Numerical study of Non-Oberbeck-Boussinesq effects on the heat transport in turbulent Rayleigh-Bénard convection in liquids

Kazuyasu Sugiyama[1], Enrico Calzavarini[1] and Detlef Lohse[1]

[1]Department of Applied Physics, University of Twente, 7500 AE Enschede, The Netherlands. `k.sugiyama@tnw.utwente.nl, e.calzavarini@tnw.utwente.nl, d.lohse@tnw.utwente.nl`

1 Introduction

In most studies on the Rayleigh-Bénard (RB) convection, the Oberbeck-Boussinesq (OB) approximation is employed, i.e., fluid material properties are assumed to be independent of temperature except for the density in the buoyancy term which is taken to be linear in T. However, in real fluids if the temperature difference, Δ, between the bottom and top plates is chosen to be large, deviations from the OB approximation may be relevant.

Here, we investigate broken symmetry features and heat-flux modifications due to the Non-Oberbeck-Boussinesq (NOB) effects in the two-dimensional RB turbulence for water and for glycerol. We perform direct numerical simulations (DNS) by solving the equation set for liquid, which consists of the incompressible ($\partial_i u_i = 0$) Navier-Stokes equation

$$\rho_m(\partial_t u_i + u_j \partial_j u_i) = -\partial_i p + \partial_j(\eta(\partial_j u_i + \partial_i u_j)) + g\,(\rho_m - \rho)\,\delta_{i3}, \quad (1)$$

and the heat-transfer equation

$$\rho_m c_{p,m}(\partial_t T + u_j \partial_j T) = \partial_j(\Lambda \partial_j T). \quad (2)$$

The dynamic viscosity, $\eta(T)$, the heat conductivity, $\Lambda(T)$, and the temperature dependent density in the buoyancy term, $\rho(T)$, are all temperature dependent, with given empirical relations [1]. As justified in [1], we assume the density and the isobaric specific heat capacity c_p in the material time derivative terms to be constant, their values (ρ_m and $c_{p,m}$) are set at the mean temperature T_m among the bottom and top plates . We vary the Rayleigh number Ra up to 10^8 and the level of the *non-Boussinesqness* Δ up to 60K. Comparison with a recent NOB Boundary Layer theory (NOB-BL) [1] is on the focus of the present study.

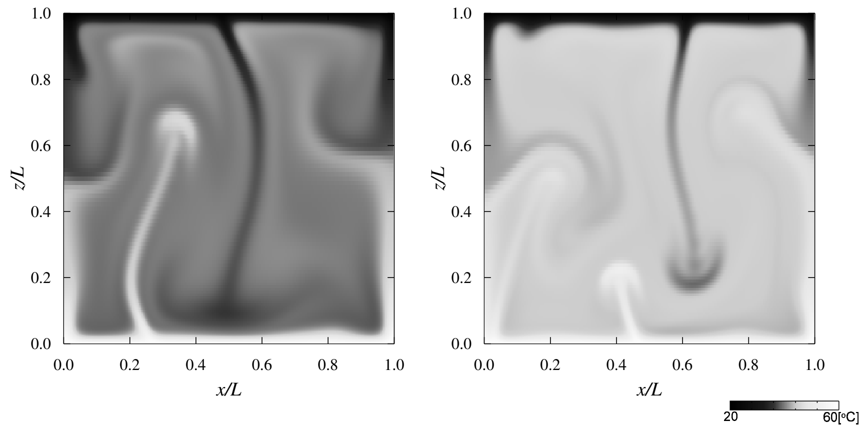

Fig. 1. Snapshots of the temperature field for glycerol at $Ra = 10^7$ and $T_m = 40^o$C. The left panel corresponds to the OB case, the right one to the NOB case with $\Delta = 40$K. Temperature color scale is the same in the two panels.

2 Results and discussion

Typical temperature snapshots from glycerol simulations are shown in figure 1. As already observed in experiments [1, 2], NOB convection is characterized by a sensible enhancement of the bulk temperature and top-bottom asymmetric variations of the thermal BLs.

In figure 2 we show the behavior of the center temperature T_c as a function of the thermal gap Δ both for water and glycerol. For water, interestingly enough, T_c beyond $Ra = 10^5$ is rather independent of Ra and shows good agreement with NOB-BL theory. In glycerol instead, only a qualitative trend is attained by the NOB-BL prediction.

