statistics in numerics and experiments (1)

R. Volk et al. Physica D (2008)

- Point-particle model & experiments

tracer
d = 1.8 η

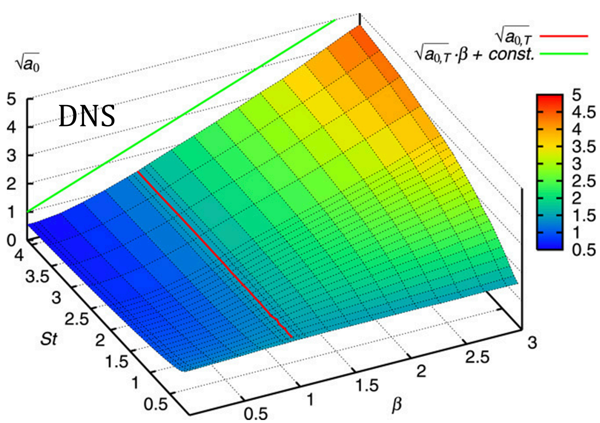
neutral
d=15 η

$$\frac{d\mathbf{v}}{dt} = \beta\frac{D\mathbf{u}}{Dt} + \frac{1}{\tau_p}(\mathbf{u}-\mathbf{v})$$

$$\beta = 3\rho_f/(\rho_f + 2\rho_p)$$

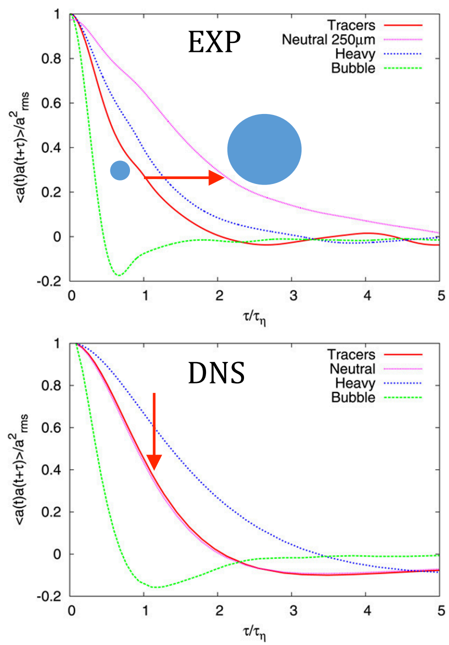
$$\tau_p = r_p^2/(3\nu\beta)$$

Variance (a_0)

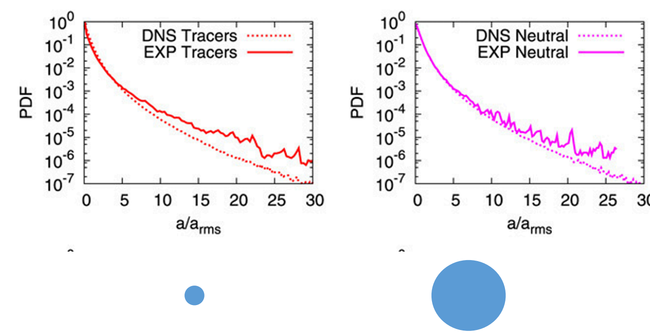


	$St = \frac{\tau_p}{\tau_\eta}$	a_0
Experiment		
Tracers	0.24	6.4 ± 1
Neutral	16	2.2 ± 1
Heavy	0.58	4.3 ± 1
Bubble	1.85	26 ± 5
Numerics		
Tracers	0.31	2.85 ± 0.07
Neutral	4.1	2.94 ± 0.07
Heavy	1.03	2.63 ± 0.12
Bubble	1.64	25.9 ± 0.46

Temporal correlation



Probability density function

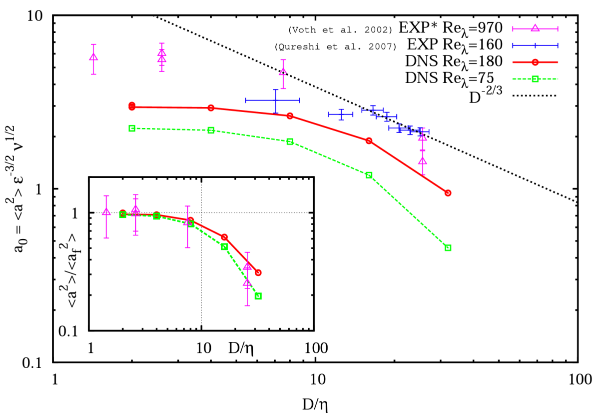


A brief recap:
Acceleration statistics in numerics and experiments (2)

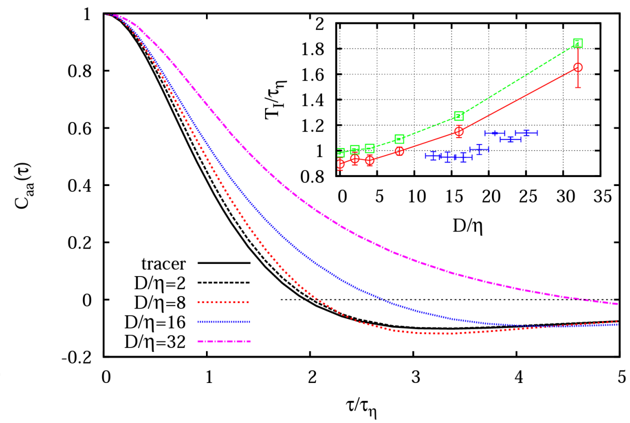
EC, Volk, Bourgoin, Leveque, Pinton, Toschi JFM (2009)

- Faxén-finite-size model & experiments $\frac{d\mathbf{v}}{dt} = \beta \left[\frac{D\mathbf{u}}{Dt}\right]_V + \frac{1}{\tau_p}([\mathbf{u}]_S - \mathbf{v})$ $\left[\frac{D\mathbf{u}}{Dt}(\mathbf{x},t)\right]_V \simeq \int_{L^3} G(\mathbf{x}') \frac{D\mathbf{u}}{Dt}(\mathbf{x}-\mathbf{x}',t)\, d^3x'$

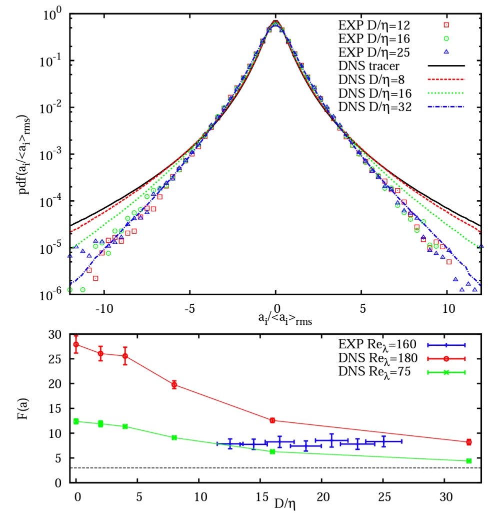
Variance



Temporal correlation



PDF and flatness



The coarse grained turbulence hypothesis

$$a(D) \sim \frac{\delta_D p}{D} \sim \frac{(\delta_D u)^2}{D} \qquad \tau_a(D) \sim \frac{D}{\delta_D u} \sim D^{2/3} \qquad \mathcal{F}_a(D) = \frac{\langle a(D)^4 \rangle}{\langle a(D)^2 \rangle^2}$$

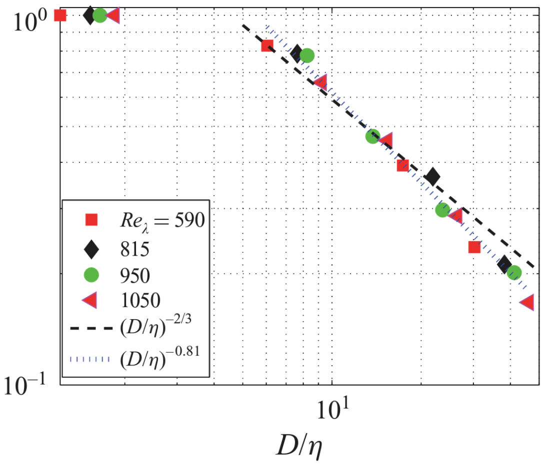
K41 $\delta_\ell u \sim \ell^{1/3}$ $\qquad \tau_u(\ell) \sim \frac{\ell}{\delta_\ell u} \sim \ell^{2/3}$ $\qquad \langle (\delta_\ell u)^p \rangle \sim \ell^{\zeta_p}$

A brief recap:
Acceleration statistics in numerics and experiments (3)

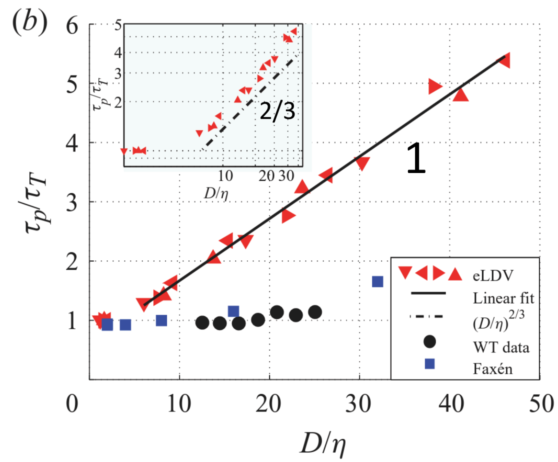
Volk, EC, Leveque, Pinton JFM (2011)

- Von Karman flow experiments (large Re)

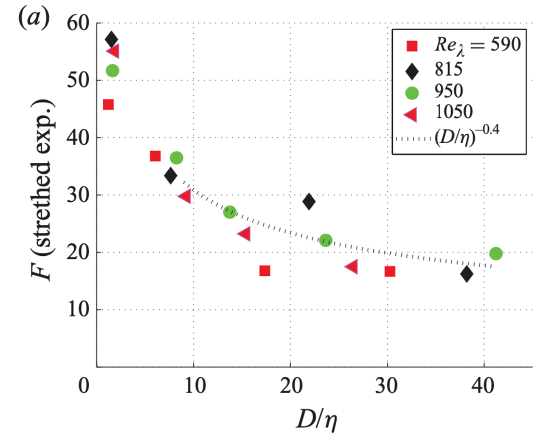
Variance



Temporal correlation



PDF and flatness



$$\langle a(D)^2 \rangle \sim D^{-0.81}$$

$$\tau_a(D) \sim D$$

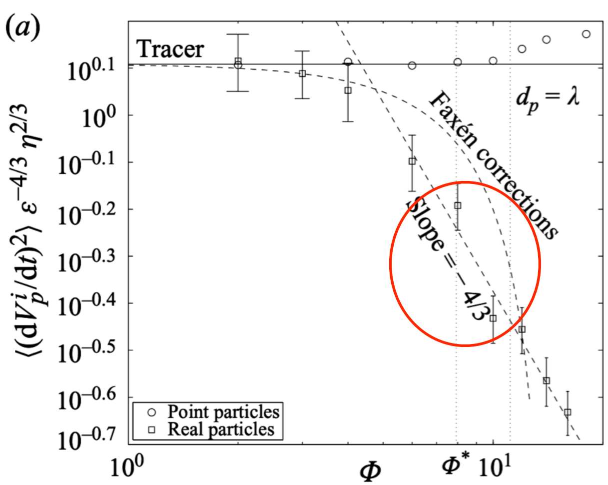
$$\mathcal{F}_a(D) \sim D^{-0.4(-0.6)}$$

