Rayleigh and Prandtl number scaling in the bulk of Rayleigh–Bénard turbulence

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The Ra and Pr number scaling of the Nusselt number Nu, the Reynolds number Re, the temperature fluctuations, and the kinetic and thermal dissipation rates is studied for (numerical) homogeneous Rayleigh–Bénard turbulence, i.e., Rayleigh–Bénard turbulence with periodic boundary conditions in all directions and a volume forcing of the temperature field by a mean gradient. This system serves as model system for the bulk of Rayleigh–Bénard flow and therefore as model for the so-called "ultimate regime of thermal convection." With respect to the Ra dependence of Nu and Re we confirm our earlier results [D. Lohse and F. Toschi, "The ultimate state of thermal convection," Phys. Rev. Lett. **90**, 034502 (2003)] which are consistent with the Kraichnan theory [R. H. Kraichnan, "Turbulent thermal convection at arbitrary Prandtl number," Phys. Fluids **5**, 1374 (1962)] and the Grossmann–Lohse (GL) theory [S. Grossmann and D. Lohse, "Scaling in thermal convection: A unifying view," J. Fluid Mech. **407**, 27 (2000); "Thermal convection for large Prandtl number," Phys. Rev. Lett. **86**, 3316 (2001); "Prandtl and Rayleigh number dependence of the Reynolds number in turbulent thermal convection," Phys. Rev. E **66**, 016305 (2002); "Fluctuations in turbulent Rayleigh–Bénard convection: The role of plumes," Phys. Fluids **16**, 4462 (2004)], which both predict $Nu \sim Ra^{1/2}$ and $Re \sim Ra^{1/2}$. However the Pr dependence within these two theories is different. Here we show that the numerical data are consistent with the GL theory $Nu \sim Pr^{1/2}$, $Re \sim Pr^{-1/2}$. For the thermal and kinetic dissipation rates we find $\epsilon_\theta/(\kappa\Delta^2 L^{-2}) \sim (Re\,Pr)^{0.87}$ and $\epsilon_u/(\nu^3 L^{-4}) \sim Re^{2.77}$, both near (but not fully consistent) the bulk dominated behavior, whereas the temperature fluctuations do not depend on Ra and Pr. Finally, the dynamics of the heat transport is studied and put into the context of a recent theoretical finding by Doering *et al.* ["Comment on ultimate state of thermal convection" (private communication)]. © *2005 American Institute of Physics*. [DOI: 10.1063/1.1884165]

I. INTRODUCTION

The scaling of large Rayleigh number (Ra) Rayleigh–Bénard (RB) convection has attracted tremendous attention in the last two decades.[1–50] There is increasing agreement that, in general, there are no clean scaling laws for Nu(Ra,Pr) and Re(Ra,Pr), apart from asymptotic cases. One of these asymptotic cases has been doped the "ultimate state of thermal convection,"[51] where the heat flux becomes independent of the kinematic viscosity ν and the thermal diffusivity κ. The physics of this regime is that the thermal and kinetic boundary layers have broken down or do not play a role any more for the heat flux and the flow is bulk dominated. The original scaling laws suggested for this regime are[51]

$$Nu \sim Ra^{1/2}(\ln Ra)^{-3/2}Pr^{1/2}, \tag{1}$$

$$Re \sim Ra^{1/2}(\ln Ra)^{-1/2}Pr^{-1/2} \tag{2}$$

for $Pr < 0.15$, while for $0.15 < Pr \lesssim 1$,

$$Nu \sim Ra^{1/2}(\ln Ra)^{-3/2}Pr^{-1/4}, \tag{3}$$

$$Re \sim Ra^{1/2}(\ln Ra)^{-1/2}Pr^{-3/4}. \tag{4}$$

The Grossmann–Lohse (GL) theory also gives such an asymptotic regime which is bulk dominated and where the plumes do not play a role[4] (regimes IV_l and IV'_l of Refs. 1–4). Apart from logarithmic corrections, it has the same Ra dependence as in Eqs. (1)–(4), but different Pr dependence, namely,

$$Nu \sim Ra^{1/2}Pr^{1/2}, \tag{5}$$

$$\text{Re} \sim \text{Ra}^{1/2}\text{Pr}^{-1/2}. \quad (6)$$

The same scaling relation of Eq. (5) first appeared in the paper by Spiegel on thermal convection in stars,[52] where it was proposed on the basis of the hypothesis that in high-turbulent conditions the dimensional heat flux shall be independent both of kinematic viscosity and thermal diffusivity. As a model of the ultimate regime we had suggested[53] homogeneous RB turbulence, i.e., RB turbulence with periodic boundary conditions in all directions and a volume forcing of the temperature field by a mean gradient,[54]

$$\frac{\partial\theta}{\partial t} + (\boldsymbol{u}\cdot\boldsymbol{\partial})\theta = \kappa\partial^2\theta + \frac{\Delta}{L}u_z. \quad (7)$$

