- EX.3

DIMENSIONNEMENT D'UNE CONDUITE $\Rightarrow 175,6\,D^4 = \dfrac{10+8D}{D}$

avec D en mètres

$175,6\,D^4 = \dfrac{10+8D}{D} \iff \underbrace{175,6\,D^5 - 8D - 10}_{f(D)} = 0$

$D \to x \Rightarrow f(x) = 0$

a) M. BISSECTION :

$x_n = \dfrac{a_n+b_n}{2}$; $I_n = [a_n, b_n]$

$f(a_n)\cdot f(x_n) < 0 \Rightarrow I_{n+1} = [\underset{a_{n+1}}{\underset{\parallel}{a_n}}, \underset{b_{n+1}}{\underset{\parallel}{x_n}}]$

autrement : $I_{n+1} = [a_{n+1}, b_{n+1}] = [x_n, b_n]$

$\varepsilon = 0,1$ (PRÉCISION) ~~XXXX~~

$I_0 = [a_0, b_0] = [0,3 ; 0,7]$

$x_0 = \dfrac{a_0+b_0}{2} = 0,5$

$f(a_0)f(x_0) > 0 \Rightarrow a_1 = x_0$; $b_1 = b_0$; $I_1 = [a_1, b_1] = [x_0, b_0] = [0,5 ; 0,7]$

$x_1 = \dfrac{a_1+b_1}{2} = 0,6$

$|x_1 - x_0| = 0,1 = \varepsilon$

$f(a_1)f(x_1) > 0 \Rightarrow a_2 = x_1$; $b_2 = b_1$; $I_2 = [a_2, b_2] = [x_1, b_1] = [0,6 ; 0,7]$

$x_2 = \dfrac{a_2+b_2}{2} = 0,65$

$|x_2 - x_1| = 0,05 < 0,1 \Rightarrow \boxed{x^* = D = 0,65\ m}$

CONVERGENCE

b) $\varepsilon = 10^{-3}\,m = 1\,mm$

$$n = \frac{\log|a_0 - b_0| - \log \varepsilon}{\log 2} \simeq 8,64 \Rightarrow \boxed{n = 9}$$

• <u>EX. 4</u>

a) $A = \begin{pmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{pmatrix}$ $\vec{q}_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $|\vec{q}| = \sup_i |q_i|$; $\varepsilon = 0,2$

$$\vec{v}_1 = A\vec{q}_0 = \begin{pmatrix} 2/3 \\ 1/3 \end{pmatrix} ; |\vec{v}_1| = \frac{2}{3} ; \vec{q}_1 = \begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix} ; \lambda_1^{(1)} = \begin{cases} \frac{(\vec{q}_1)_1}{(\vec{q}_0)_1} = 1 \\ \frac{(\vec{q}_1)_2}{(\vec{q}_0)_2} = / \end{cases}$$

$$\vec{v}_2 = A\vec{q}_1 = \begin{pmatrix} 5/6 \\ 2/3 \end{pmatrix} ; |\vec{v}_2| = \frac{5}{6} ; \vec{q}_2 = \begin{pmatrix} 1 \\ \frac{4}{5} \end{pmatrix} ; \lambda_1^{(2)} = \begin{cases} \frac{(\vec{q}_2)_1}{(\vec{q}_1)_1} = 1 \\ \frac{(\vec{q}_2)_2}{(\vec{q}_1)_2} = \frac{8}{5} = 1,6 \end{cases}$$

$$|\lambda_1^{(2)} - \lambda_1^{(1)}| = \begin{cases} 0 \\ 0,6 > \varepsilon \end{cases}$$

$$\vec{v}_3 = A\vec{q}_2 = \begin{pmatrix} \frac{2}{3} + \frac{4}{15} \\ \frac{1}{3} + \frac{8}{15} \end{pmatrix} = \begin{pmatrix} \frac{14}{15} \\ \frac{13}{15} \end{pmatrix} ; |\vec{v}_3| = \frac{14}{15} ; \vec{q}_3 = \begin{pmatrix} 1 \\ \frac{13}{14} \end{pmatrix} ; \lambda_1^{(3)} = \begin{cases} \frac{(\vec{q}_3)_1}{(\vec{q}_2)_1} = 1 \\ \frac{(\vec{q}_3)_2}{(\vec{q}_2)_1} \simeq 1,16 \end{cases}$$

$$|\lambda_1^{(3)} - \lambda_1^{(2)}| = \begin{cases} 0 \\ 0,44 > \varepsilon \end{cases}$$

