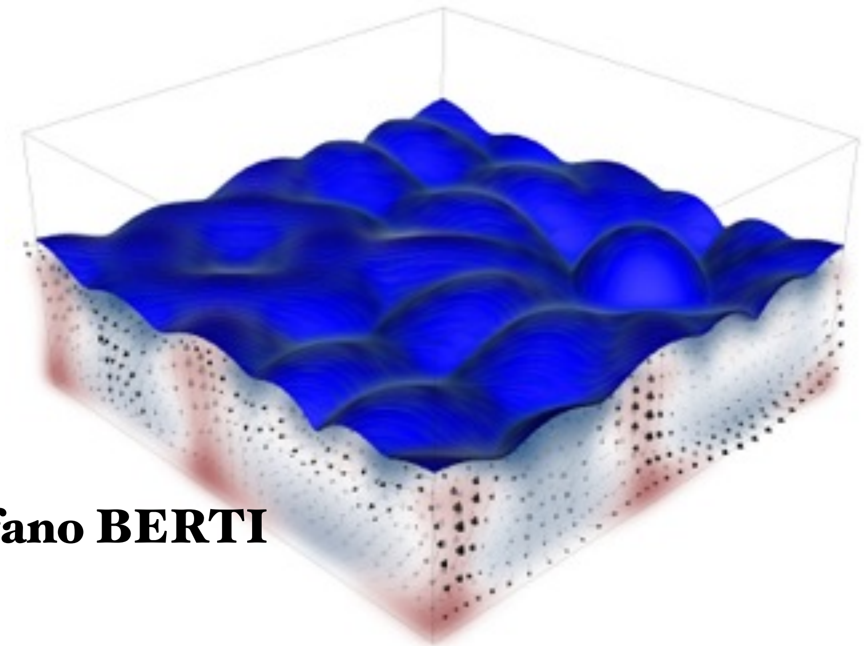
# Basal Melting Driven by Turbulent Thermal Convection



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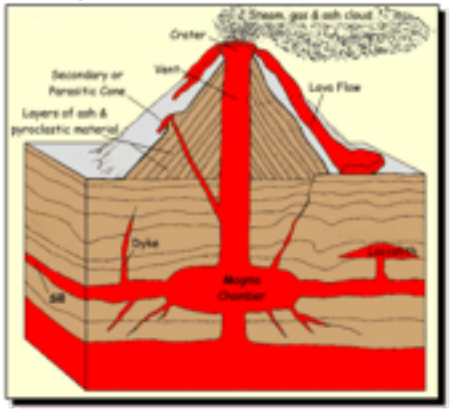
14-18 May 2018
University of Twente – Enschede – The Netherlands

# Convective flows and phase change

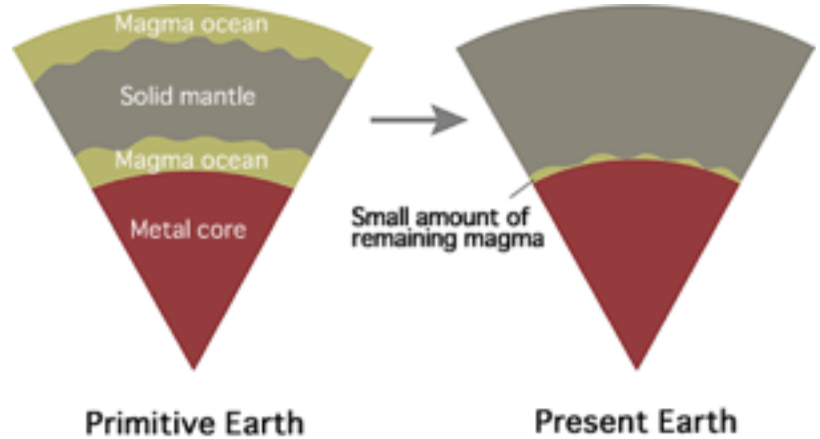
## Lava lakes

Persistent lava lake of Mount Nyiragongo

## Magma cambers




## Magma ocean




## Ice melt ponds

Arctic ICESCAPE mission, NASA, July 12, 2011

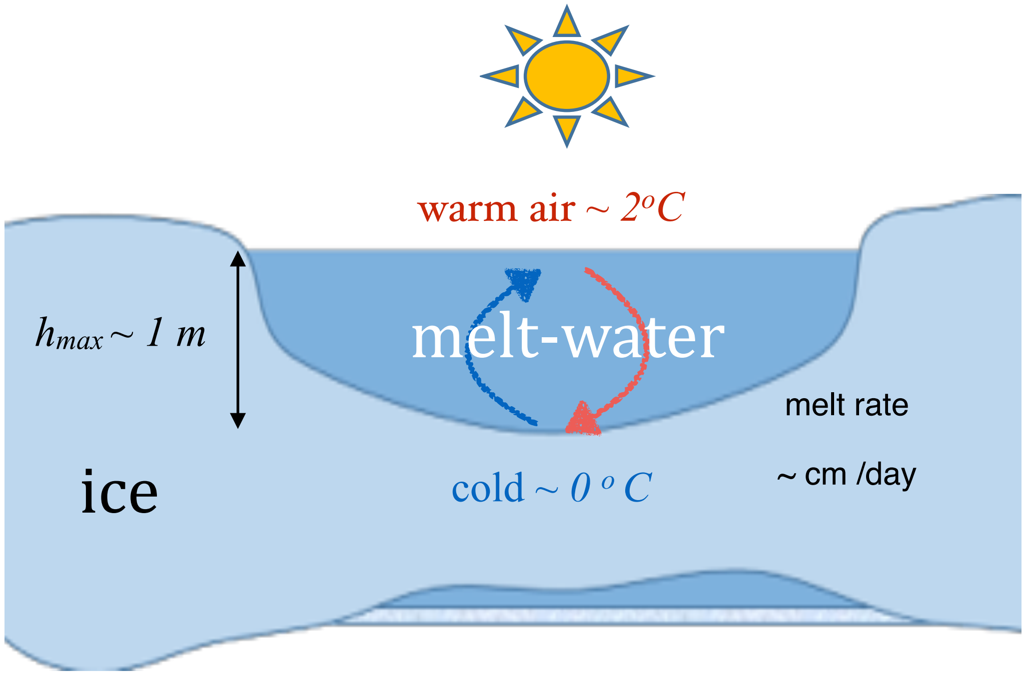
**Ice-albedo positive feedback mechanism**

Melt ponds formation

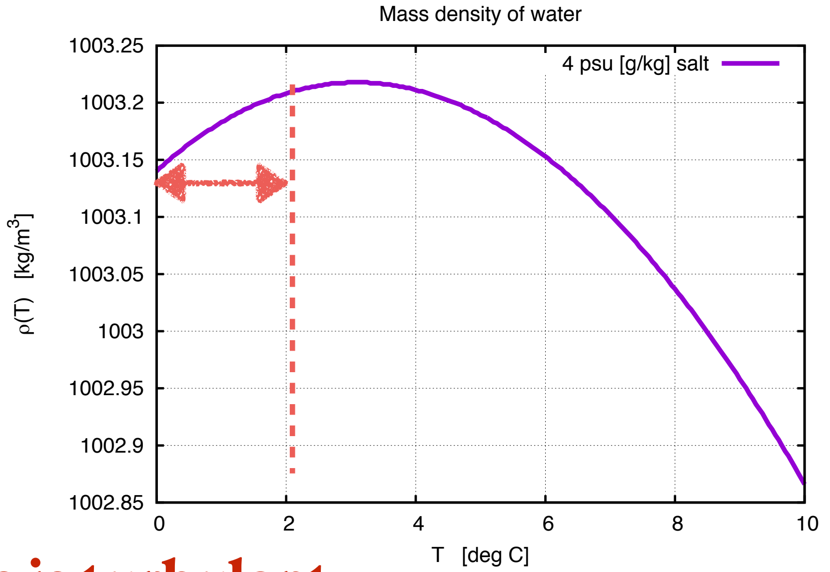
reduced albedo

increased Earth temperature

# Thermal convection in Arctic ice melt ponds




Unstable stratification due to water density anomaly below ~$4^oC$




**Thermal convection in ponds is turbulent**

$$\left.\begin{array}{l}\Delta T = 0.2\ K \\ H = 10\ cm\end{array}\right\} Ra = 10^6$$ up to $Ra \sim 10^9$ with $Pr \sim 10$