Here $\theta = T + (\Delta/L)z$ is the deviation of the temperature from the linear temperature profile $-(\Delta/L)z$. The velocity field $\boldsymbol{u}(\boldsymbol{x},t)$ obeys the standard Boussinesq equation,

$$\frac{\partial\boldsymbol{u}}{\partial t} + (\boldsymbol{u}\cdot\boldsymbol{\partial})\boldsymbol{u} = -\boldsymbol{\nabla} p + \nu\partial^2\boldsymbol{u} + \beta g\hat{z}\theta. \quad (8)$$

Here, β is the thermal expansion coefficient, g the gravity, p the pressure, and $\theta(\boldsymbol{x},t)$ and $u_i(\boldsymbol{x},t)$ are temperature and velocity field, respectively. This model has been previously studied by Borue and Orszag[55] by means of a spectral numerical simulation with built-in hyperviscosity. They focused mainly on turbulent spectra and second-order correlation functions behavior, but not on scaling of integral quantities with respect to Ra and Pr. Actually, their results suggested a dependency of dimensional heat flux Q on the Ra number which was not compatible with the asymptotic predictions (1), (3), and (5). Furthermore they noticed for the first time large scale structures in the temperature field (called "jets" in that paper) similar to the ones we observe in our simulation.

How to connect the homogeneous Rayleigh–Bénard system studied here with the standard top/bottom bounded Rayleigh–Bénard system, and, in particular, how to connect the respective Rayleigh numbers? We stress that such a relation is nontrivial. Let us denote the standard Rayleigh number of the top/bottom bounded system with temperature difference Δ_{tb} between the top and bottom wall Ra_{tb}. We define the *bulk-Rayleigh* Ra_{bulk} for the bulk with the temperature drop Δ_{bulk} across the bulk and the bulk height $H-2\lambda_\theta$, where λ_θ is the thickness of the thermal boundary layer. Then Ra_{bulk} and Ra_{tb} are related through

$$\text{Ra}_{bulk} \equiv \frac{\Delta_{bulk}}{\Delta_{tb}}\left(1 - \frac{2\lambda_\theta}{H}\right)^3 \text{Ra}_{tb} \simeq \frac{\Delta_{bulk}}{\Delta_{tb}}\text{Ra}_{tb}. \quad (9)$$

The ratio λ_θ/H becomes rapidly negligible for highly turbulent conditions. We think that from the available experimental and numerical data it is difficult to extract the $\Delta_{bulk}/\Delta_{tb}$ dependency on Ra_{tb} (or alternatively the ratio among mean thermal gradient in the bulk respect to the imposed thermal gradient vs Ra_{tb}), we only can guess that, if it is constant, it shall be a very small number of order 10^{-2} or less, as can be deduced, for example, from Ref. 23, Fig. 3(b). Trying to relate the bulk-Rayleigh number to Ra_{tb} for a complete cell is out of the scope of the present work. Nevertheless it is worthwhile to note that if in the asymptotic limit the relation linking Ra_{tb} to Ra_{bulk} reveals to be nonlinear, the global results presented for the homogenous Rayleigh–Bénard model cannot be directly translated to a real RB cell. Indeed, in Ref. 53 we showed that the numerical results from Eqs. (7) and (8) are consistent with the suggested[51,1–4] Ra dependence of Nu and Re, $\text{Nu} \sim \text{Ra}^{1/2}$ and $\text{Re} \sim \text{Ra}^{1/2}$. However, the Pr dependences of Nu and Re, for which the predictions of Kraichnan[51] and GL (Refs. 1–4) are different, has not yet been tested for homogeneous turbulence: this is the first aim of this paper (Sec. III). Section II contains details of the numerics. In Sec. IV we study the bulk scaling laws for the thermal and kinetic dissipation rates and compare them with the GL theory. In that section we study the temperature fluctuations $\theta' = \langle\theta^2\rangle^{1/2}$. The dynamics of the flow, including $\text{Nu}(t)$ and its PDF (probability density function), is studied in Sec. V and put into the context of a recent analytical finding by Doering and co-workers.[56] Section VI contains our conclusions.

II. DETAILS OF THE NUMERICS

Our numerical simulation is based on a lattice Boltzman equation algorithm on a cubic 240^3 grid. The same scheme and resolution has already been used in Refs. 54 and 57. We run two sets of simulations in statistically stationary conditions. The first fixed at $\text{Pr}=1$ varying the Ra number between 9.6×10^4 and 1.4×10^7. The second fixed at $\text{Ra}=1.4\times10^7$. This, the highest value we can reach at the present resolution, was studied for five different Pr numbers, 1/10, 1/3, 1, 3, and 4. We recorded shortly spaced time series of Nu and root mean squared (rms) values of temperature and velocity, and we stored a collection of the whole field configurations with a coarse time spacing. The length of each different run ranges between 64 and 166 eddy turnover times. Our simulation was performed on an APEmille machine in a 128 processor configuration.[58,59] Each eddy turnover time requires on average 4 h of computation. The total computational time required for the whole set of simulations is roughly 150 days. The total number of stored configurations is around 2000.

