Effect of Faxén forces on acceleration statistics of material particles in turbulent flow

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The statistical description of particle dispersion in turbulent flow constitutes nowadays an active research area. Such a field is linked to the Lagrangian description of fluid turbulence and poses long standing questions on modeling of hydrodynamical forces on objects in unsteady nonuniform flows [1]. Even in the simplest conditions of highly diluted suspensions of particles whose sizes are smaller than the dissipative scale of the carrier flow, the presently available models are highly idealized. Therefore, theoretical results and predictions need systematic verifications with experimental measures. Particle tracking experiments are technically challenging for the high time and space resolution demanded and for the need to precisely estimate and control the intensity of turbulent flow. Our recent works have been devoted to comparisons between experiments and numerical simulations of simple dynamical particle models [2, 3]. We have widely investigated Lagrangian particle models evolving in a turbulent environment which is described from an Eulerian point of view. In this abstract we shortly review the methods employed: the models for particle dynamics together with the numerical methodology, and we detail on some recent progresses and results.

Lagrangian models for particle dynamics

Lagrangian models for particle dynamics build-up on the equation of motion for a parcel of fluid (fluid tracer) in a flow. The trajectory of such an ideal particle can be described by the equation $d\mathbf{x}/dt = \mathbf{u}(\mathbf{x}(t), t)$, where $\mathbf{x}(t)$ denotes tracer's position at time t and $\mathbf{u}$ the velocity of the fluid at that location. When a spherical massive particle (with diameter d much smaller than the dissipative scale η) is considered, the so called point-particle (PP) model can be employed:

$$\frac{d\mathbf{x}}{dt} = \boldsymbol{v}; \quad \frac{d\boldsymbol{v}}{dt} = \frac{3\ \rho_f}{\rho_f + 2\ \rho_p} \left(\frac{D\mathbf{u}}{Dt} + \frac{12\nu}{d^2} (\mathbf{u} - \boldsymbol{v}) \right), \tag{1}$$

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ρ_f and ρ_p denote respectively the fluid and the particle density, ν the fluid kinematic viscosity and $\boldsymbol{v}$ is the particle velocity. While $\mathbf{u}$ and $D\mathbf{u}/Dt$ are the fluid velocity and acceleration evaluated at the particle center postion. The above dynamical model, when coupled to Navier-Stokes equations to describe the evolution of $\mathbf{u}$, may be characterized by two dimensionless parameters, the modified density ratio $\beta = 3\rho_f/(\rho_f + 2\rho_p)$ and the Stokes number $St = \tau_p/\tau_\eta$, where $\tau_p = d^2/(12\beta\nu)$ is the particle response time and τ_η is dissipative time-scale of turbulence. Despite its simplicity - only Stokes drag force and inertial added-mass effects are accounted for - eq.(1) produces non trivial effects on particle concentrations, velocity and acceleration statistics [4]. When the particle size is larger than η - but still the slip velocity of the particle is much smaller than the mean fluid velocity - we have recently proposed that Faxén forces become relevant and must be included [3]:

$$\frac{d\mathbf{x}}{dt} = \boldsymbol{v}; \quad \frac{d\boldsymbol{v}}{dt} = \frac{3\ \rho_f}{\rho_f + 2\ \rho_p} \left(\left\langle \frac{D\mathbf{u}}{dt} \right\rangle_{V_p} + \frac{12\nu}{d^2} \left(\langle \mathbf{u} \rangle_{S_p} - \boldsymbol{v} \right) \right), \qquad (2)$$

where $\langle \ldots \rangle_{V_p}$ and $\langle \ldots \rangle_{S_p}$ denote volume and surface average over the (spherical) particle. These averages constitute the Faxén correction (FC) terms, which account for the non-uniformity of the flow at the particle-scale. Finally, we note that this modeling approach assumes the particles' induced perturbation on the fluid flow negligible, and also inter-particle collisions and gravity are neglected.

Eulerian-Lagrangian Numerical simulations

The Lagrangian particle models (1),(2) are evolved in a statistically stationary, homogeneous and isotropic turbulent flow. Turbulence is simulated on a periodic cubic domain by integrating Navier-Stokes (NS) equations forced by a large-scale volume term. In our numerical implementation NS is discretized on a regular grid, integrated using pseudo-spectral algorithm and second-order time marching scheme. The idea sustaining this approach is that small-scale Eulerian as well as Lagrangian statistics is universal, i.e., independent of the type of forcing applied at large scale, this has been recently tested in [5]. In order to validate and benchmarking our simulations we have performed a test on acceleration statistics of fluid tracers. We have run two independently developed codes which implements slightly different algorithms: i) Verlet time-stepping algorithm vs. second order Adams-Bashfort, ii) tri-cubic vs. tri-linear interpolation, iii) forcing term at constant energy input vs. constant energy at large-scales. We obtain an excellent level of universality for comparable turbulence levels (see fig. 1).