**How does the heat-flux scale in a pond?**

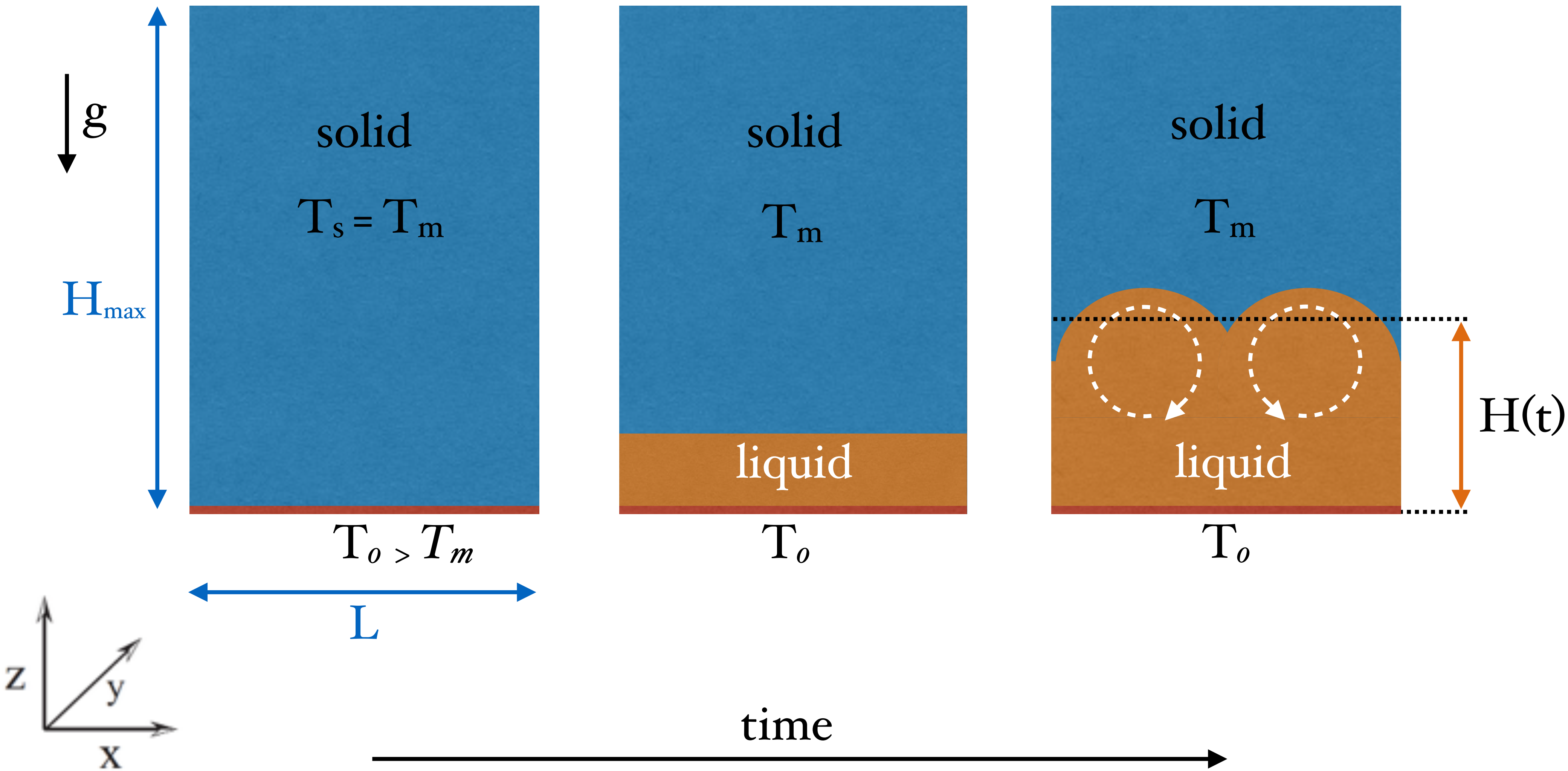
$$Nu \simeq 0.04\ Ra^{1/3}$$ Malkus (1954) scaling


Taylor & Feltham, "A model of melt pond evolution on sea ice", *J. Geophys. Res. 109,(2004).*

Lüthje, Mikael, et al. "Modeling the summertime evolution of sea-ice melt ponds." *J. Geophys. Res.* 111 (2006).

# A model system

**Basal melting driven by natural thermal convection (CM)**



# Equations for the model system

**Boussinesq equations**

$$\rho_0 \left( \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \boldsymbol{\nabla} \mathbf{u} \right) = -\boldsymbol{\nabla} p + \mu \, \Delta \mathbf{u} + \rho_L \mathbf{g},$$

$$\boldsymbol{\nabla} \cdot \mathbf{u} = \mathbf{0},$$

$$\rho_L = \rho_0 \left(1 - \beta(T - T_0)\right),$$

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \boldsymbol{\nabla} T = \kappa \, \Delta T,$$

**+ Boundary conditions**

$$-\kappa \, \boldsymbol{\nabla} T|_{\boldsymbol{x}=\boldsymbol{x}_m(t)} = \frac{\mathcal{L}}{c_p} \dot{\boldsymbol{x}}_m(t)$$

$$\mathbf{u}|_{\boldsymbol{x}=\boldsymbol{x}_m(t)} = \mathbf{0}$$

phase-change interface

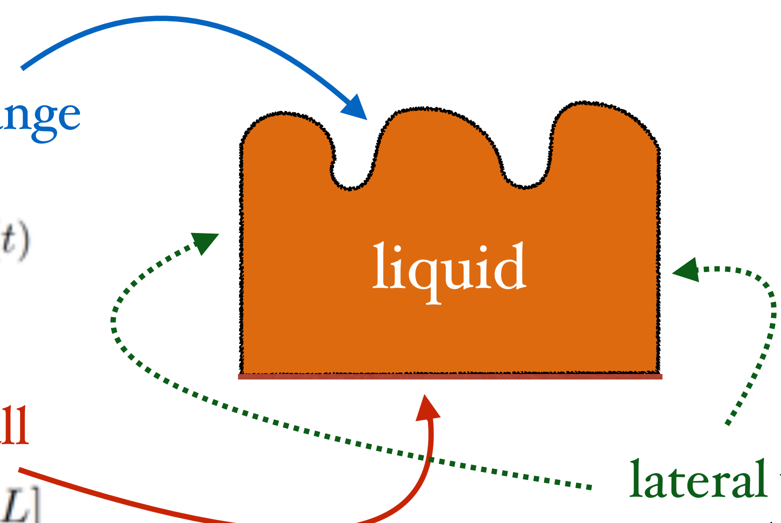
$\forall \boldsymbol{x}_m(t) \in \mathcal{I}(t)$

$$T|_{\boldsymbol{x}=(x,y,0)} = T_0$$

$$\mathbf{u}|_{\boldsymbol{x}=(x,y,0)} = \mathbf{0}$$

bottom wall

$\forall x, y \in [0, L][0, L]$




lateral walls

periodic bc

# Control parameters

**Reference scales**

**length** $H_{max}$

**time** $H_{max}^2/\kappa$

**temperature** $\Delta T = T_0 - T_m$

**Dimensionelss form**

$$\frac{\partial \tilde{\mathbf{u}}}{\partial \tilde{t}} + \tilde{\mathbf{u}} \cdot \nabla \tilde{\mathbf{u}} = -\nabla \tilde{p} + Pr\,\Delta \tilde{\mathbf{u}} + Ra_{max} Pr\,\hat{\tilde{\mathbf{z}}},$$

$$\nabla \cdot \tilde{\mathbf{u}} = \mathbf{0},$$

local liquid fraction

$$\frac{\partial \tilde{T}}{\partial \tilde{t}} + \tilde{\mathbf{u}} \cdot \nabla \tilde{T} = \kappa\,\Delta \tilde{T} - \frac{1}{St}\frac{\partial \phi_l}{\partial \tilde{t}}.$$