III. Nu(Ra,Pr) AND Re(Ra,Pr)

The Nusselt number is defined as the dimensionless heat flux

$$\text{Nu} = \frac{1}{\kappa\Delta L^{-1}}[\langle u_3 T\rangle_{A,t}(z) - \kappa\langle\partial_3 T\rangle_{A,t}(z)] = \frac{\langle u_3\theta\rangle_{A,t}(z)}{\kappa\Delta L^{-1}} - 1, \quad (10)$$

where the average $\langle\cdots\rangle_{A,t}$ is over a horizontal plane and over time. From Eqs. (7)–(10) one can derive two exact relations for the volume averaged thermal dissipation rate $\epsilon_\theta = \kappa\langle(\partial_i\theta)^2\rangle_V$ and the volume averaged kinetic dissipation rate $\epsilon_u = \nu\langle(\partial_i u_j)^2\rangle_V$, namely,

$$\epsilon_u = \frac{\nu^3}{L^4}\text{Nu Ra Pr}^{-2}, \quad (11)$$

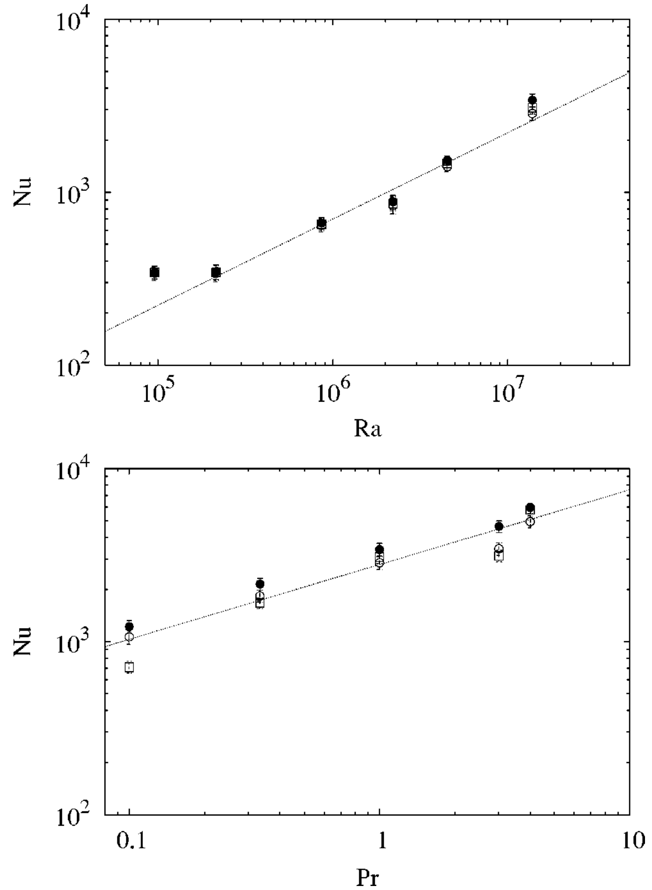

FIG. 1. (a) Nu(Ra) for Pr=1, computed in three different ways: (●) using Eq. (10), (□) using Eq. (11), and (○) from Eq. (12). The power law fits, performed on the mean value of the three different estimates and for $\mathrm{Ra}>10^5$, give a slope 0.50 ± 0.05. (b) Nu(Pr) for $\mathrm{Ra}=1.4\times10^7$, fit performed as before, with a resulting slope of 0.43 ± 0.07.