Results

Recent experimental studies on the acceleration of neutrally buoyant particles (ρ_p=ρ_f) have highlighted statistical effects linked to the particle size (d),

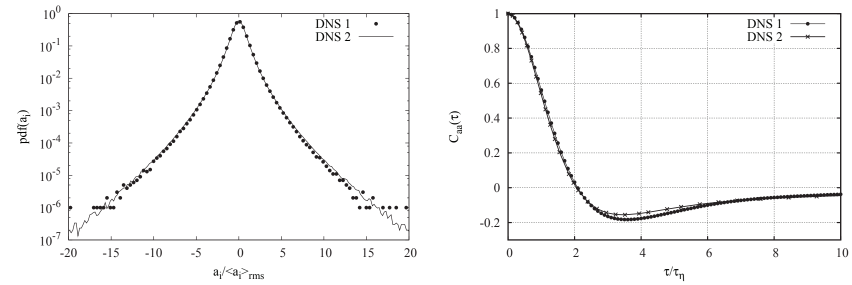

Fig. 1. Single-component acceleration p.d.f. (left) and time correlation function (right) for lagrangian fluid tracers. Results from two different simulations, with different large-scale forcing but comparable level of turbulence ($Re_\lambda = 50$ and 75).

namely a decrease of acceleration variance and increase of correlation times at increasing d, and independence on d of the probability density function of the acceleration once normalized by its variance [2, 6]. We showed that these effects are not captured by PP model (1), [2], while the FC one adds important and necessary physical corrections (see [3] for a detailed comparison with experimental data). The differences between PP and FC on the single-component acceleration r.m.s. value as a function of d/η - also when $\rho_p \neq \rho_f$ - are shown in fig. 2. Faxén terms always reduce the acceleration amplitude for particles larger than about 10η. Alternatively, one can look at the behavior of acceleration r.m.s. in the β-St parameter space. Fig. 3 shows the different behavior of $\langle a_i \rangle_{rms}$ values measured in PP and FC simulations for the same flow conditions. FC significatively reduce the acceleration values for particles lighter than the fluid, such as air bubbles in water. In fig. 4 we report the value of acceleration flatness, $F(a_i) = \langle a_i^4 \rangle / \langle a_i^2 \rangle^2$, showing that light particles ($\beta > 3$), which can have a highly intermittent statistics at small St both in PP and FC model, in the large St limit become gradually less intermittent only if Faxén corrections are included. However, the PP vs. FC acceleration scenario changes much less for heavier particles ($\beta < 1$). We wish these predictions to be tested against experimental measurements in the near future.

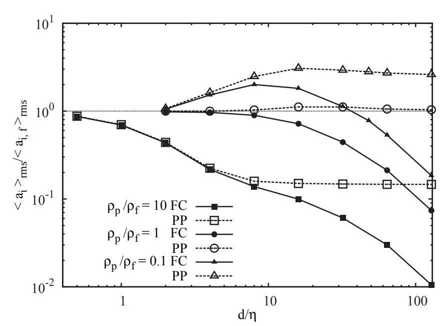

Fig. 2. Particle acceleration r.m.s. normalized by the fluid one, $\langle a_i \rangle_{rms} / \langle a_{i,f} \rangle_{rms}$, vs. d/η for particle types with densities $\rho_p/\rho_f = 0.1, 1, 10$, at $Re_\lambda = 75$.

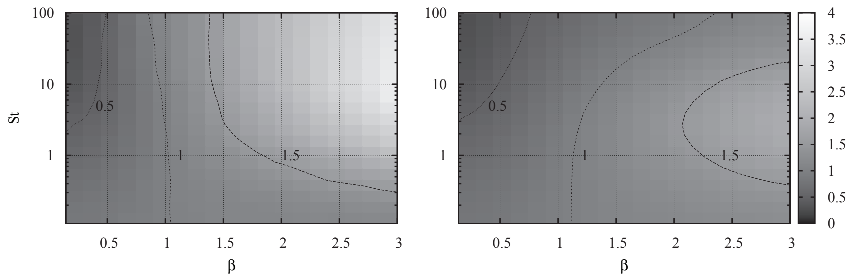

Fig. 3. $\langle a_i \rangle_{rms}/\langle a_{i,f} \rangle_{rms}$, in the β-St plane for PP (left) and FC (right) model. Isolines are traced at 0.5,1,1.5 values.

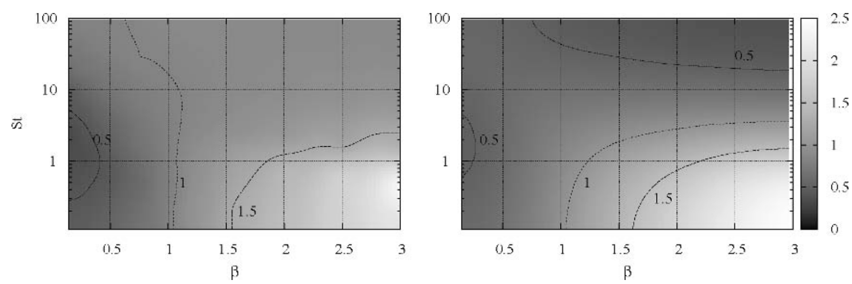

Fig. 4. Ratio of particles' to fluid acceleration flatness, $F(a_i)/F(a_{i,f})$, in the β-St plane for PP model (left) and FC (right). Isolines are traced at 0.5,1,1.5 values.

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