**Stefan**

$$St = \frac{c_p \Delta T}{\mathcal{L}}$$

**Prandtl**

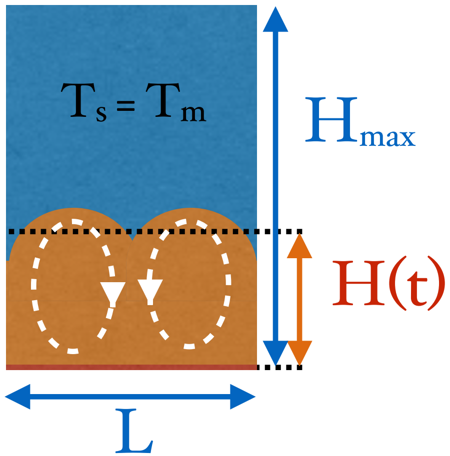
$$Pr = \frac{\nu}{\kappa}$$

**Rayleigh**

$$Ra_{max} = \frac{\beta \rho_0 g \Delta T H_{max}^3}{\mu \kappa}$$

**Aspect ratio**

$$\Gamma_{min} = \frac{L}{H_{max}}$$




**Effective Rayleigh**

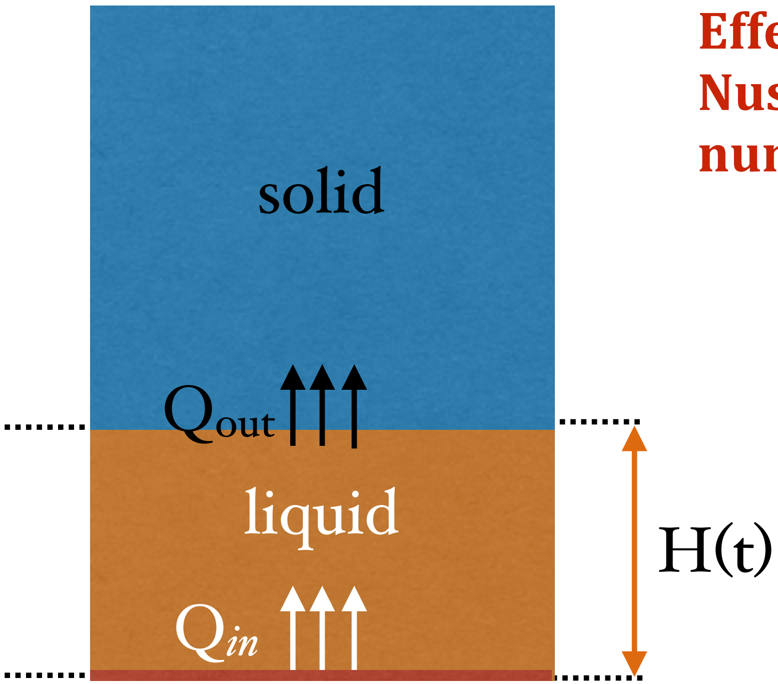
$$Ra_{eff} = \frac{\beta \rho_0 g \Delta T H(t)^3}{\mu \kappa}$$

**Effective Aspect ratio**

$$\Gamma_{eff} = \frac{L}{H(t)}$$

Davis, S.H. *et al.* *J. Fluid Mech.* 144, 133–151 (1984).

# Global heat budget




**Effective Nusselt number**

$$Nu_{eff}(t) = \frac{Q(t)}{\rho_0 c_p \kappa \frac{\Delta T}{H(t)}}$$

*Ulvrová, Labrosse, et al. Phys. Earth and Planetary Interiors (2012)*

$$Nu^{in}_{eff} = Nu^{out}_{eff} + \underbrace{\langle \phi_l \rangle^2 \langle \partial_{\tilde{t}} \tilde{T} \rangle_{V_l}}_{>0}$$

**Conductive regime:**

$$\mathrm{H}(t) = 2\lambda\sqrt{\kappa t} \qquad \lambda \exp(\lambda^2)\, \mathrm{erf}(\lambda) = \frac{\mathrm{St}}{\sqrt{\pi}}$$

$$Nu^{in}_{eff} = \frac{2\lambda^2}{St} e^{\lambda^2} \qquad Nu^{out}_{eff} = \frac{2\lambda^2}{St}$$

independent of time and > 1

Convective regime?

# Global heat flux: convection

$$Nu_{eff}(Ra_{eff}, Pr, St) = ?$$

$$Nu^{out}_{eff} \sim Ra^{\alpha}_{eff}\; Pr^{\delta}\; St^{\gamma} \longrightarrow \langle \phi_l \rangle \sim \tilde{t}^{\frac{1}{2-3\alpha}}\; Pr^{\frac{\delta}{2-3\alpha}}\; St^{\frac{\gamma+1}{2-3\alpha}}$$

**global liquid fraction**

**Conductive case and St small**

$$\alpha = \delta = \gamma = 0 \longrightarrow \langle \phi_l \rangle \sim \tilde{t}^{1/2}\; St^{1/2}$$

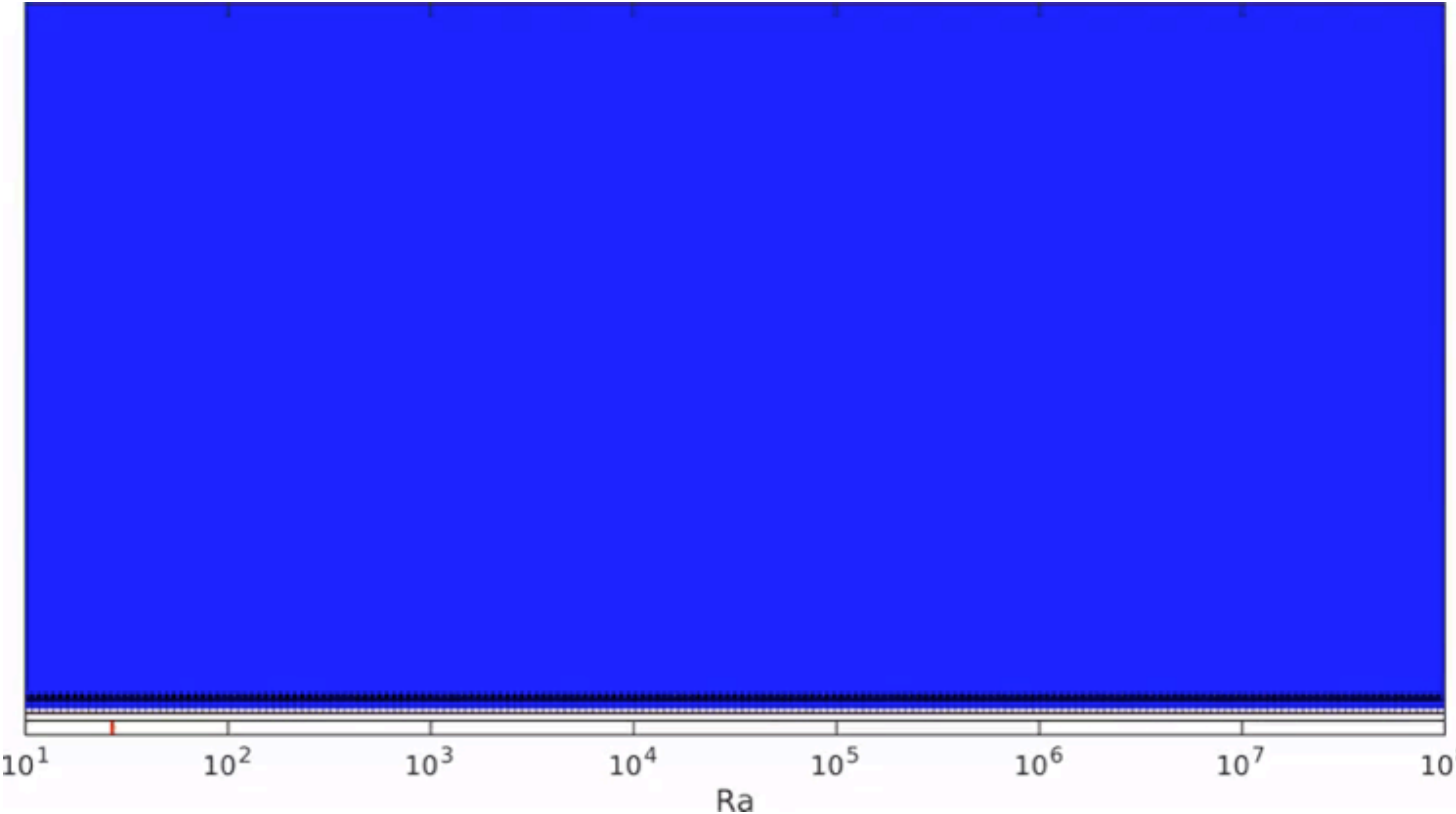
**RB Malkus scaling** — **constant melt front speed**