The mean heat-flux behavior in NOB case as compared to OB is addressed by looking at the Nusselt ratio behavior Nu_{NOB}/Nu_{OB} at changing the level of non-Boussinesqness (Δ). In particular, we decompose Nu_{NOB}/Nu_{OB} into the product of two terms, corresponding to different effects. (i) the relative change of the thermal BL thicknesses λ^{sl} based on the temperature slope at the plate

$$F_\lambda = 2\lambda^{sl}_{OB}/(\lambda^{sl}_t + \lambda^{sl}_b), \tag{3}$$

and (ii) the T_c shift

$$F_\Delta = (\kappa_t(T_c - T_t) + \kappa_b(T_b - T_c))/(\kappa_m \Delta), \tag{4}$$

where the subscripts t and b represent the top and bottom plates respectively. The relations between $Nu_{NOB}/Nu_{OB} = F_\lambda \cdot F_\Delta$, F_λ, F_Δ and Δ at $Ra = 10^8$ are reported in figure 3. Particularly for water, the DNS results are in quite good agreement with experiments [1], which were indeed carried out at higher Rayleigh numbers ($10^8 \leq Ra \leq 10^{10}$). In the water case $F_\lambda \approx 1$, which is a basic phenomenological assumption of NOB-BL theory, appears to be here a good approximation. $F_\lambda \approx 1$ indicates that the NOB Nusselt number variation in water is mainly due to the T_c shift. On the other hand, for glycerol, the Nusselt number modification is governed by the change of boundary layers, and F_λ sensibly depends on Δ.

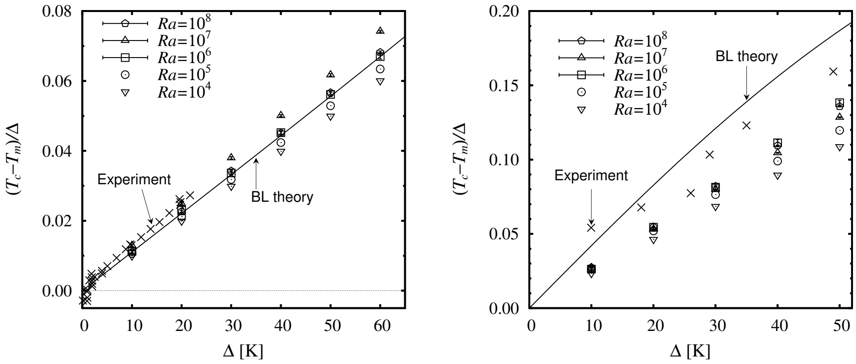

Fig. 2. Deviation of the center (T_c) from the mean (T_m) temperature normalized by the thermal gap (Δ), $(T_c - T_m)/\Delta$ versus Δ for water (left) and for glycerol (right), both at fixed $T_m = 40^oC$ for various values of Ra. The symbols $\times$ show the available experimental data for water [1] and for glycerol [2]. The line is NOB-BL prediction.

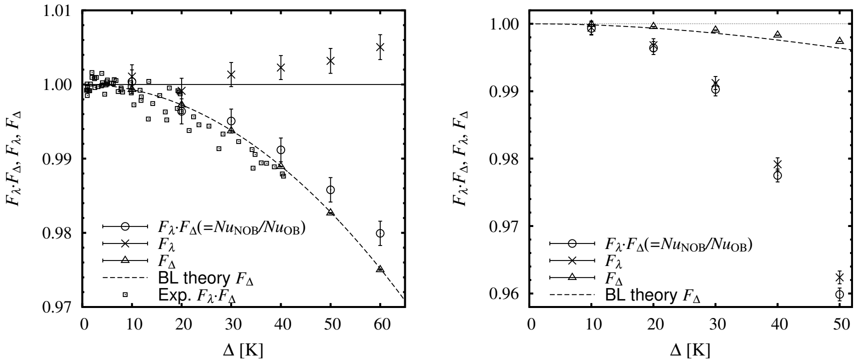

Fig. 3. Nusselt number ratio $Nu_{NOB}/Nu_{OB} = F_\lambda \cdot F_\Delta$, F_λ, and F_Δ versus Δ for water (left) and for glycerol (right), both at fixed $Ra = 10^8$ and $T_m = 40°$C. Symbols $\bigcirc$, $\times$ and $\triangle$ show the DNS results. Lines show the NOB-BL prediction. On right panel also the experimental data for water ($\boxdot$) are shown [1].

References

1. Ahlers, G., Brown, E., Fontenele Araujo, F., Funfschilling, D., Grossmann, S. and Lohse, D.: J. Fluid Mech. **569**, 409–445 (2006).
2. Zhang, J., Childress, S. and Libchaber, A.: Phys. Fluids **9**, 1034–1042 (1997).