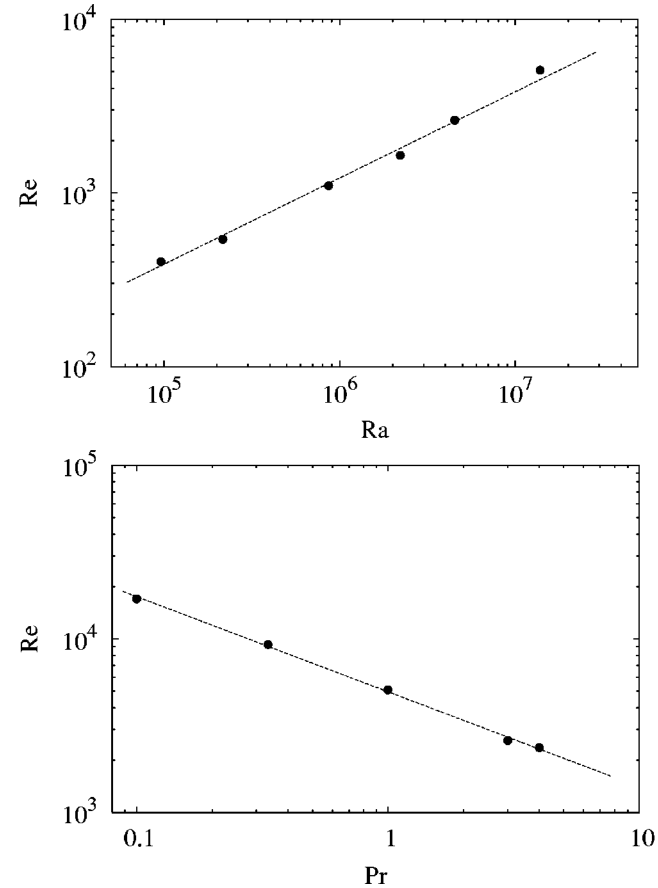

FIG. 2. (a) Re(Ra) for Pr=1, with a fitted slope 0.50 ± 0.02. (b) Re(Pr) for $\mathrm{Ra}=1.4\times10^7$, with a fitted slope of -0.55 ± 0.01.

$$\epsilon_\theta = \kappa\frac{\Delta^2}{L^2}\mathrm{Nu}. \tag{12}$$

One can therefore numerically compute Nu in three different ways: (i) from its direct definition (10), (ii) from the volume averaged kinetic dissipation rate (11), and (iii) from the volume averaged thermal dissipation rate (12).

The results are shown in Fig. 1(a) as a function of Ra for Pr=1. There is very good agreement of Nu obtained from the three different methods for all Ra, giving us further confidence in the convergence of the numerics. If we fit all data points beyond $\mathrm{Ra}=10^5$ with an effective power law, we obtain $\mathrm{Nu}\sim\mathrm{Ra}^{0.50\pm0.05}$, consistent with the asymptotically expected law $\mathrm{Nu}\sim\mathrm{Ra}^{1/2}$.[61]

In Fig. 1(b) we display Nu as function of Pr for fixed $\mathrm{Ra}=1.4\times10^7$. For the cases with $\mathrm{Pr}\neq1$ the convergence of the three different methods to calculate Nu is not perfect. This may be due to numerical errors in the resolution of the small scale differences, especially when ν and κ are considerably different. However, one can clearly notice a strong increase of Nu with Pr. A fit with an effective power law gives $\mathrm{Nu}\sim\mathrm{Pr}^{0.43\pm0.07}$, which is consistent with the asymptotic power law $\mathrm{Nu}\sim\mathrm{Pr}^{1/2}$ suggested by the GL theory and by the small Pr regime (1) proposed by Kraichnan, but not with Kraichnan's large Pr regime (3). Increasing Pr further (at fixed Ra) the flow will eventually laminarize, i.e., can no longer be considered as model system for the bulk of turbulence. This also follows from Fig. 2(b), in which we show the Reynolds number

$$\mathrm{Re} = \frac{u'L}{\nu} \tag{13}$$

as function of Pr for fixed $\mathrm{Ra}=1.4\times10^7$. Note that this is the *fluctuation* Reynolds number, defined by the rms velocity fluctuation $u'=\langle u^2\rangle^{1/2}$: in homogeneous RB no large scale wind exists. Re(Pr) displays an effective scaling law $\mathrm{Re}\sim\mathrm{Pr}^{-0.55\pm0.01}$, consistent with the GL prediction $\mathrm{Pr}^{-1/2}$ for the ultimate regime (if one identifies the wind Reynolds number in GL with the fluctuation Reynolds number here) and also with the Kraichnan prediction (2). Also the Ra scaling of Re is consistent with GL (and also with Kraichnan), $\mathrm{Re}\sim\mathrm{Ra}^{1/2}$, as seen from Fig. 2(a) and as already shown in Ref. 53.

IV. SCALING LAWS FOR ϵ_u,ϵ_θ AND THE TEMPERATURE FLUCTUATIONS

A. Kinetic and thermal dissipations

The homogeneous RB turbulence offers the opportunity to numerically test one of the basic assumptions of the GL theory, namely, that the energy dissipation rate in the bulk scales like