$$\vec{v}_4 = A\vec{q}_3 = \begin{pmatrix} \frac{2}{3} + \frac{13}{14\cdot 3} \\ \frac{1}{3} + \frac{26}{14\cdot 3} \end{pmatrix} = \begin{pmatrix} \frac{41}{14\cdot 3} \\ \frac{40}{14\cdot 3} \end{pmatrix} ; |\vec{v}_4| = \frac{41}{14\cdot 3} ; \vec{q}_4 = \begin{pmatrix} 1 \\ \frac{40}{41} \end{pmatrix} ; \lambda_1^{(4)} = \begin{cases} \frac{(\vec{q}_4)_1}{(\vec{q}_3)_1} = 1 \\ \frac{(\vec{q}_4)_2}{(\vec{q}_3)_2} = \frac{40}{41} \cdot \frac{14}{13} \simeq \\ \simeq 1,05 \end{cases}$$

$$|\lambda_1^{(4)} - \lambda_1^{(3)}| = \begin{cases} 0 \\ 0,11 < \varepsilon \end{cases}$$ <u>CONVERGENCE</u> $\Rightarrow \boxed{\lambda_1 = 1}$

b) $\lambda_1 = \dfrac{\vec{q}^T \cdot A\vec{q}}{\vec{q}\cdot\vec{q}}$ COEFFICIENT (QUOTIENT) de RAYLEIGH

$$\lambda_1^{(0)} = \frac{\vec{q}_0^{\,T}\cdot A\vec{q}_0}{\vec{q}_0^{\,T}\cdot\vec{q}_0} = \frac{2/3}{1} = \frac{2}{3}$$

$$\lambda_1^{(1)} = \frac{\vec{q}_1^{\,T}\cdot A\vec{q}_1}{\vec{q}_1^{\,T}\cdot\vec{q}_1} = \frac{\frac{5}{6}+\frac{1}{3}}{1+\frac{1}{4}} = \frac{7}{\cancel{6}_3}\,\frac{\cancel{4}^{2}}{5} = \frac{14}{15}$$

$$|\lambda_1^{(1)} - \lambda_1^{(0)}| = \left|\frac{14}{15} - \frac{2}{3}\right| = \frac{14-10}{15} = \frac{4}{15} \cong 0{,}27 > \varepsilon$$

$$\lambda_1^{(2)} = \frac{\vec{q}_2^{\,T}\cdot A\vec{q}_2}{\vec{q}_2^{\,T}\cdot\vec{q}_2} = \frac{\frac{14}{15}+\frac{13}{15}\frac{4}{5}}{1+\frac{16}{25}} = \frac{\frac{14\cdot5+13\cdot4}{75}}{\frac{41}{25}} = \frac{122}{\cancel{75}_3}\,\frac{\cancel{25}}{41} = \frac{122}{123} \simeq 0{,}99$$

$$|\lambda_1^{(2)} - \lambda_1^{(1)}| = \left|\frac{122}{123} - \frac{14}{15}\right| \simeq 0{,}058 < \varepsilon \Rightarrow \text{CONVERGENCE et } \lambda_1 = 1$$

NOMBRE D'ITÉRATIONS INFÉRIEURE (par rapport au cas précédent); en effet ~~[illegible]~~ la technique de calcul basée sur le coefficient de Rayleigh est conçue pour accélérer la convergence.

c) $B = A - \lambda_1 \dfrac{\vec{x}_1\vec{x}_1^{\,T}}{\vec{x}_1^{\,T}\cdot\vec{x}_1}$

$\left.\begin{array}{l}\lambda_1 = 1\\ \vec{x}_1 = \begin{pmatrix}1\\ \frac{40}{41}\end{pmatrix}\end{array}\right\}$ cf. point a) $\Rightarrow$ $\vec{x}_1^{\,T}\cdot\vec{x}_1 = 1+\left(\frac{40}{41}\right)^2 = \frac{3281}{1681}$

$$\vec{x}_1\vec{x}_1^{\,T} = \begin{pmatrix}1\\ \frac{40}{41}\end{pmatrix}\left(1,\ \frac{40}{41}\right) = \begin{pmatrix}1 & \frac{40}{41}\\ \frac{40}{41} & \left(\frac{40}{41}\right)^2\end{pmatrix} \Rightarrow$$

$$\Rightarrow B = \begin{pmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix} - 1 \cdot \frac{41^2}{40^2+41^2} \begin{pmatrix} 1 & \frac{40}{41} \\ \frac{40}{41} & \frac{40^2}{41^2} \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{2}{3} - \frac{41^2}{40^2+41^2} & \frac{1}{3} - \frac{41^{\cancel{2}} \cdot 40}{(40^2+41^2)\cancel{41}} \\ \frac{1}{3} - \frac{41^{\cancel{2}}}{40^2+41^2} \frac{40}{\cancel{41}} & \frac{2}{3} - \frac{\cancel{41^2}}{40^2+41^2} \frac{40^2}{\cancel{41^2}} \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{2}{3} - \frac{1681}{3281} & \frac{1}{3} - \frac{1640}{3281} \\ \frac{1}{3} - \frac{1640}{3281} & \frac{2}{3} - \frac{1600}{3281} \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{1519}{9843} & -\frac{1639}{9843} \\ -\frac{1639}{9843} & \frac{1762}{9843} \end{pmatrix}$$