$$\alpha = 1/3 \longrightarrow \tilde{v}_m = \frac{d}{dt}\langle \phi_l \rangle = const$$

**RB Ultimate regime** — **constant melt front acceleration**

$$\alpha = 1/2 \text{ and } \delta = 1/2 \longrightarrow \tilde{a}_m = \frac{d^2}{dt^2}\langle \phi_l \rangle = const$$

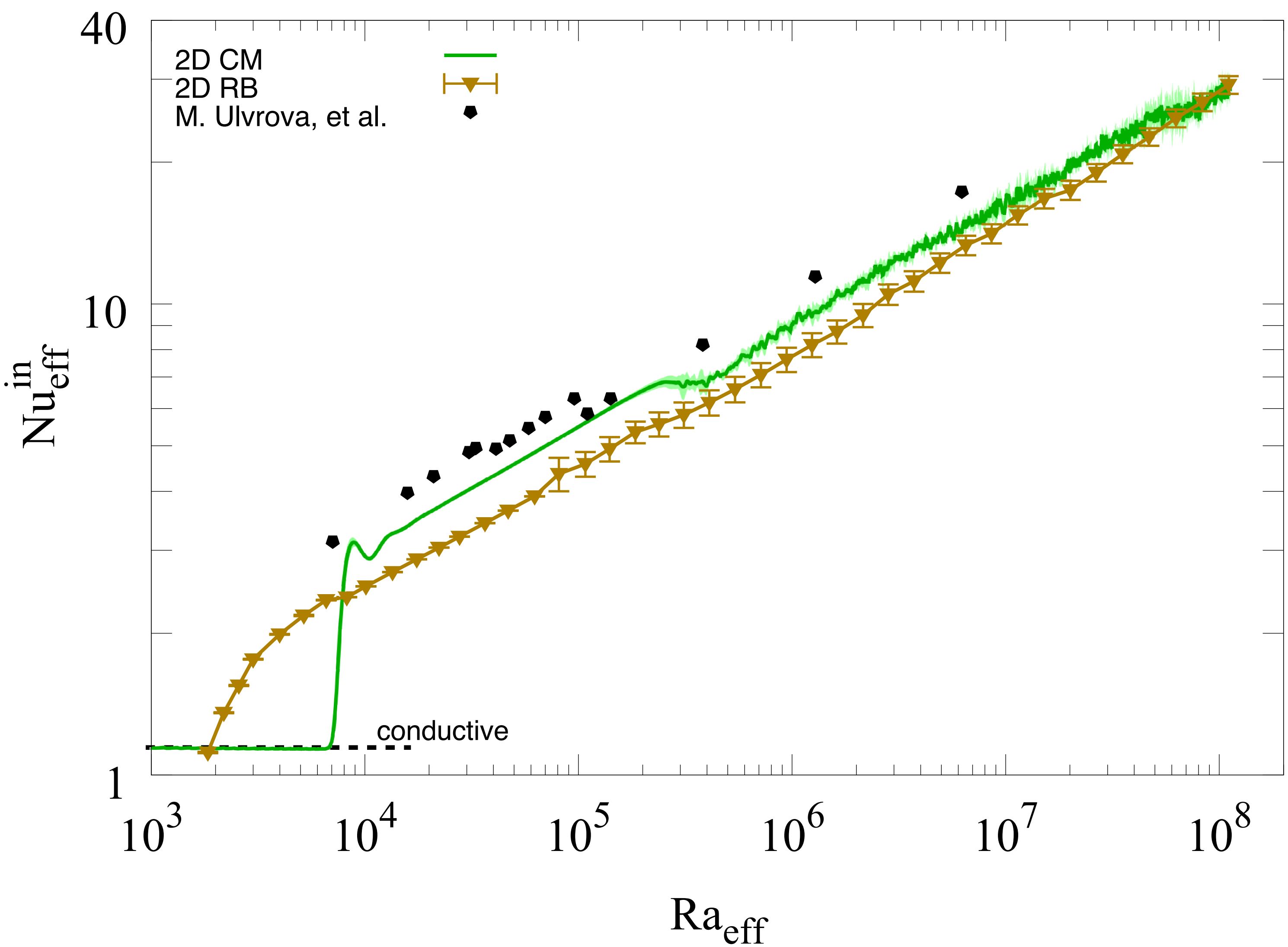
# DNS results: convective melting in 2D




St=1 , Pr= 10 , $\Gamma_{min} = 2$

# Nusselt *vs* Rayleigh (2D)

$Nu_{eff}^{in}$ inflowing heat flux




- Delayed onset compared to RB and rapid growth (Kim, Lee , Choi 2008) (Vasil & Proctor JFM 2011)
- $\mathrm{Nu}_{\mathrm{eff}}$ > Nu @RB (~20%) but vanish at large Ra
- Consistent with *Ulvrova et al. (2012)* (although different conditions: Pr = 7 , St =1.1, free-slip walls + adiabatic lateral walls )