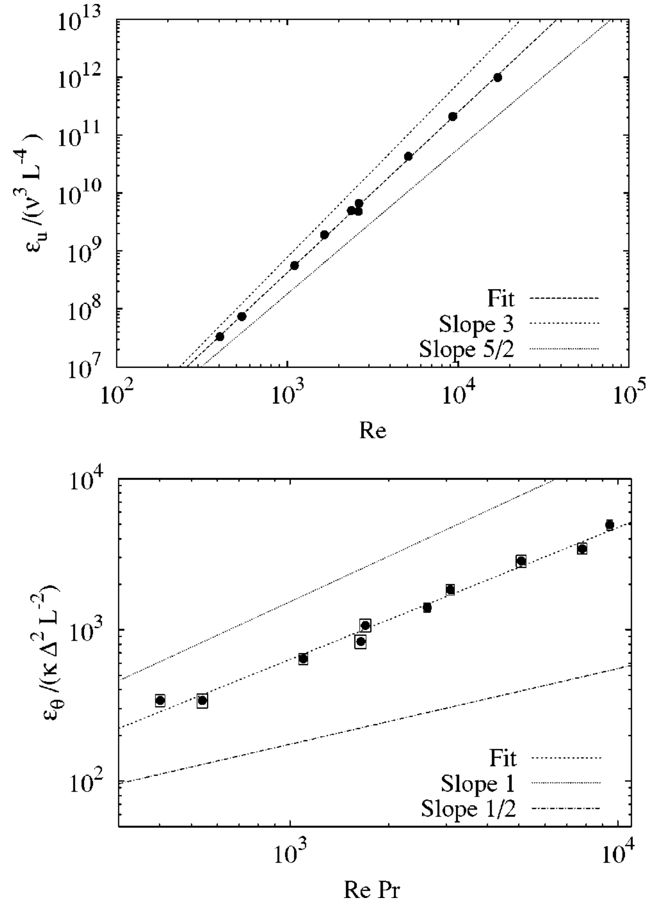

FIG. 3. (a) We show $\epsilon_u/(\nu^3 L^{-4})$ vs Re. The fit gives a slope of 2.77 ± 0.03, slope 3 and 5/2 are shown for comparison. (b) $\epsilon_\theta/(\kappa\Delta^2 L^{-2})$ vs Re Pr. We obtained a fitted slope of 0.87 ± 0.04 while slopes 1 and 1/2 are also shown for comparison.

$$\epsilon_{u,\text{bulk}} \sim \frac{\nu^3}{L^4}\text{Re}^3. \tag{14}$$

In contrast if energy dissipation is dominated by the boundary layer (BL) region GL predicts the $\epsilon_{u,\text{BL}} \sim \nu^3 L^{-4}\text{Re}^{5/2}$ behavior. In Fig. 3(a) we plot $\epsilon_u/(\nu^3 L^{-4})$ vs Re for all Ra and Pr, and find $\epsilon_u/(\nu^3 L^{-4}) \sim \text{Re}^{2.77\pm0.03}$ closer to the expectation (14) but not fully compatible with it.

The disentanglement of the thermal dissipation rate ϵ_θ into two different scaling contributions is, in principle, less straightforward. The GL theory decombines

$$\epsilon_\theta/(\kappa\Delta^2 L^{-2}) = c_3(\text{Re Pr})^{1/2} + c_4(\text{Re Pr}), \tag{15}$$

where the first term has been interpreted as boundary layer and plume contribution $\epsilon_{\theta,\text{pl}}$ and the second one as background contribution $\epsilon_{\theta,\text{bg}}$.[4] The prefactors c_3 and c_4 are given in Ref. 4. Plumes are interpreted as detached boundary layer.[4] Our simulation gives, see Fig. 3(b), the $\epsilon_\theta/(\kappa\Delta^2 L^{-2}) \sim (\text{Re Pr})^{0.87\pm0.04}$ behavior again closer to the background scaling, but not fully compatible with it, and consistent with the kinetic energy dissipation result. These unexpected deviations from the bulk behavior can be due to the presence of layers characterized by strong gradients both in the velocity and in the temperature field, i.e., to the formation of dynamical BL in the flow. It has been already observed in Ref. 55 and it is confirmed by our visualizations that the temperature field patterns often lead to the appearance of some nearby large vertical jets (see Sec. V of this paper). These jets are associated to the formation of strong vertical temperature gradients ($\partial_3\theta$) on the surfaces at their boundaries. We think this feature of homogeneous convection is of basic importance to explain the observed deviations from pure bulk scaling.

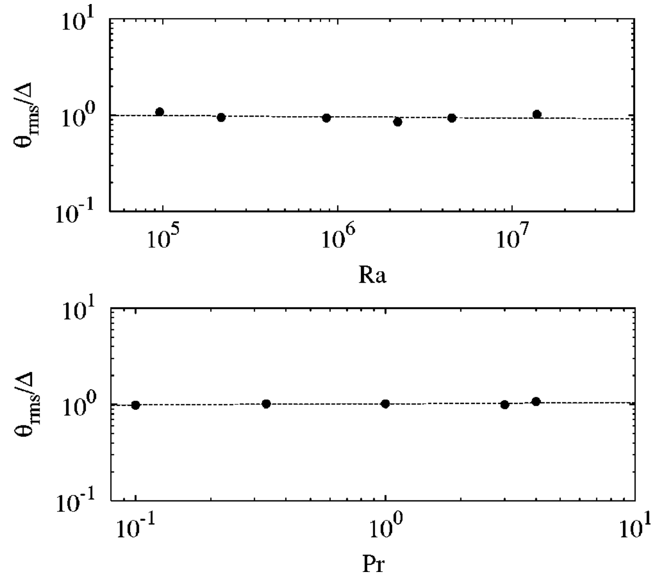

FIG. 4. (a) Normalized temperature variance θ'/Δ vs Ra at fixed Pr=1. (b) θ'/Δ vs Pr at fixed $\text{Ra}=1.4\times10^7$.