A brief recap:
Acceleration statistics in numerics and experiments (4)

- Fully resolved simulations (low Re) $Re_\lambda = 32$

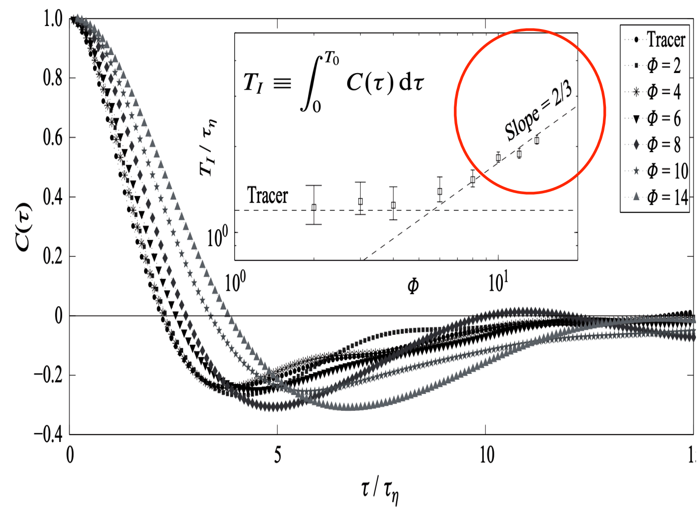
Homann & Bec JFM 2010

Variance



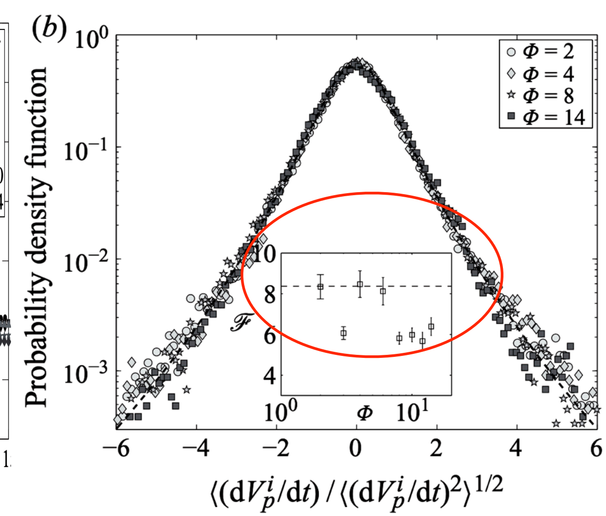
$$\langle a(D)^2\rangle \sim D^{-4/3}$$

Temporal correlation



$$\tau_a(D) \sim D^{2/3}$$

PDF and flatness



$$\mathcal{F}_a(D) \sim const.$$

Cisse, Homann & Bec JFM 2013 -> slipping motion and particle boundary layer

Uhlmann & Chouippe JFM 2017 -> particle preferential concentration

Interesting questions

- Do **finite-sized particles** experience coarse-grained turbulent flow **accelerations**?

$$a(D) \sim \frac{\delta_D p}{D} \sim \frac{(\delta_D u)^2}{D} \sim D^{-1/3} \qquad \tau_a(D) \sim \frac{D}{\delta_D u} \sim D^{2/3} \qquad \mathcal{F}_a(D) \sim D^{\zeta_8 - 2\zeta_4}$$

or does particle-flow coupling create specific statistical signatures?

- How is the particle finite-size reflected on particle **rotations**?

$$\Omega(D) \sim \frac{1}{\tau(D)} \sim \frac{\delta_D u}{D} \sim D^{-2/3}$$

- What is the effect of particle **anisotropy (shape)**?

Small *anisotropic particles align with flow structures: For large particles is a randomization of rotations to be expected?*

- What do we learn of turbulence when we look to a moving particle?

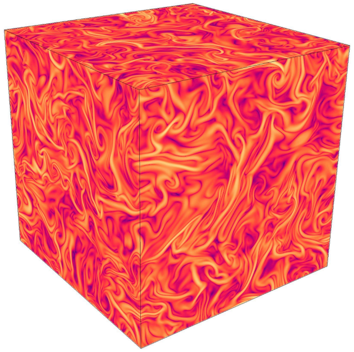
A new numerical study

L. Jiang et al. JFM (2022)
https://arxiv.org/abs/2202.03937

Navier-Stokes equations

$$\partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u} = -\rho^{-1}\nabla p + \nu\,\nabla^2\mathbf{u} + \mathbf{f},$$
$$\nabla\cdot\mathbf{u} = 0,$$

+ no-slip bc at particle interface



Code:
https://github.com/ecalzavarini/ch4-project

Methods :
Lattice Boltzmann +
Immersed boundaries

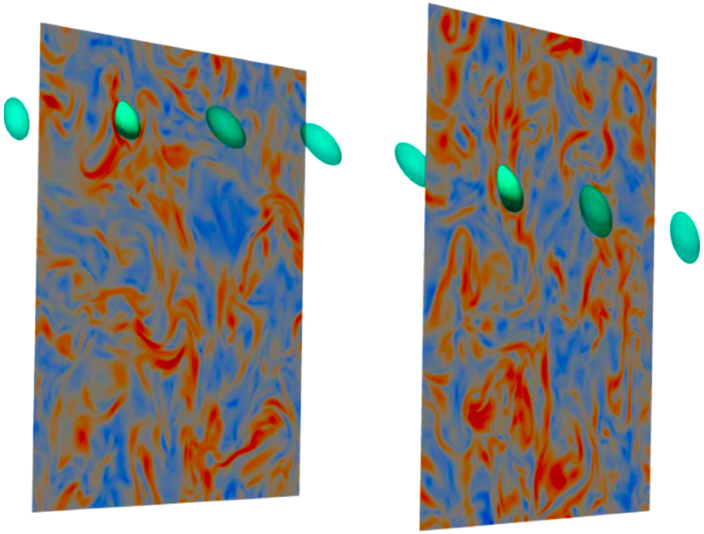
Newton- Euler equations (NEE)

$$m_p \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \mathbf{F} \qquad \mathbf{F} = \oint_{S_p} \boldsymbol{\sigma}\cdot\mathbf{n}\, dS$$

$$\frac{\mathrm{d}\boldsymbol{\mathcal{I}\Omega}}{\mathrm{d}t} = \mathbf{T} \qquad \mathbf{T} = \oint_{S_p} (\mathbf{x}-\mathbf{r})\times(\boldsymbol{\sigma}\cdot\mathbf{n})\, dS$$

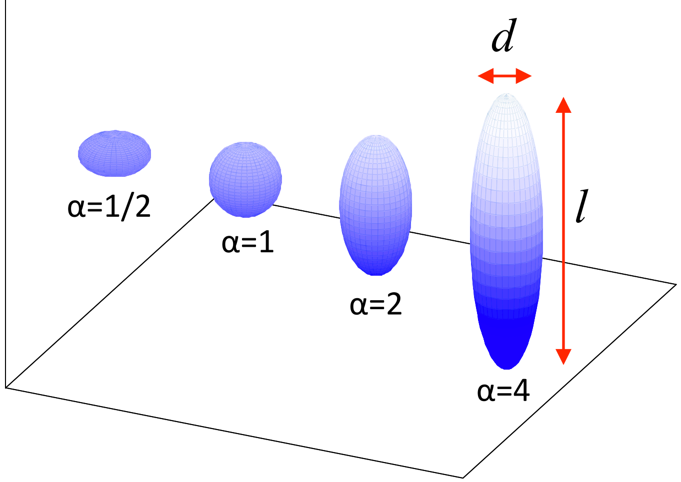
$$\boldsymbol{\sigma} = -p\boldsymbol{I} + \rho\nu(\nabla\mathbf{u} + \nabla\mathbf{u}^T)$$

Real Particles



Parameter space

axisymmetric spheroids



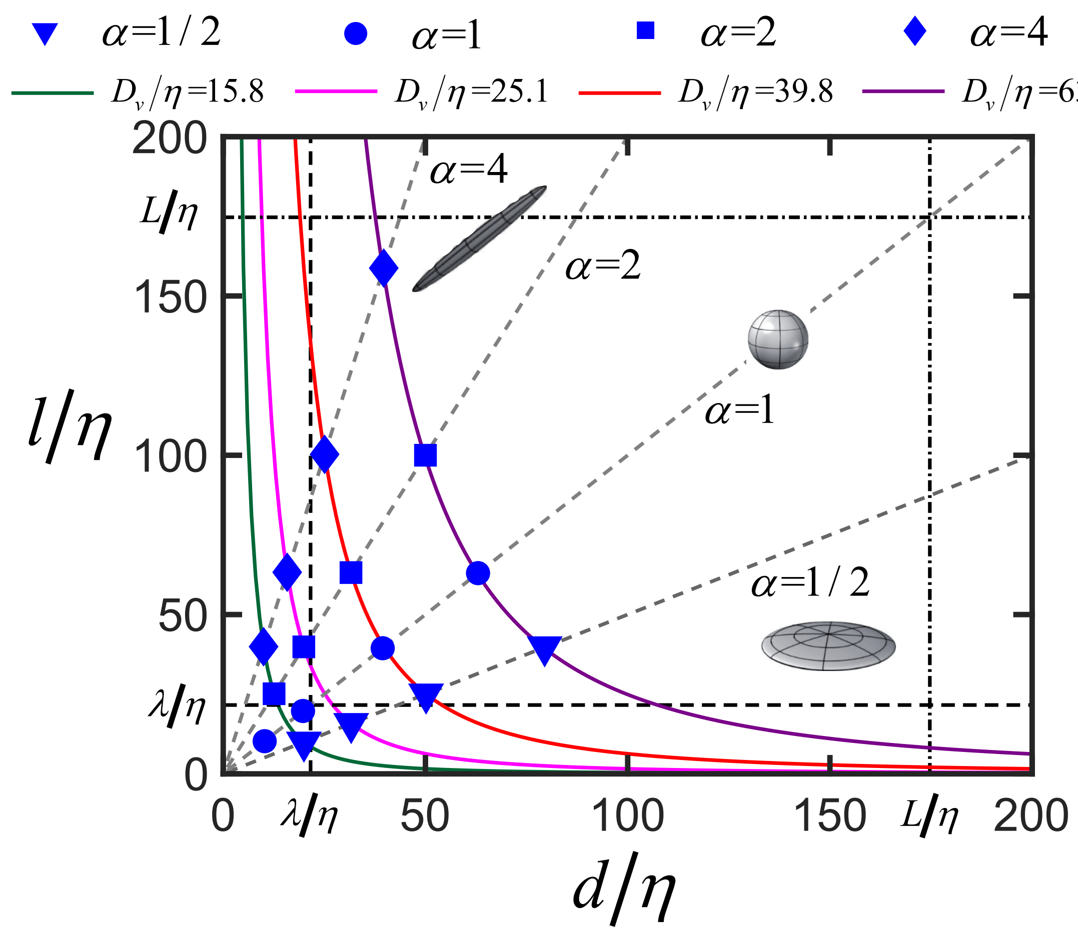
aspect ratio

$$\alpha = \frac{l}{d}$$

Volume equivalent diameter

$$D_{\mathrm{v}} = (d^2 l)^{1/3} = d\alpha^{1/3}$$

$$Re_\lambda = 120$$

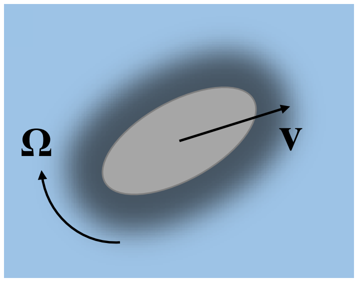


Auxiliary particle models

$$\mathbf{F} = \oint_{S_p} \boldsymbol{\sigma}\cdot\mathbf{n}\, dS$$

$$\mathbf{T} = \oint_{S_p} (\mathbf{x}-\mathbf{r})\times(\boldsymbol{\sigma}\cdot\mathbf{n})\, dS$$