# Reynolds *vs* Rayleigh (2D)

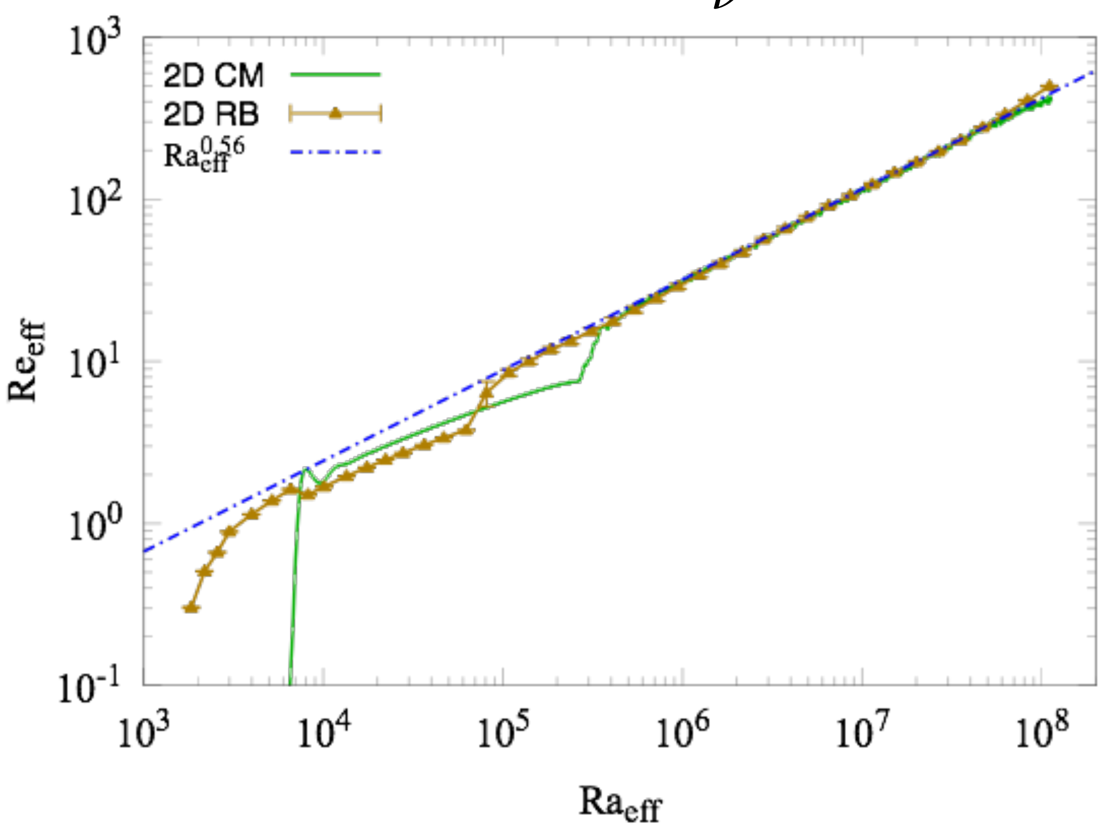
$$Re_{eff} = \frac{u_{rms}\ H(t)}{\nu}$$

typical fluid velocity amplitude $u_{rms}$

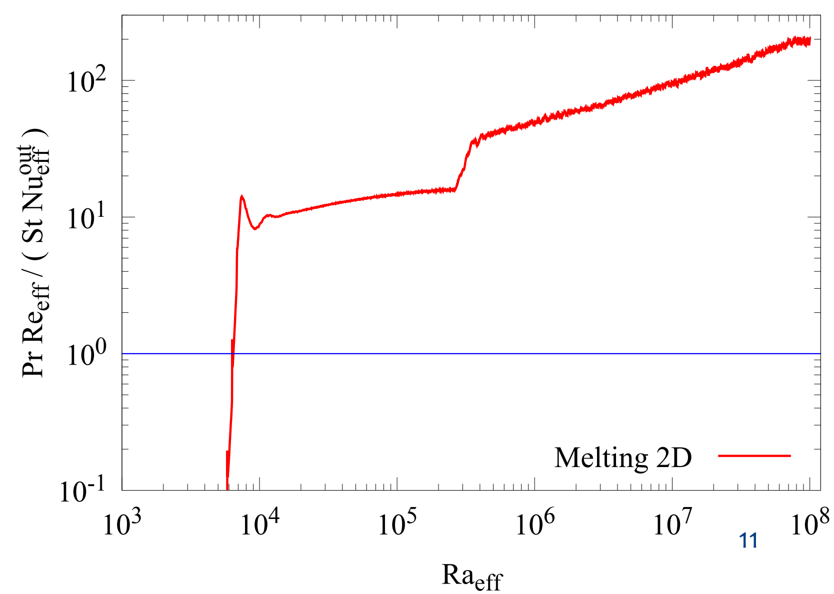
mean melting front speed $v_m = dH(t)/dt$