B. Temperature fluctuations

In our numerics we find the temperature fluctuations $\theta' = \langle\theta^2\rangle^{1/2}$ to be independent from Ra and Pr, see Fig. 4. These figures show that we have $\theta' \simeq \Delta$ for all Ra and Pr within our numerical precision. In contrast, Ref. 4 predicted a dependence of the thermal fluctuations on both Ra and Pr, namely, $\theta'/\Delta \sim (\text{Pr Ra})^{-1/8}$ for the regimes IV_l and IV_l' which correspond to the bulk of turbulence analyzed here. Our interpretation of Fig. 4 is that the bulk turbulence only has one temperature scale, namely, Δ. For real RB turbulence it is the boundary layer dynamics which introduces further temperature scales, leading to the Ra and Pr number dependence of the temperature fluctuations observed in experiments.[5,16,47,48]

V. DYNAMICS OF THE FLOW

In this section we provide an insight into the dynamics of the periodic Rayleigh–Bénard flow. A bidimensional vertical snapshot of the flow is shown in Fig. 5. Already from this pictorial view the presence of an upward moving hot column and a downward moving cold column is clearly evident.

Indeed these large scale structure can be related to the presence of "elevator modes" (or jets, forming in the flow) growing in time until finally breaking down due to some instability mechanisms.

As proposed by Doering and co-workers in Ref. 56 it is possible to predict the presence of these modes directly start-

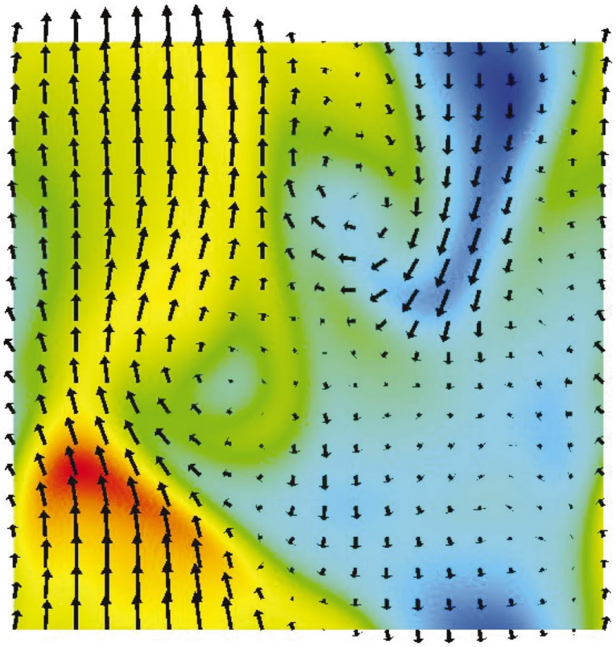

FIG. 5. (Color). Snapshot of the flow, at Ra$\sim 10^5$ showing elevator modes and jets. Here θ is shown in colors: red and yellow encode for positive values, with red greater in amplitude than yellow; green is for small values around zero; while blue stands for the negative values; dark blue stands for the more negative values. Velocity in the same plane is shown with arrows.