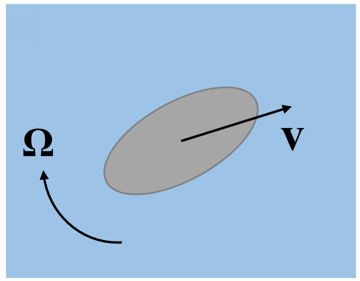
Real Particles (RP)



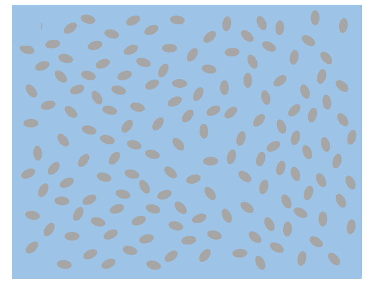
$$\mathbf{F}_{VP} = \int_{V_p} \rho \frac{D\mathbf{u}}{Dt} d^3x$$

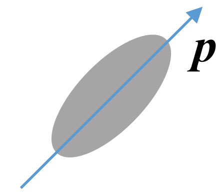
$$\mathbf{T}_{VP} = \int_{V_p} (\mathbf{x}-\mathbf{r})\times \rho \frac{D\mathbf{u}}{Dt} d^3x$$

Virtual Particles (VP)



Jeffery fluid tracers (JFT)



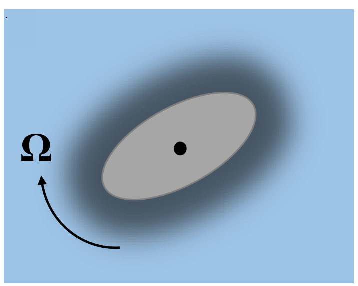


$$\dot{\mathbf{r}} = \mathbf{u}(\mathbf{r}(t),t),$$

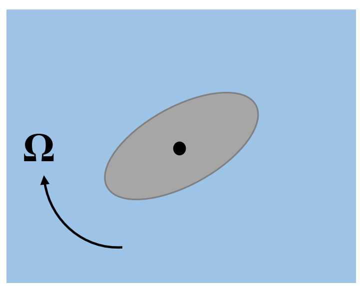
$$\dot{\mathbf{p}} = \boldsymbol{\Omega}\times\mathbf{p}$$ Jeffery eq. 1922

$$\boldsymbol{\Omega} = \frac{1}{2}\boldsymbol{\omega}(\mathbf{r}(t),t) + \Lambda\, \mathbf{p}\times\mathcal{S}(\mathbf{r}(t),t)\mathbf{p}$$

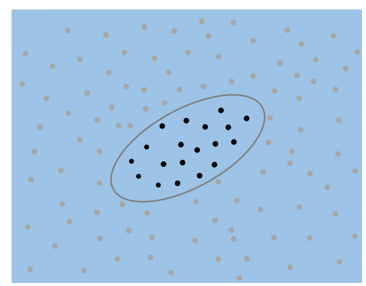
$$\Lambda = \frac{\alpha^2-1}{\alpha^2+1}$$



Real Particles Fixed Location (RPFL)



Virtual Particles Fixed Location (VPFL)



Volume Averages (VA)

$$\mathbf{a}_{VA} = \frac{1}{V_p}\int_{V_p} \frac{D\mathbf{u}}{Dt} d^3x$$

$$\boldsymbol{\Omega}_{VA} = \frac{1}{V_p}\int_{V_p} \nabla\times\mathbf{u}\, d^3x$$

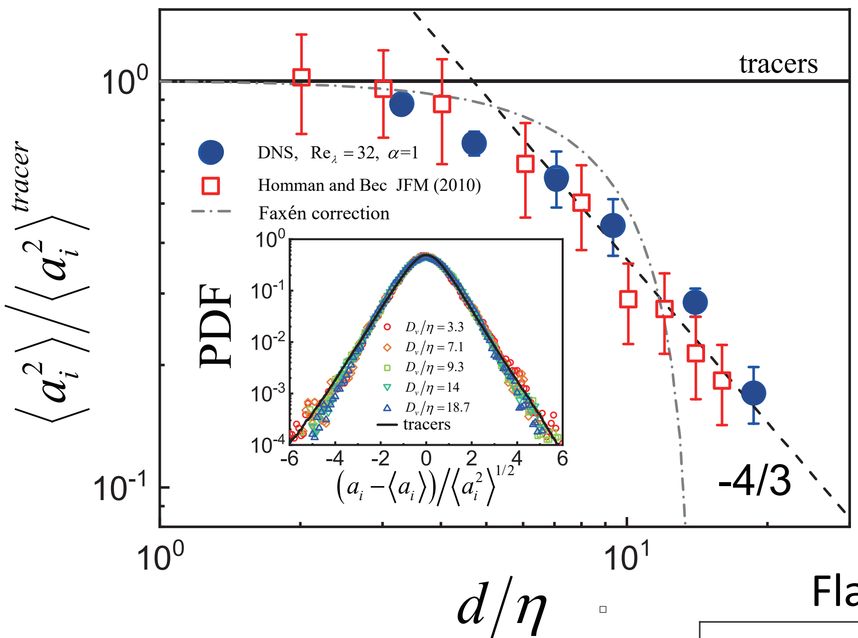
Numerical Experiments	Model Equations
Real Particles (RP)	NSE + NEE + IBM
Volume Averages (VA)	NSE
Virtual Particles (VP)	NSE + NEE
Real Particles with Fixed Locations (RPFL)	NSE + EE + IBM
Virtual Particles with Fixed Locations (VPFL)	NSE + EE
Jeffery Fluid Tracers (JFT)	NSE + tracer eq.+ Jeffery eq.