**RB-like behaviour if** $v_m \ll u_{rms}$

$$St\ Nu_{eff}^{out} \ll Pr\ Re_{eff}$$



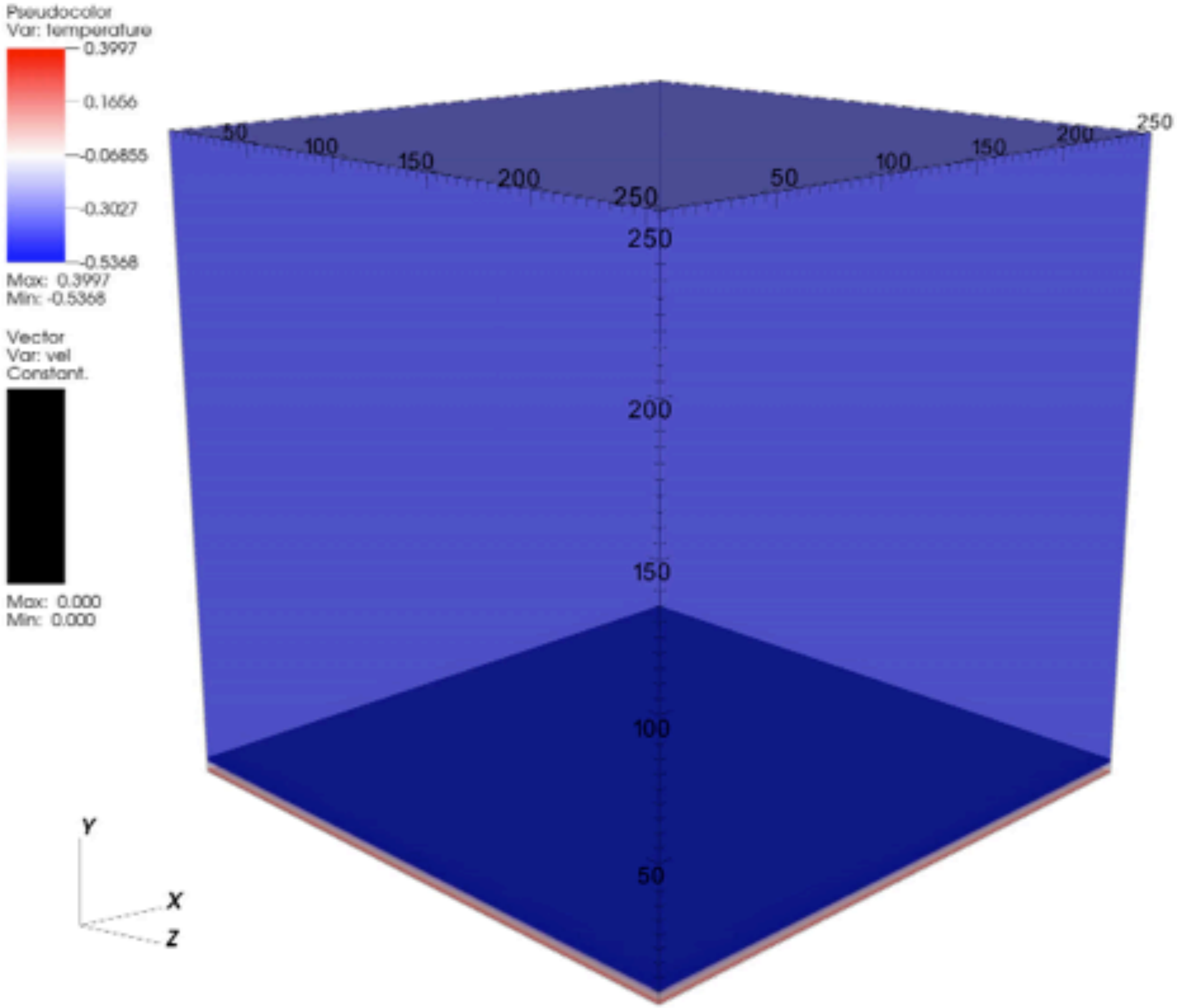

Similar scaling as in RB system



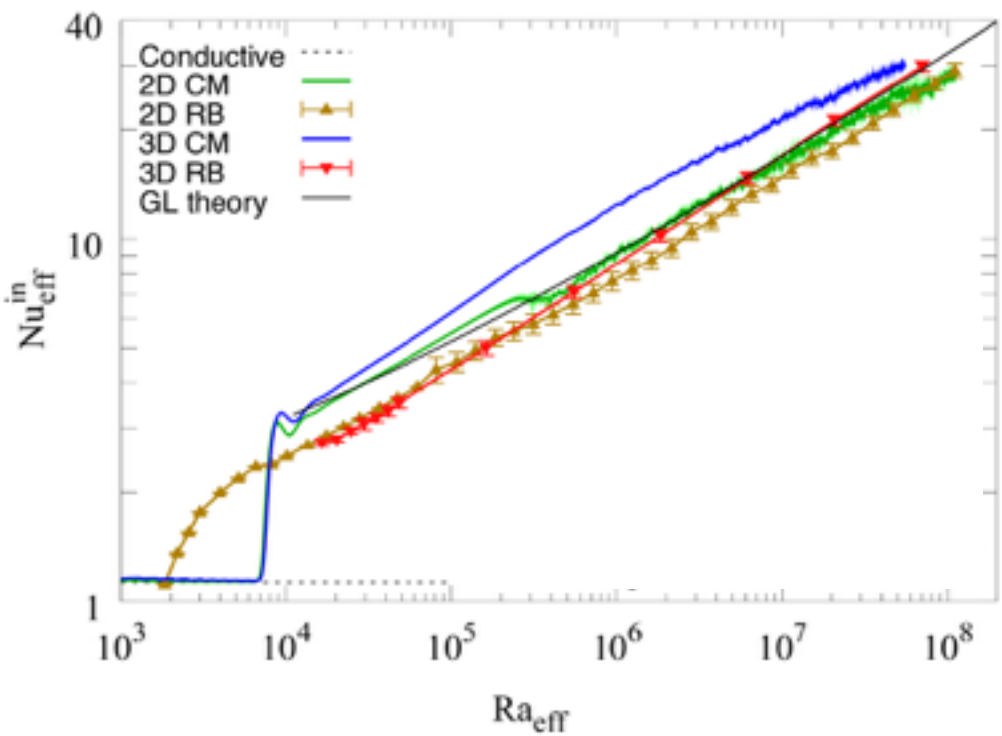

**Why convective melting is so similar to RB?**

# Convective melting in 3D

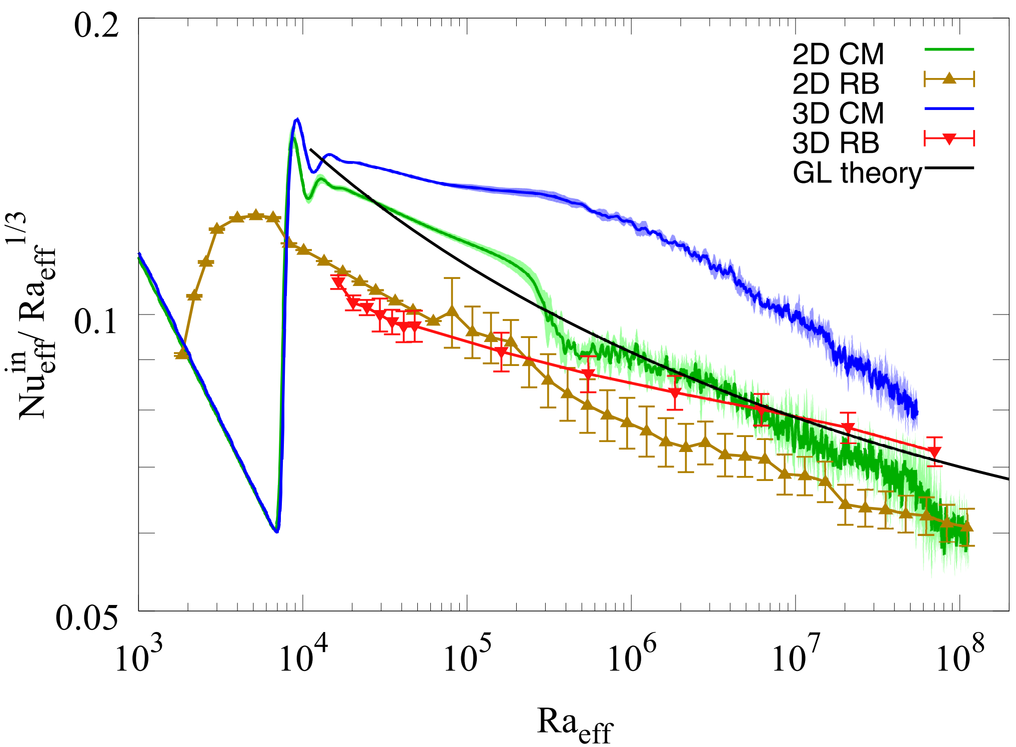
$Ra_{max} = 8x10^6$
St = 1
Pr = 10
$\Gamma_{min} = 1$



# Nusselt *vs* Rayleigh (3D)



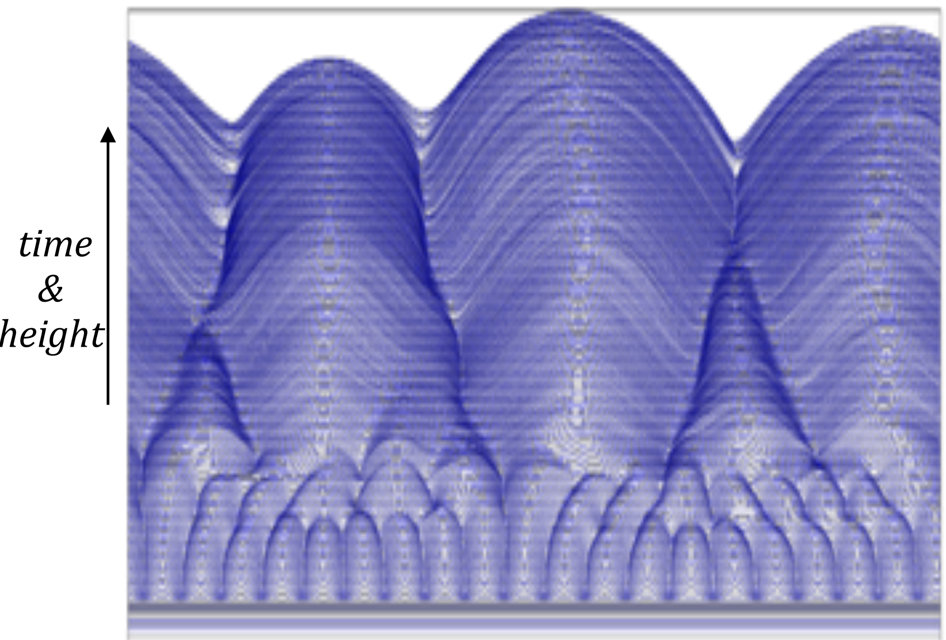

Ra$^{1/3}$ compensated




- $Nu_{eff}$ 3D > $Nu_{eff}$ 2D
  same trend as in 2D-3D RB system ( E.P. van der Poel *et al.* JFM 2013)
- $Nu_{eff}$ 3D > Nu @RB 3D (max increase 47% but transient effect)
- Ra exponent < 1/3