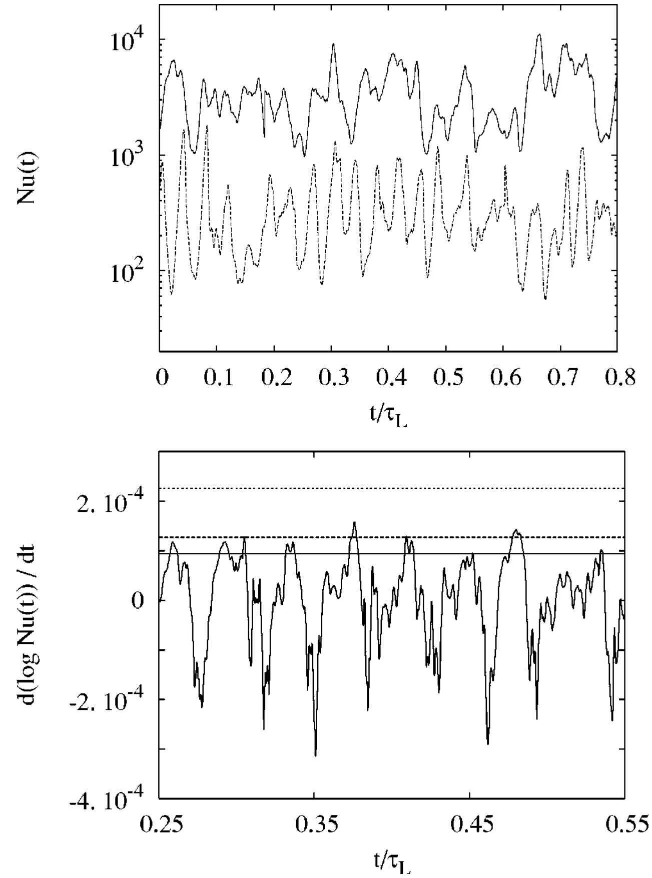

FIG. 6. (a) Time series Nu(t) for Ra$=1.4\times 10^7$ (top) and Ra$=9.6\times 10^4$ (bottom), in both case Pr$=1$. (b) Logarithmic derivative of Nu(t) for Ra $=9.6\times 10^4$, here reproduced only for a small time section of the data in (a). The series of horizontal lines represent the exponential rate of growing respectively (top to bottom) for the mode $\lambda(0,1)$, $\lambda(0,2)$, and $\lambda(1,2)$.

ing from Eqs. (7) and (8). Doering *et al.* showed that, due to the periodic boundary conditions, this coupled system of equations admits a particular solution $\theta=\theta_0 e^{\kappa\lambda t}\sin(\boldsymbol{k}\cdot\boldsymbol{x})$, $u_3 = u_0 e^{\kappa\lambda t}\sin(\boldsymbol{k}\cdot\boldsymbol{x})$, $u_2=u_1=0$, which is independent from the vertical coordinate z [here $\boldsymbol{k}=(k_x,k_y)$] and with

$$\lambda = -\frac{1}{2}(\mathrm{Pr}+1)k^2 + \frac{1}{2}\sqrt{(\mathrm{Pr}+1)^2k^4 + 4\,\mathrm{Pr}\left(\frac{\mathrm{Ra}}{L^4} - k^4\right)}. \tag{16}$$

From Eq. (16) one finds that the first unstable mode appears for $\mathrm{Ra}\geqslant \mathrm{Ra}_c=(2\pi)^4\sim 1558.54$, corresponding to the instability of the smallest possible wavenumber in the system, i.e., $k^2=(2\pi/L)^2 n^2$ with $n=(1,0)$.

The presence of accelerating modes with growth rate controlled by λ can also be seen from Fig. 6 where we show Nu(t) on log scale (notice the huge range over which Nu fluctuates), and its logarithmic derivative.

In Fig. 7 we show the PDF of Nu(t) which is strongly skewed towards large Nu values. This asymmetry reflects the periods of exponential growth (also visible in Fig. 6). As can be seen in Fig. 8, for all Ra and Pr the system typically spends 54% of the time in growing modes.

Also the relative fluctuations of Nu on the Ra and Pr numbers (see Fig. 9) seem to indicate no dependencies, at least in the range of parameters studied.

Despite the presence of exact exploding solutions, our system clearly shows that in the turbulent regime these solutions become unstable due to some yet to be explored instability mechanism. The interplay between exploding modes and destabilization sets the value of the Nusselt number, i.e., the heat transfer through the cell.

We stress that the study of the dynamics of the explosive solutions and of their successive collaps in a turbulent cell is crucial for the understanding the behavior of "integral" quantities, like, for example, the heat transfer.

VI. CONCLUSIONS

In conclusion, we confirmed that both the Ra and the Pr scaling of Nu and Re in homogeneous Rayleigh–Bénard convection are consistent with the suggested scaling laws of the Grossmann–Lohse theory for the bulk-dominated regime (regime IV_l of Refs. 1–3), which is the so-called ultimate regime of thermal convection. We also showed that the thermal and kinetic dissipations scale roughly as assumed in that theory. The temperature fluctuations do not show any Ra or Pr dependence for homogeneous Rayleigh–Benard convection. From the dynamics the heat transport and flow visualizations we identify elevator modes which are brought into the context of a recent analytical finding by Doering *et al.* In future work we plan to further clarify the flow organization and, in particular, the instability mechanisms of the elevator modes which set the Nusselt number in homogeneous RB flow and therefore presumably also in the ultimate regime of thermal convection.