Spheres: acceleration at low Re

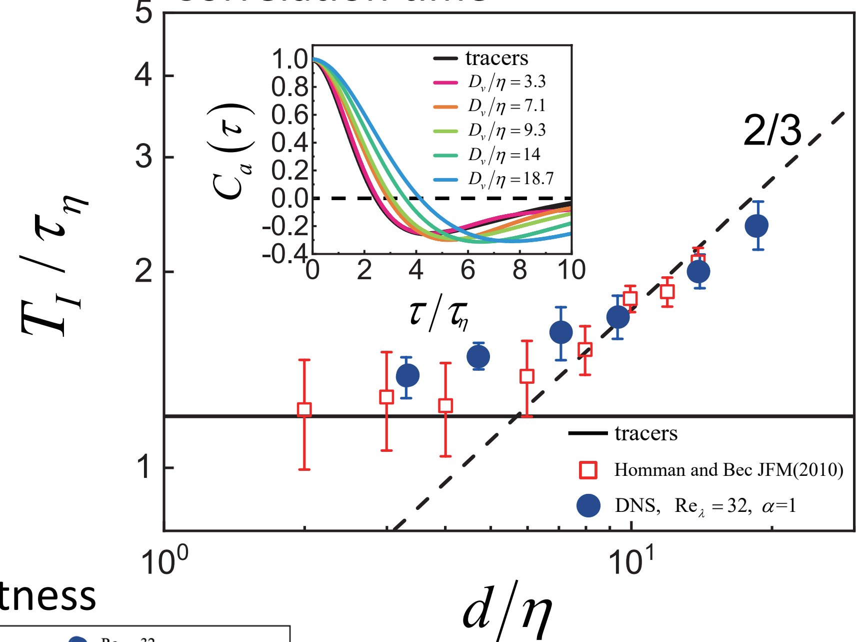
Comparison with Homann & Bec JFM 2010

$Re_\lambda = 32$

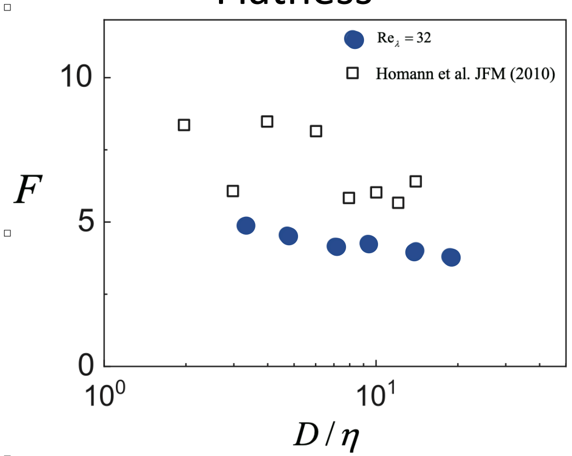
Variance



Correlation time



Flatness

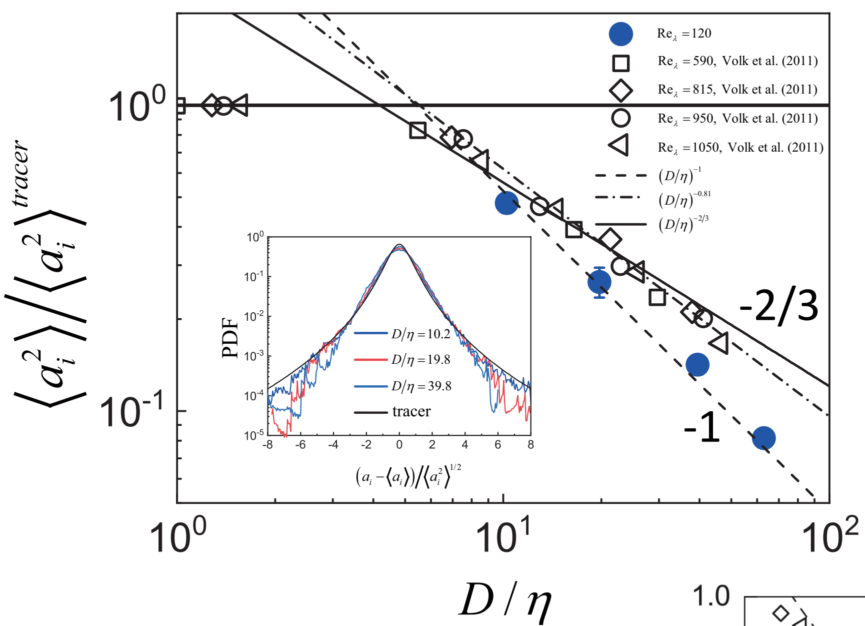


Spheres: acceleration at larger Re

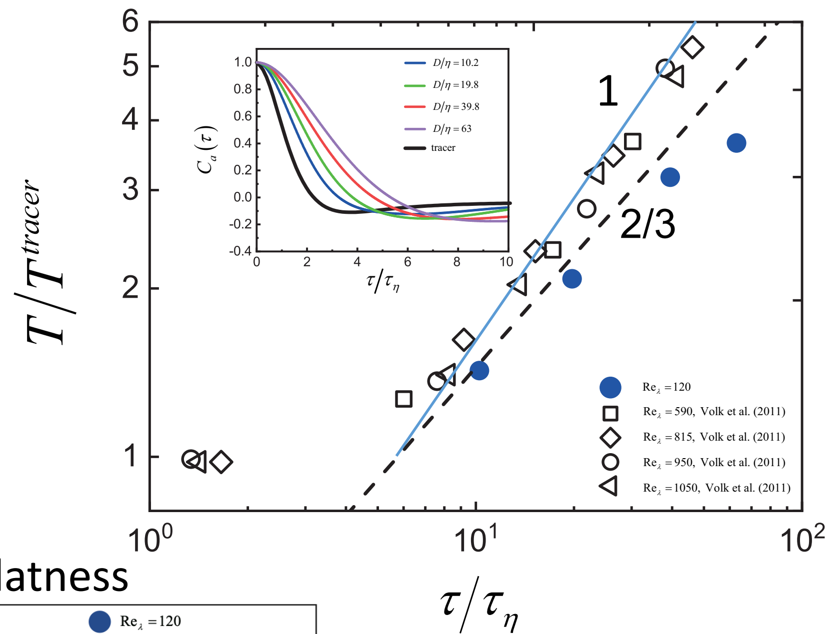
Comparison with Volk et al. JFM 2011

$Re_\lambda = 120$

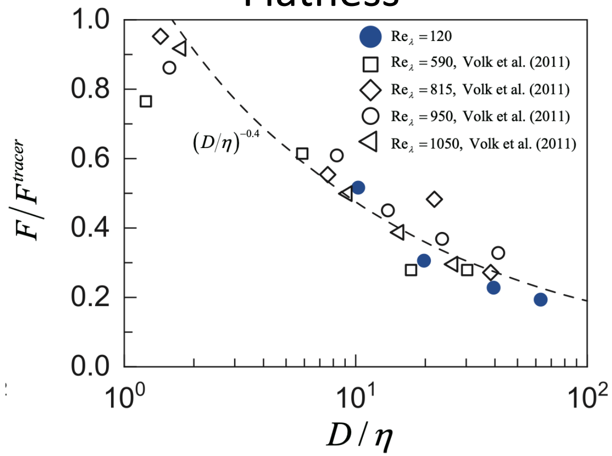
Variance



Correlation time

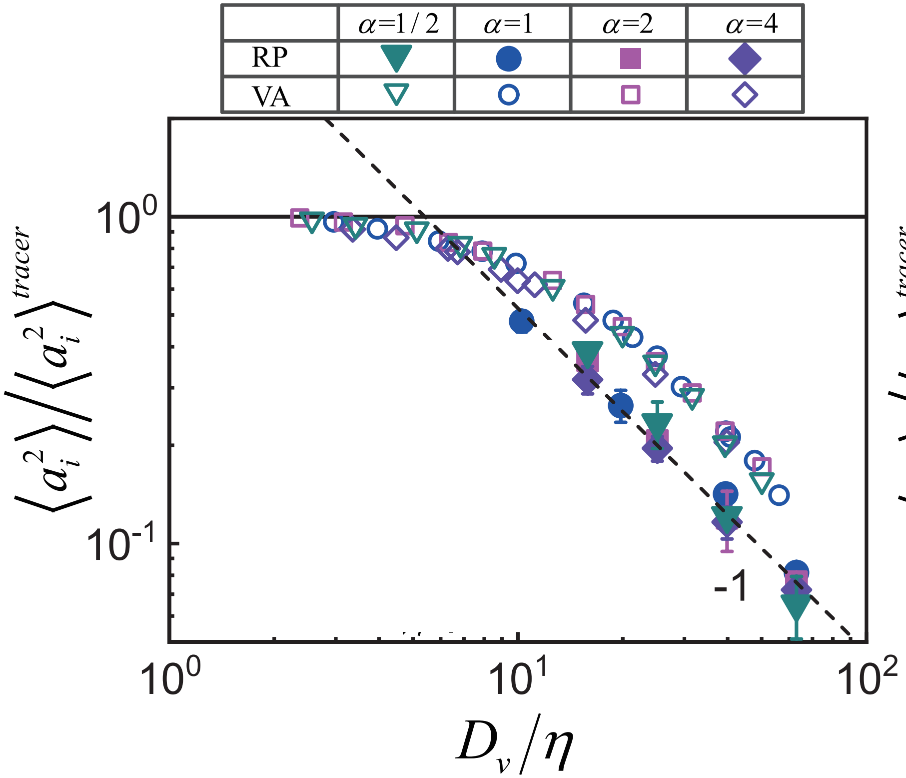


Flatness

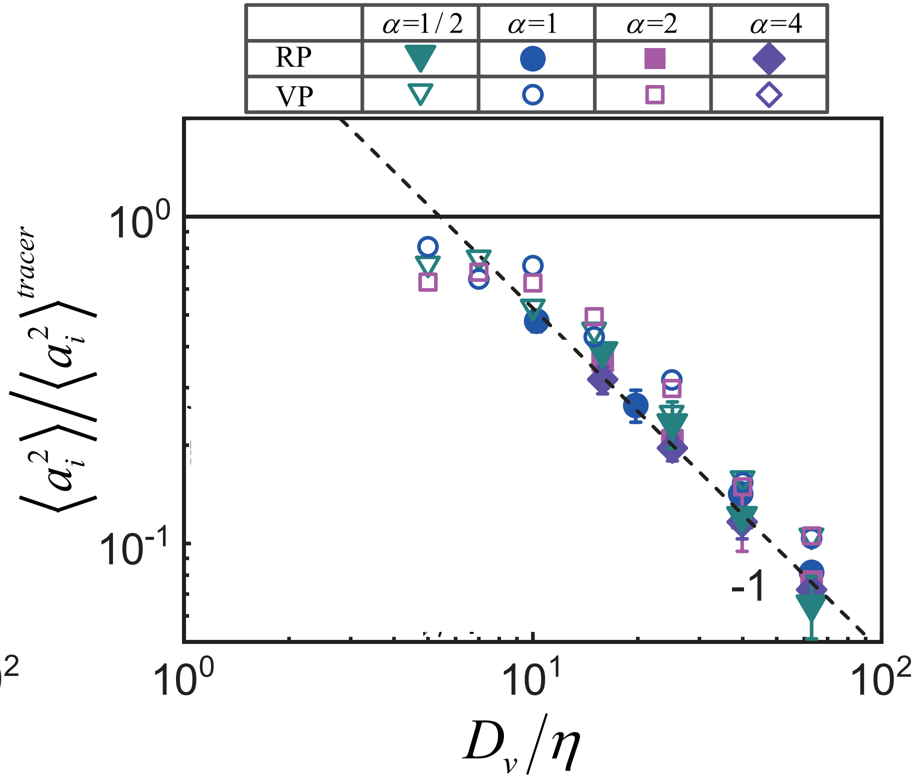


Spheroids: Acceleration variance

Real Particles vs Volume Averages



Real Particles vs Virtual Particles

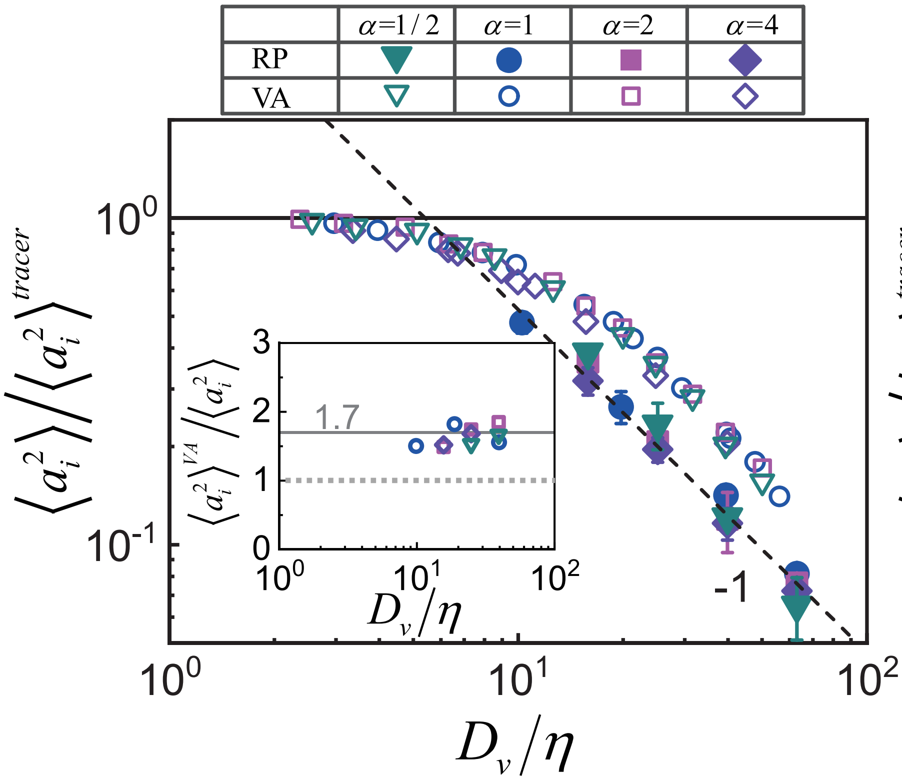


$$D_{\mathrm{v}} = (d^2 l)^{1/3} = d\alpha^{1/3}$$

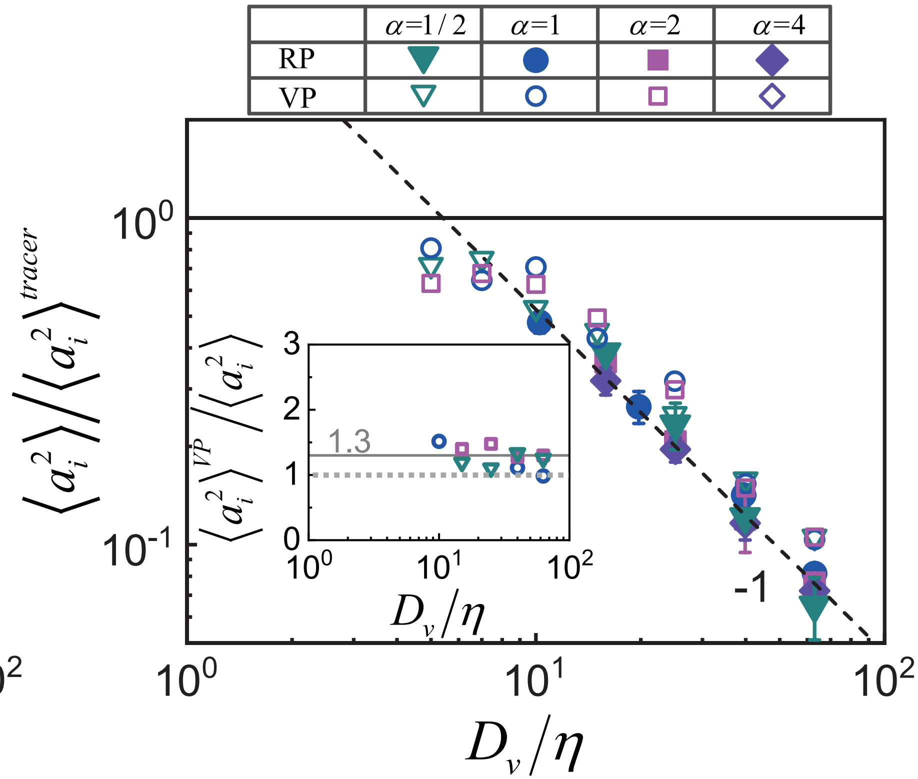
Better overlap for VP particles

Spheroids: Acceleration variance

Real Particles vs Volume Averages



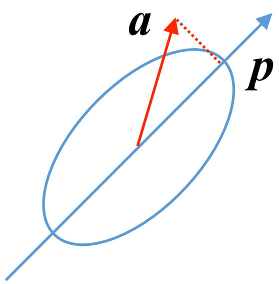
Real Particles vs Virtual Particles