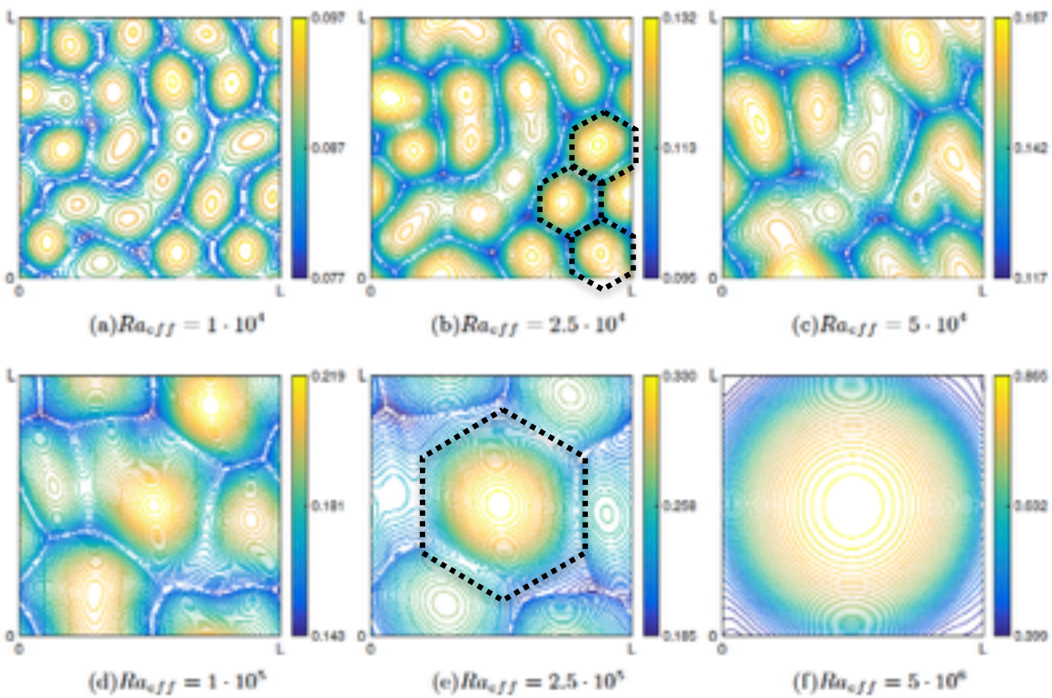
# Interface shape analysis (1)

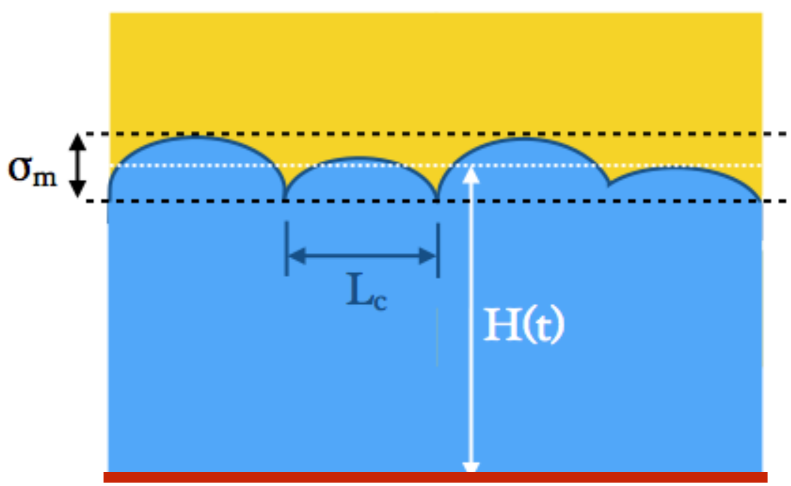
2D




$$Ra_{max} = 1.5 \times 10^7, Pr = 10, St = 1$$

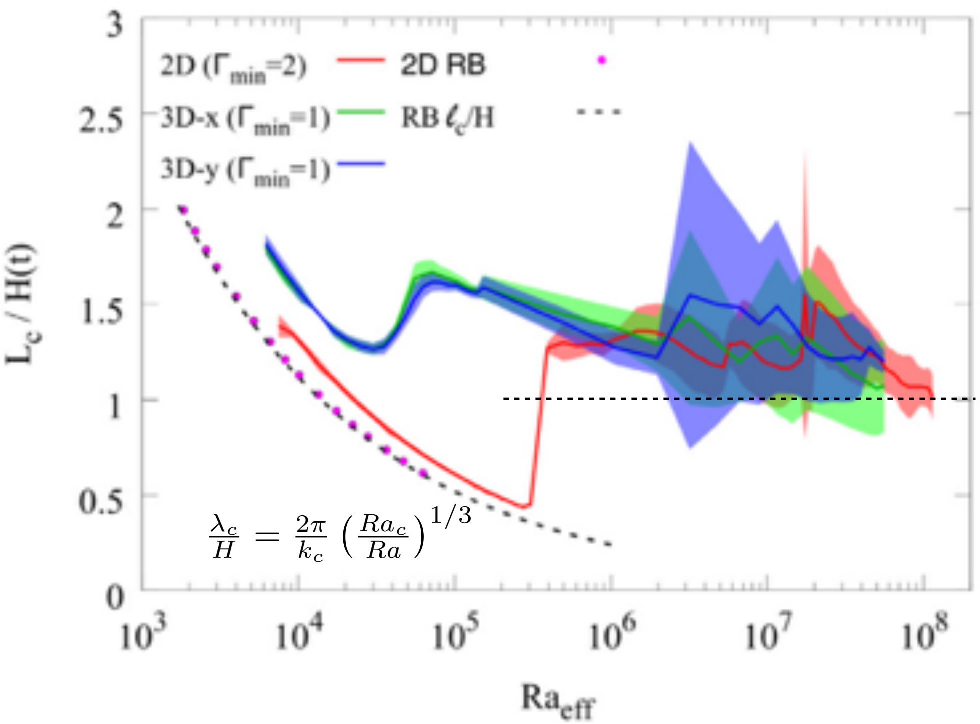
3D



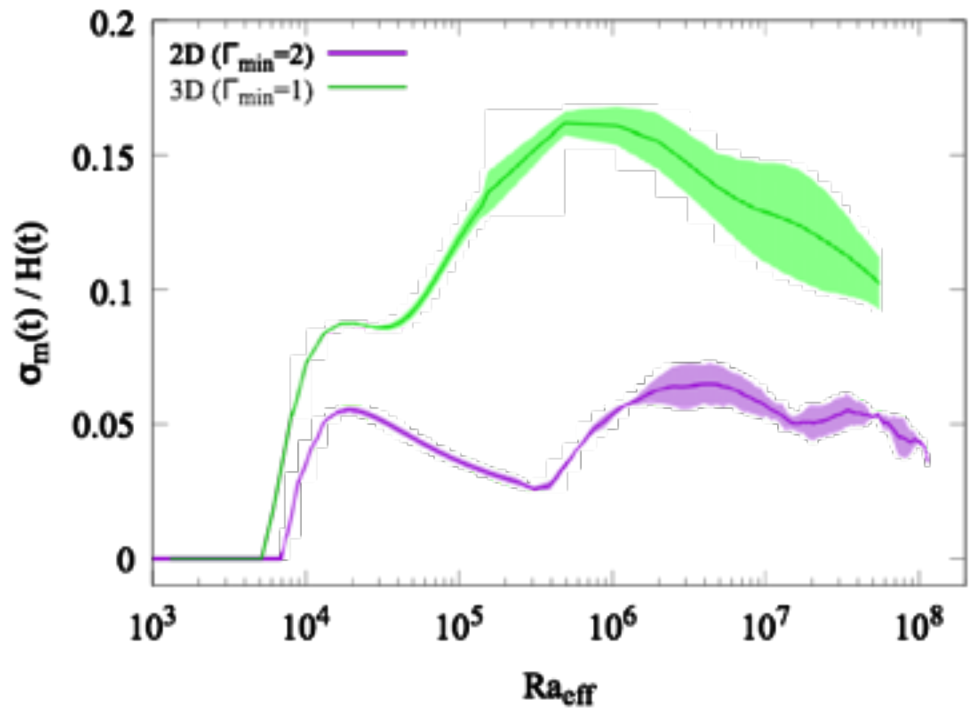
# Interface shape analysis (2)