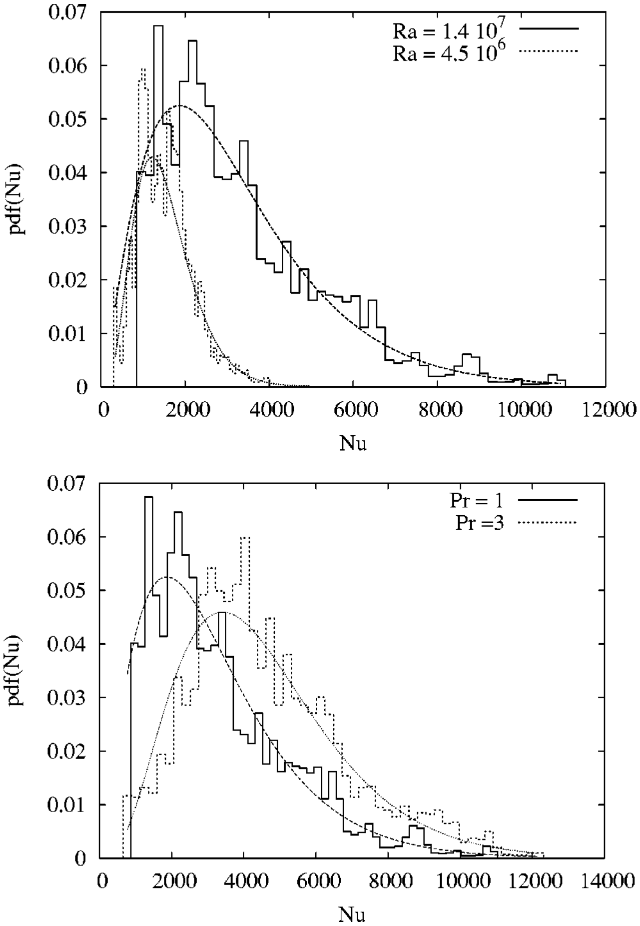

FIG. 7. (a) and (b) PDF of Nu(t) for different Ra and Pr. The superimposed curves correspond to a two parameter Γ distribution fit, $\mathrm{Nu}^a \exp(-b\,\mathrm{Nu})$ (Ref. 60).

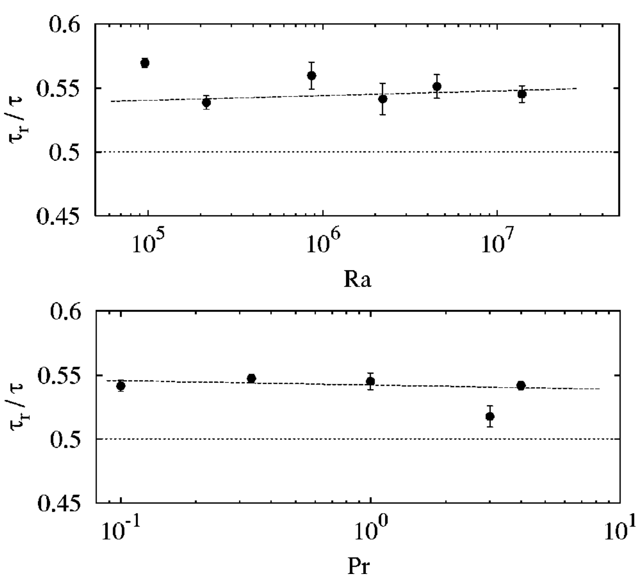

FIG. 8. Normalized rising time τ_r/τ as a function of (a) Ra for Pr=1 and (b) Pr for Ra=1.4×10^7. The time τ_r is the total time with positive slope of Nu(t), whereas the time τ is the total time of the run. The slope of the two fits in the shown graphs is compatible with zero, the overall mean value for τ_r/τ is 0.54.

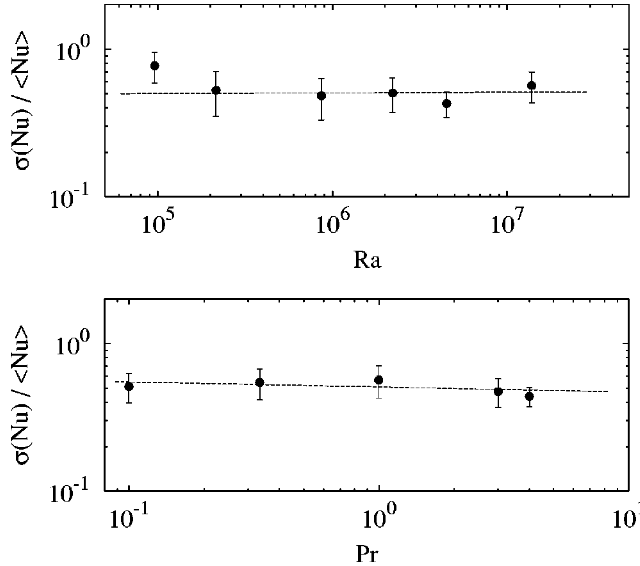

FIG. 9. Relative fluctuations $\sigma(\mathrm{Nu})/\langle \mathrm{Nu}\rangle$, where $\sigma(\mathrm{Nu}) \equiv \langle[\mathrm{Nu}(t) - \langle \mathrm{Nu}\rangle]^2\rangle^{1/2}$, as function of (a) Ra for Pr=1 and (b) Pr for Ra=1.4×10^7.

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[61] In our previous paper (Ref. 53) the overall magnitude of Nu was affected by a normalization error, hence all points of Fig. 1 of that paper should be multiplied by a factor 240 (corresponding to the grid size of our simulation). This of course does not affect the scaling exponent given there.