Better overlap for VP particles

$$D_{\mathrm{v}} = (d^2 l)^{1/3} = d\alpha^{1/3}$$

Spheroids: Acceleration alignment

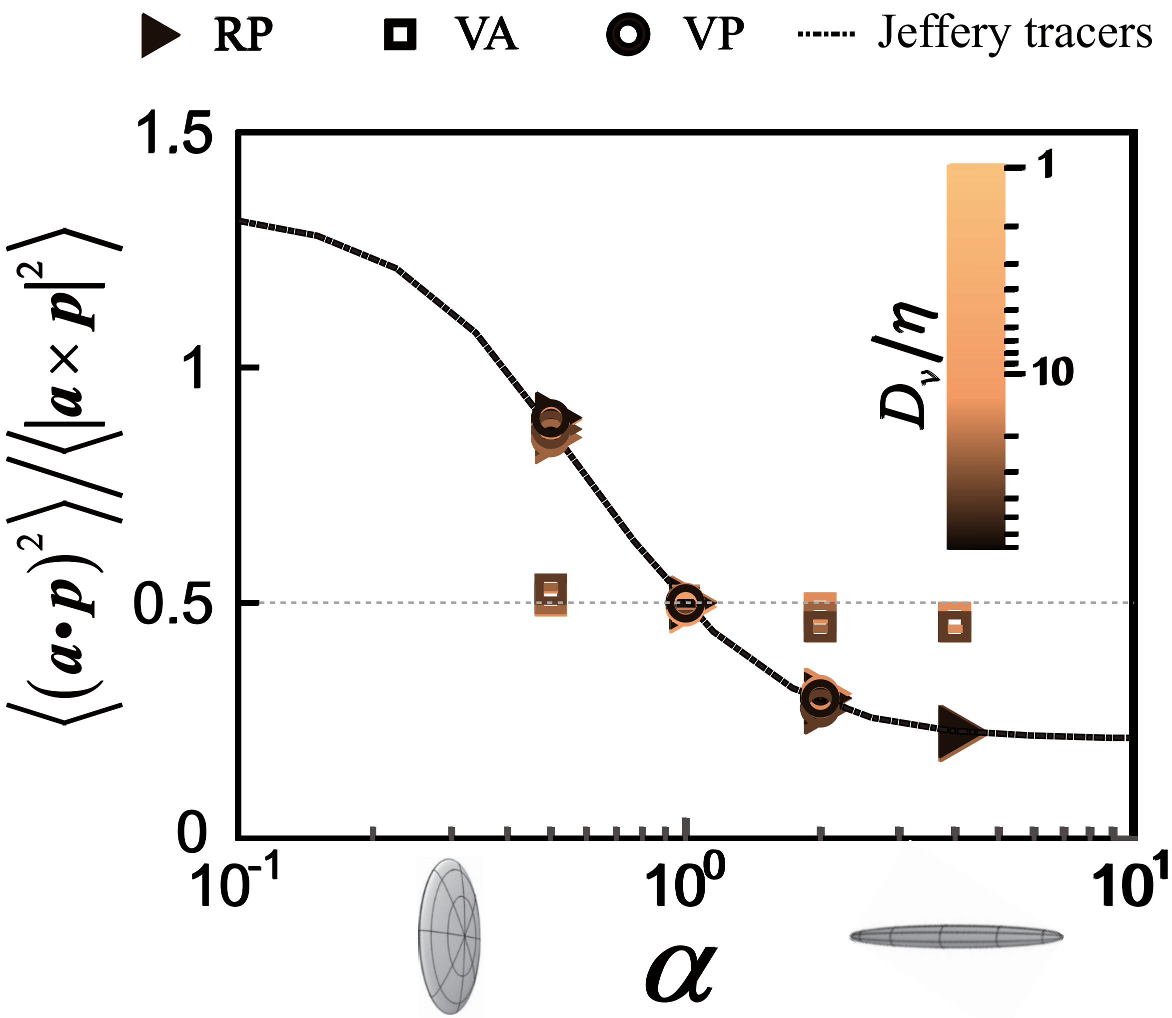


Axial acceleration

$$a^{\parallel} = (a \cdot \mathrm{p})\mathrm{p}$$

Transverse acc

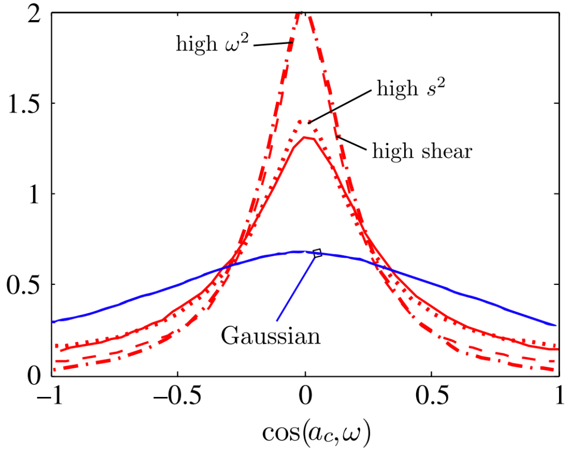
$$a^{\perp} = a \times \mathrm{p}$$



Spheroids: Acceleration alignment

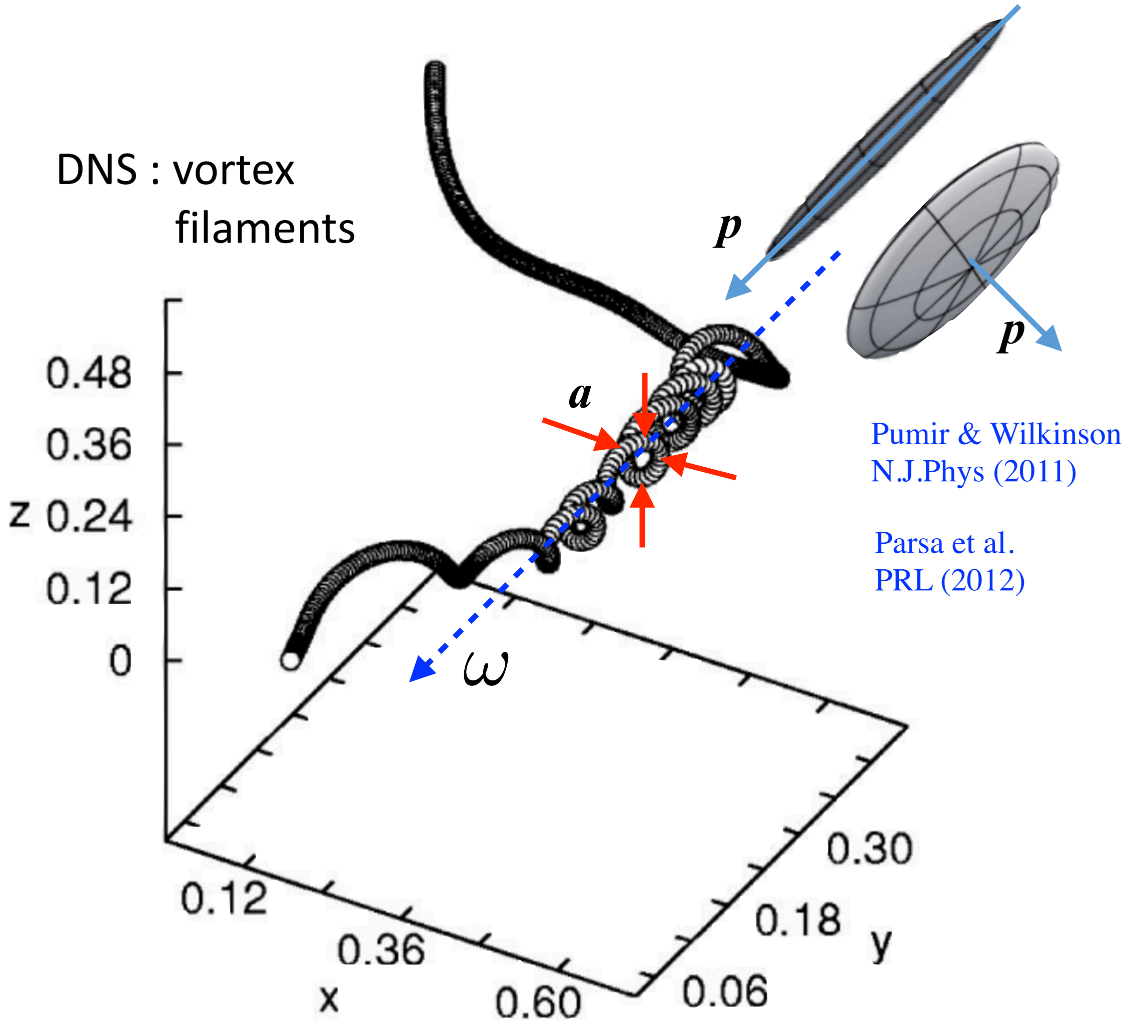
A qualitative explanation

EXP: $\mathbf{a} \perp \boldsymbol{\omega}$



A.Liberzon et al. Physics (2012)

DNS : vortex filaments



Pumir & Wilkinson N.J.Phys (2011)

Parsa et al. PRL (2012)

L.Biferale et al. Phys Fluids (2005)

Summary

Acceleration

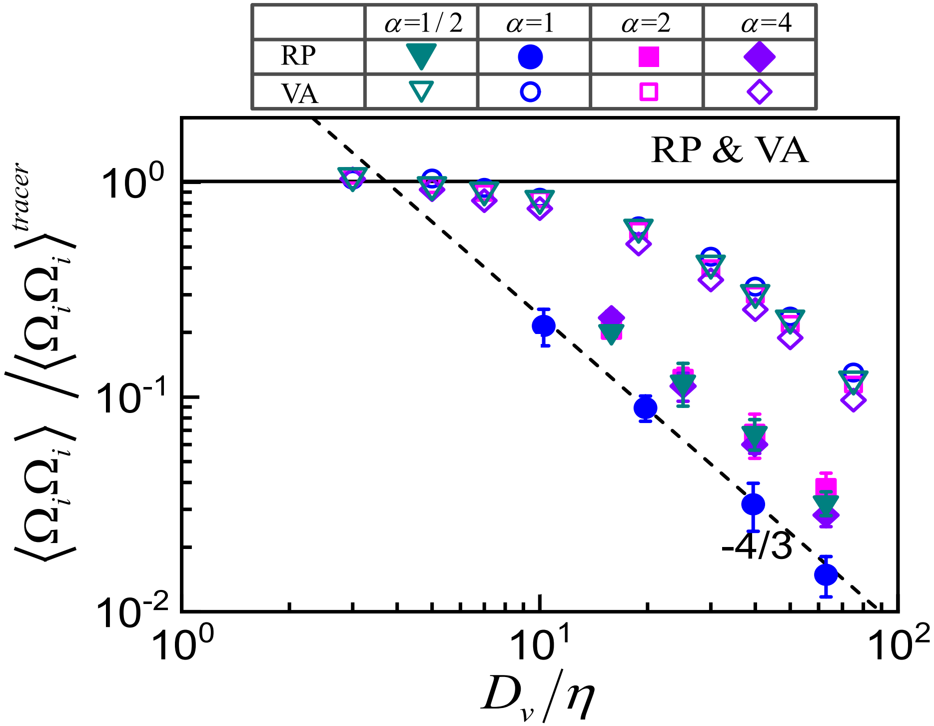
Size-dependence explained by **fluid acceleration volume averages** (VA or VP)
Amplitude better accounted for **dynamical** (Lagrangian) **volume averages** (VP)

Importance of **flow structures**: preferential alignment even for inertial-scale particles!

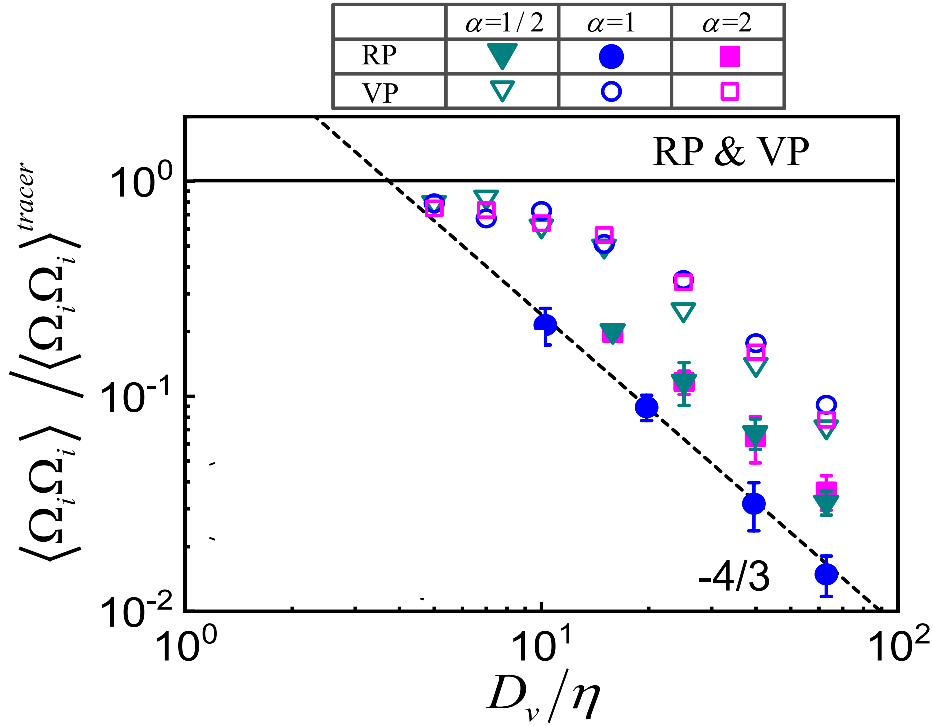
Rotation

Spheroids: Angular velocity

Real Particles vs Volume Averages



Real Particles vs Virtual Particles

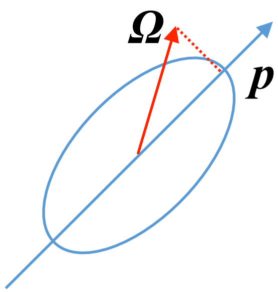


$$\Omega(D) \sim \frac{1}{\tau(D)} \sim \frac{\delta_D u}{D} \sim D^{-2/3}$$

Coarse grained scaling relation ok!

But no overlap of RP with VA or VP

Spheroids: spinning and tumbling



Axial rotation rate (spin)

$$\boldsymbol{\Omega}^s = (\boldsymbol{\Omega}\cdot\mathbf{p})\mathbf{p}$$

Transverse rotation (tumbling)

$$\dot{\mathbf{p}} = \boldsymbol{\Omega}\times\mathbf{p}$$

Two predictions:

1) Randomly oriented model in HIT flow

Parsa et al., PRL 2012

$$\langle \Omega_i^s \Omega_i^s \rangle = \frac{1}{12}\frac{1}{\tau_\eta^2}$$

$$\langle \dot{p}_i \dot{p}_i \rangle = \left(\frac{1}{6} + \frac{1}{10}\Lambda^2\right)\frac{1}{\tau_\eta^2}$$

$$\Lambda = \frac{\alpha^2-1}{\alpha^2+1}$$

$$\boxed{\frac{\langle \Omega_i^s \Omega_i^s \rangle}{\langle \dot{p}_i \dot{p}_i \rangle} = \frac{\frac{1}{12}\langle \omega^2\rangle}{\frac{1}{6}\langle\omega^2\rangle + \frac{1}{5}\Lambda^2\langle \mathcal{S}:\mathcal{S}\rangle} = \frac{5}{10+6\Lambda^2}}$$

2)Coarse-grained (K41) model

Oehmke et al., PRF 2021

$$\langle \Omega_i^s \Omega_i^s \rangle \sim d^{-4/3}$$

$$\langle \dot{p}_i \dot{p}_i \rangle \sim l^{-4/3}$$

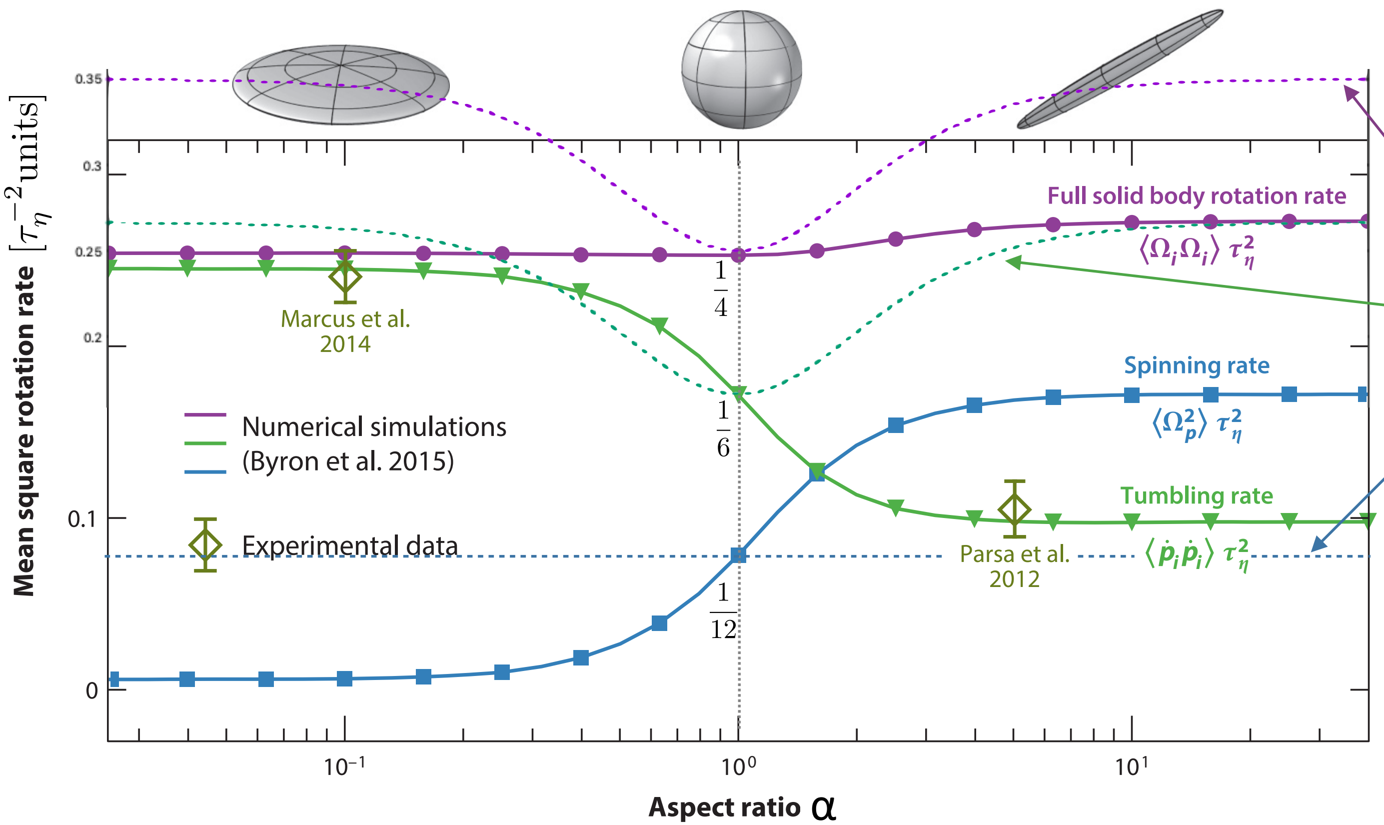
$$\boxed{\frac{\langle \Omega_i^s \Omega_i^s \rangle}{\langle \dot{p}_i \dot{p}_i \rangle} \sim \frac{\alpha^{4/3}}{2}}$$

Always no size dependence!

Review:
Voth & Soldati, Ann Rev Fluid Mech 2017

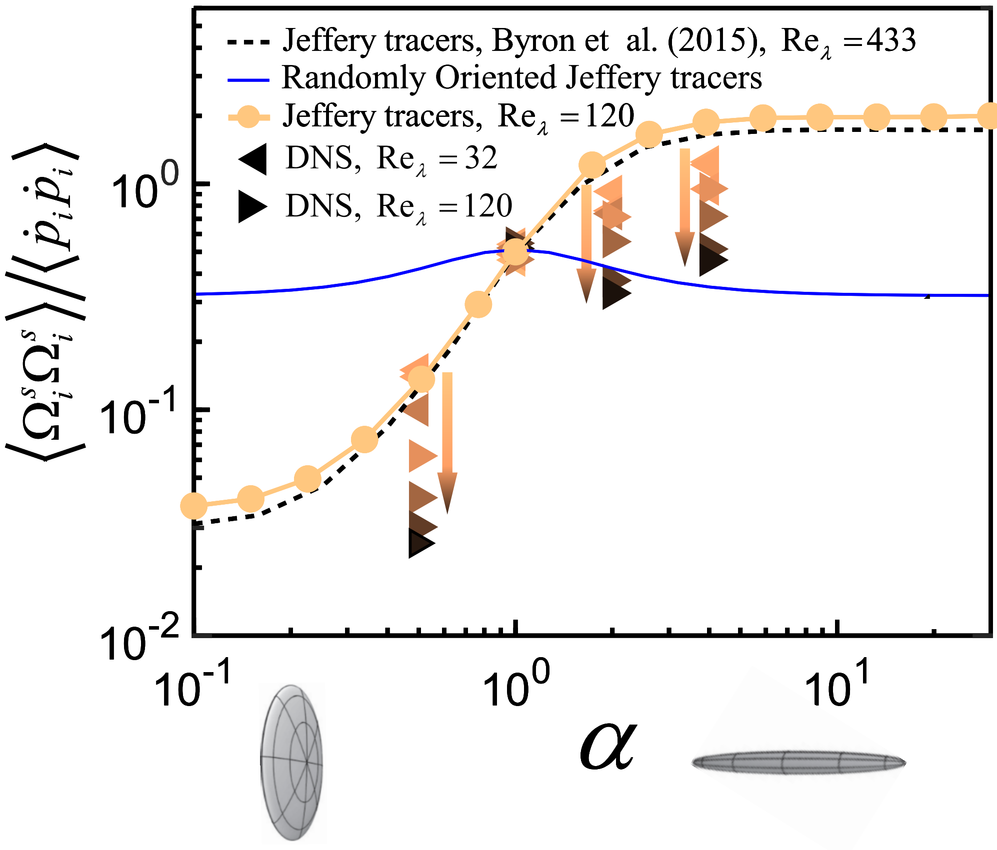
Rotation rate variances in homogenous isotropic turbulence (HIT)

Byron et al., Phys Fluids 2015
Review: Voth & Soldati, Ann Rev Fluid Mech 2017

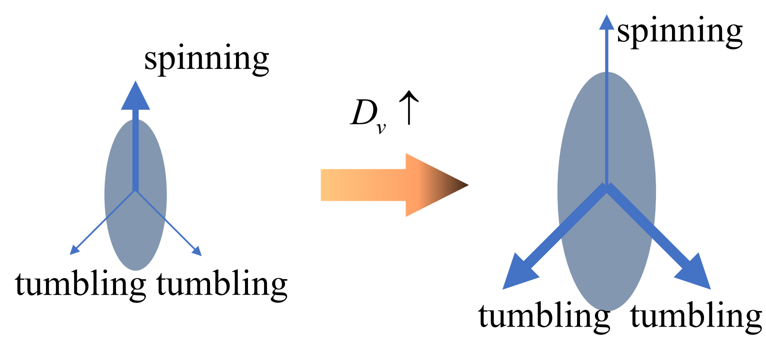


Random orientation

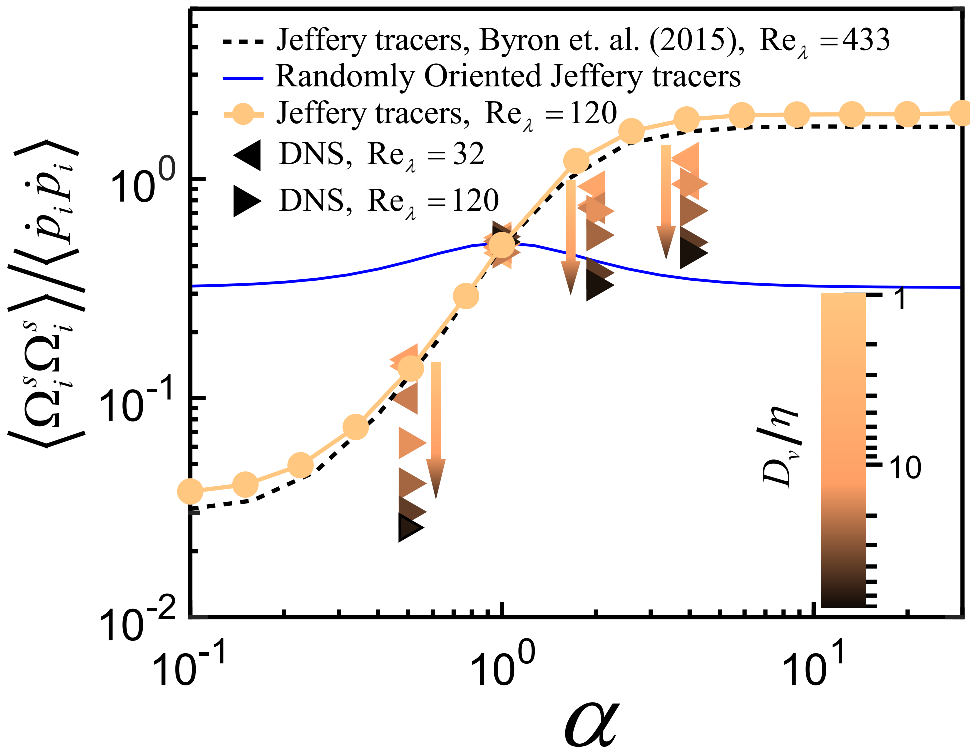
Spinning - tumbling variance ratio



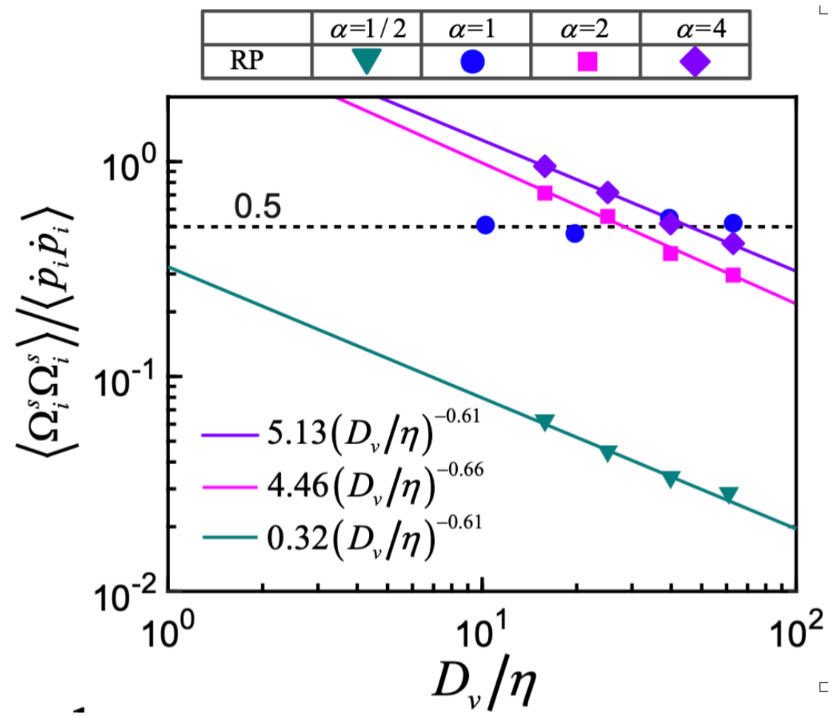
Large particles
spin less and tumble more



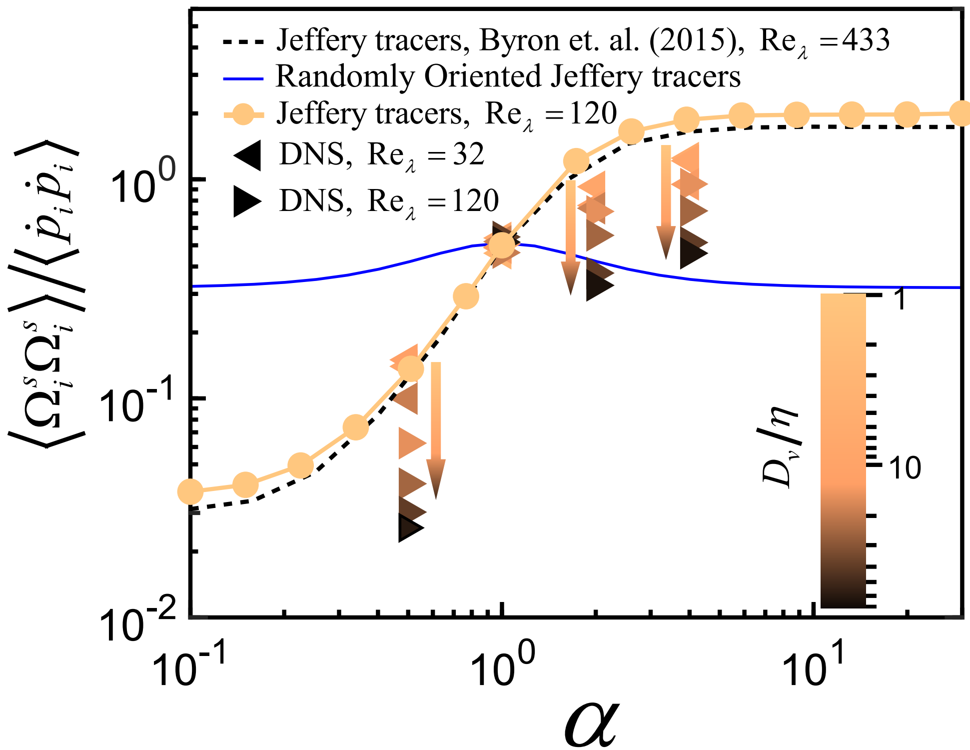
Spinning - tumbling variance ratio



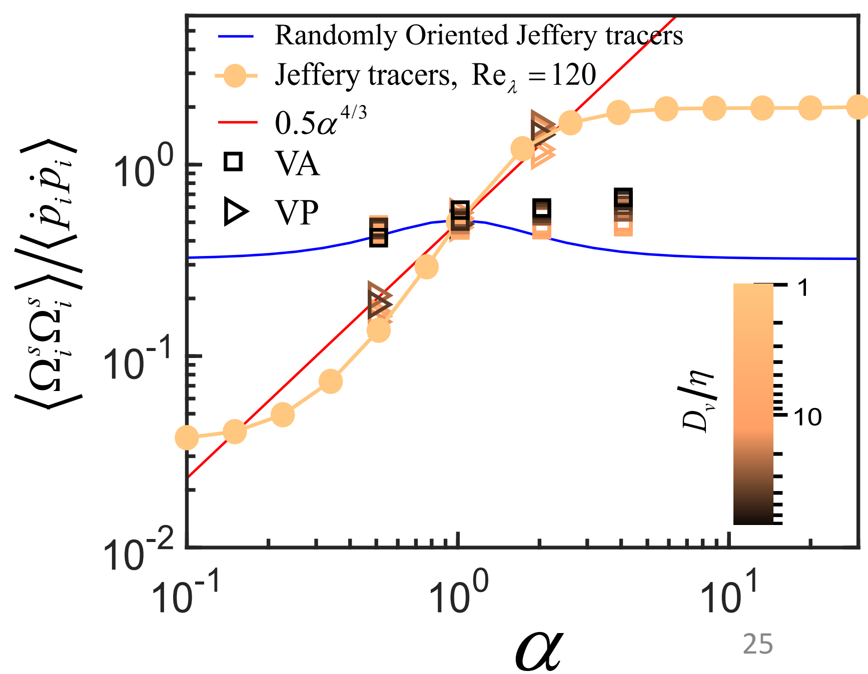
$$\frac{\langle \Omega_i^s \Omega_i^s \rangle}{\langle \dot{p}_i \dot{p}_i \rangle} \sim D_v^{-0.6}$$



Spinning - tumbling variance ratio



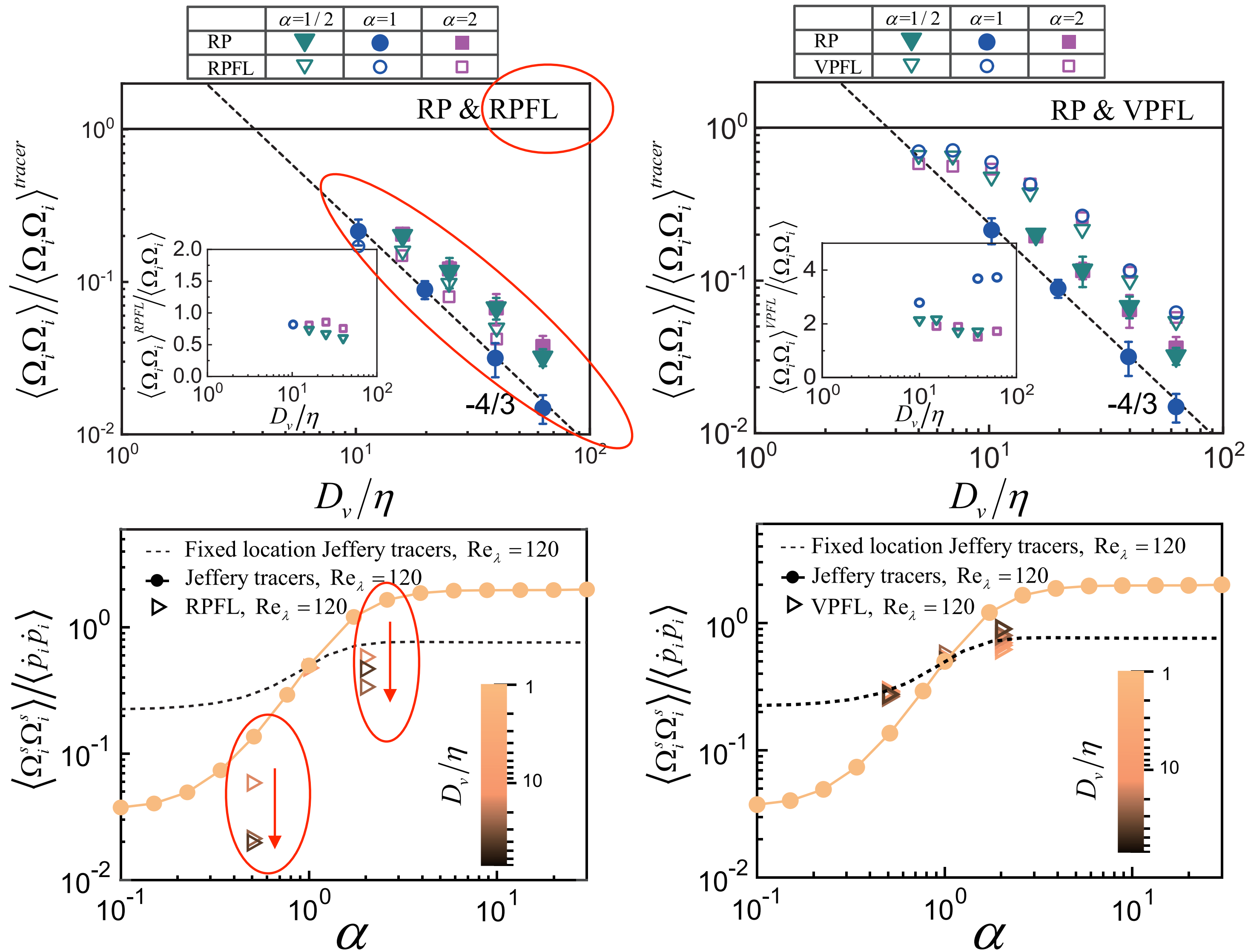
VA and VP do not explain the trends



Spinning - tumbling variance ratio

Real Particles Fixed Location

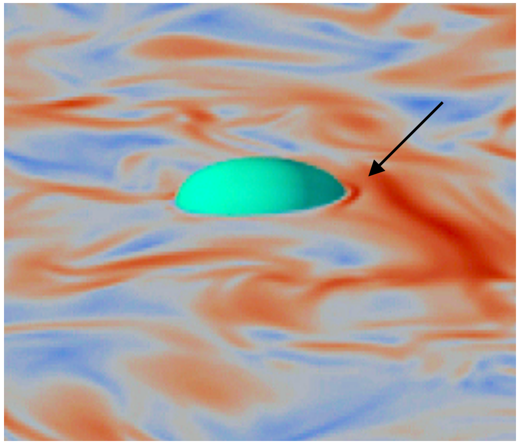
Virtual Particles Fixed Location



Spinning - tumbling variance ratio

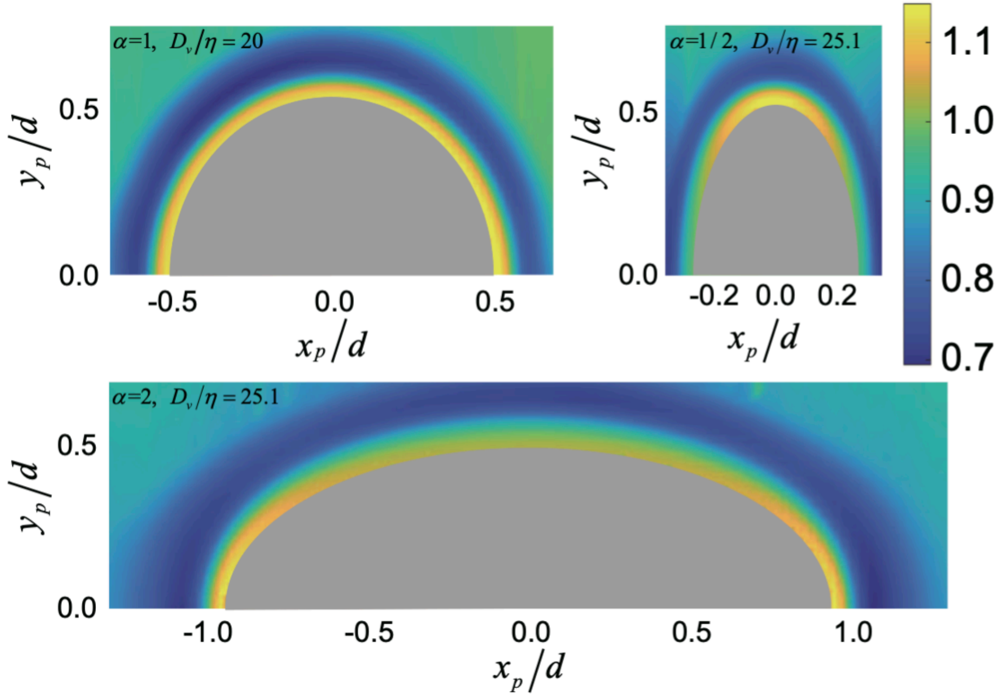
A boundary layer effect?

$$Re_p = \frac{|\mathbf{u}-\mathbf{v}|D_v}{\nu} \simeq \frac{D_v}{\lambda} Re_\lambda \sim O(10^2)$$



Dissipation field surrounding
a fully resolved particle

$$\frac{1}{4}\overline{\omega^2}/\overline{S:S}$$



Overall increase in strain-rate as compared to vorticity —> this promote tumbling

Cisse, Homann & Bec JFM 2013

Summary / conclusion

Acceleration

Size-dependence explained by **fluid acceleration volume averages** (VA or VP)
Amplitude better accounted for **dynamical** (Lagrangian) **volume averages** (VP)

Importance of **flow structures**: preferential alignment even for inertial-scale particles!

Rotation

Size-dependence explained in terms of volume averages but not its **amplitude**

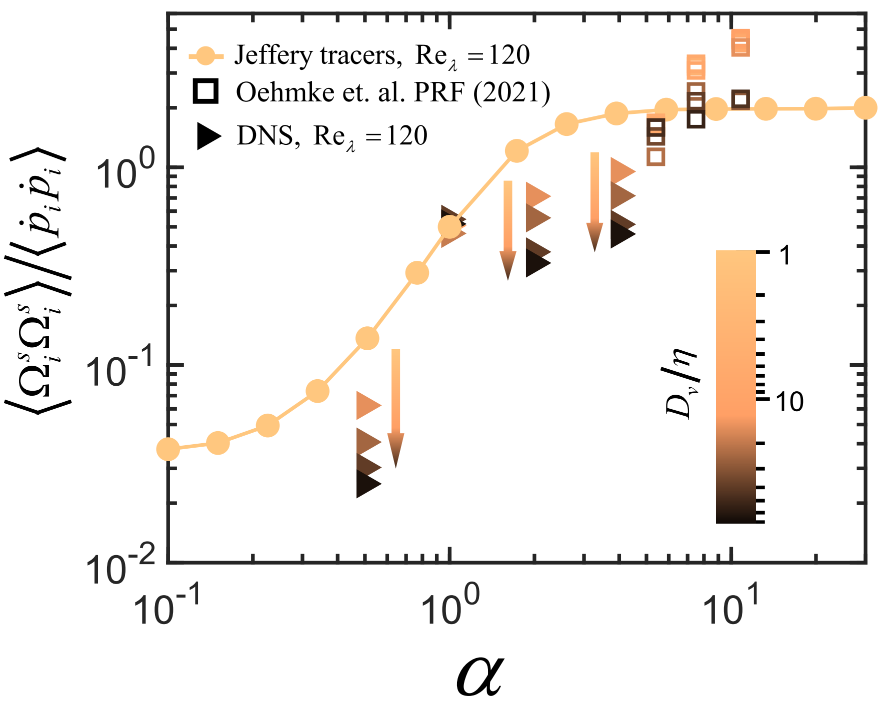
Importance of **two-way coupling** and **boundary layers**: decrease of spinning vs. tumbling!

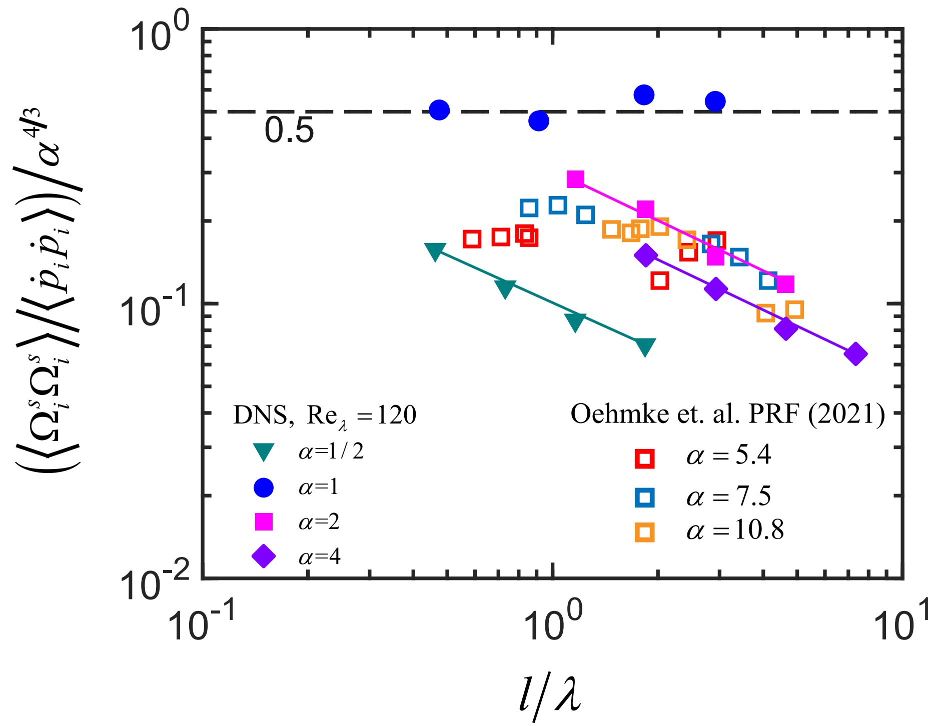
Experimental verifications are needed!

Dynamics of finite-size spheroids in turbulent flow: the roles of flow structures and particle boundary layers,
L. Jiang, C. Wang, S. Liu, C. Sun, E. Calzavarini, [http://arxiv.org/abs/2202.03937] **J. Fluid Mech** (2022)

Spinning - tumbling variance ratio

Comparison with Oehmke, Bordoloi, Variano, Verhille PRF (2021)





More experimental measures of tumbling & spinning needed.