Longitudinal correlation length $L_c$ *vs* $Ra_{eff}$




Asymptotic aspect ratio
of flows patterns = 1

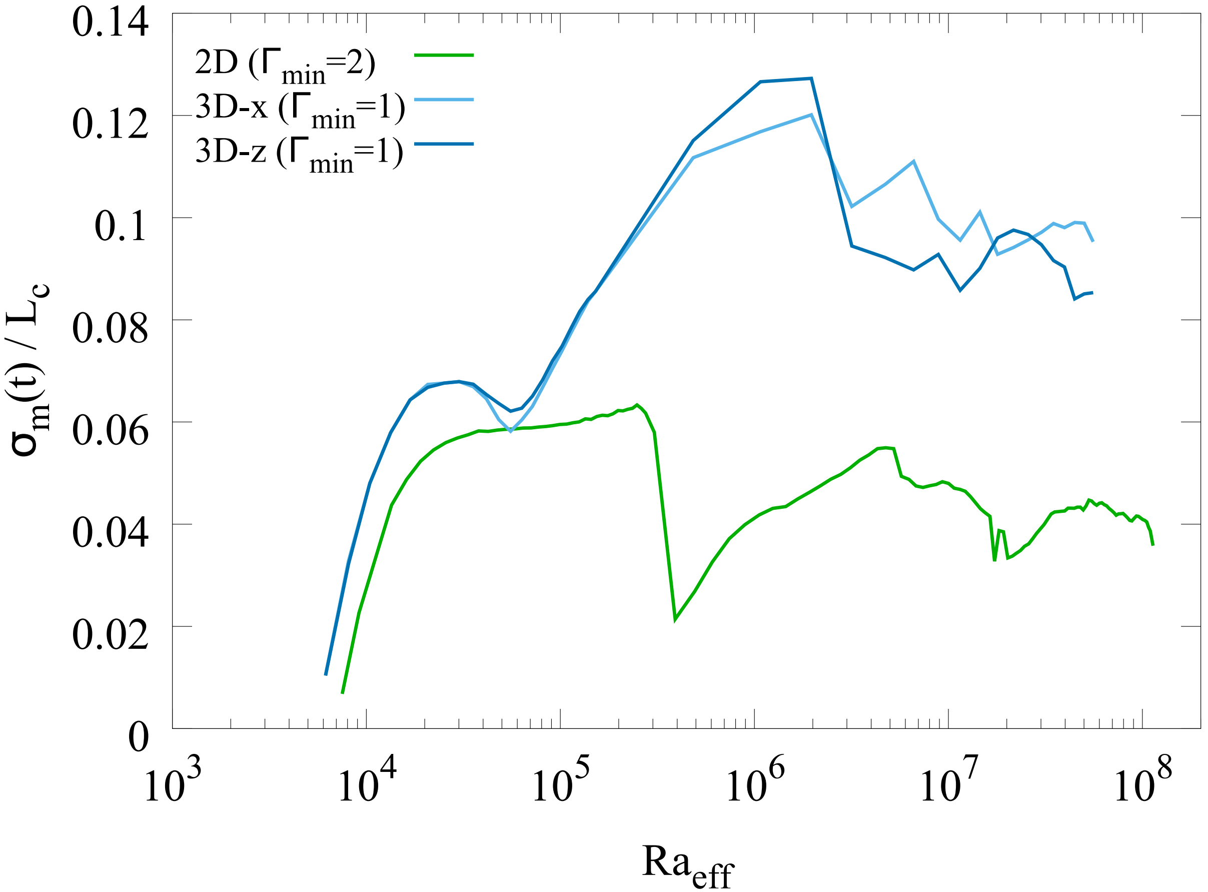
Height standard deviation *vs* $Ra_{eff}$




Higher roughness in 3D

# Interface shape analysis (3)

Roughness aspect ratio $\sigma_m / L_c$



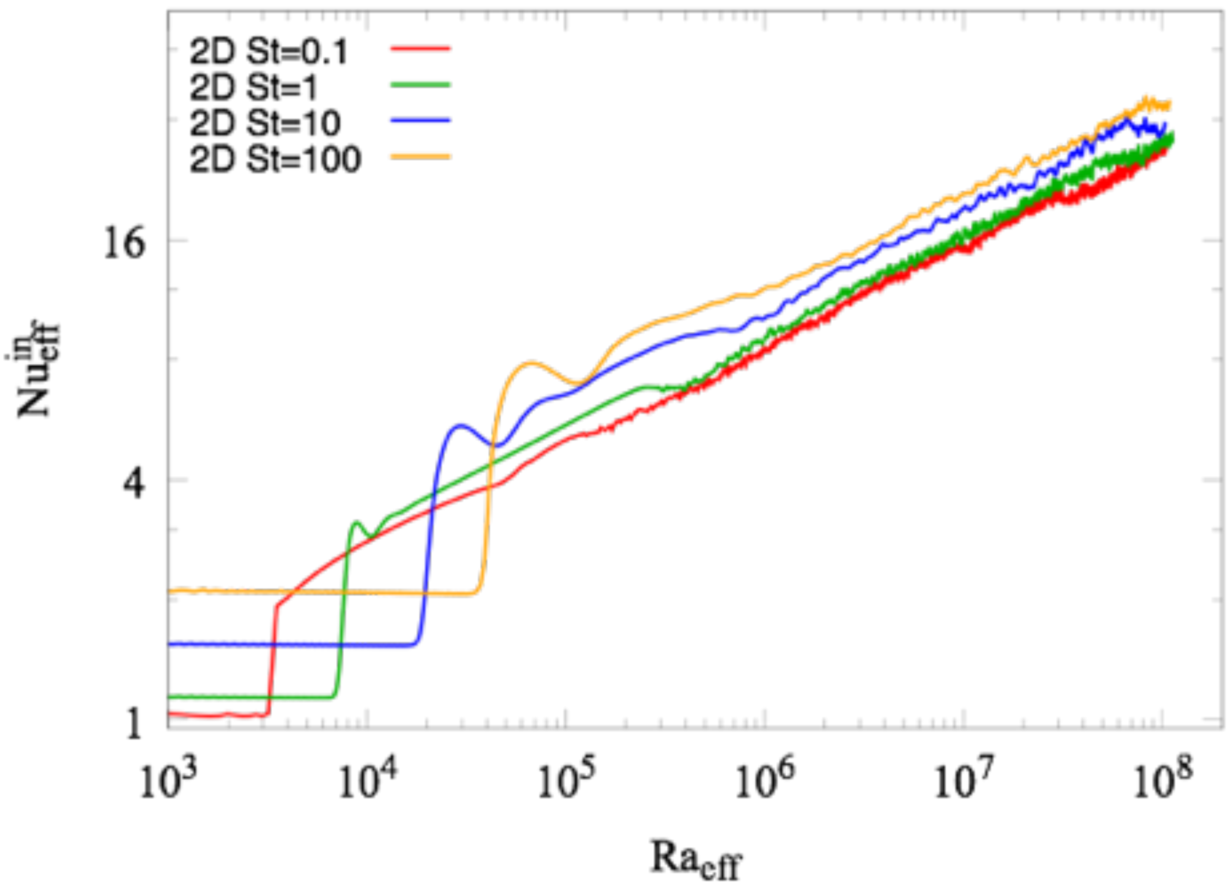

Very small roughness,

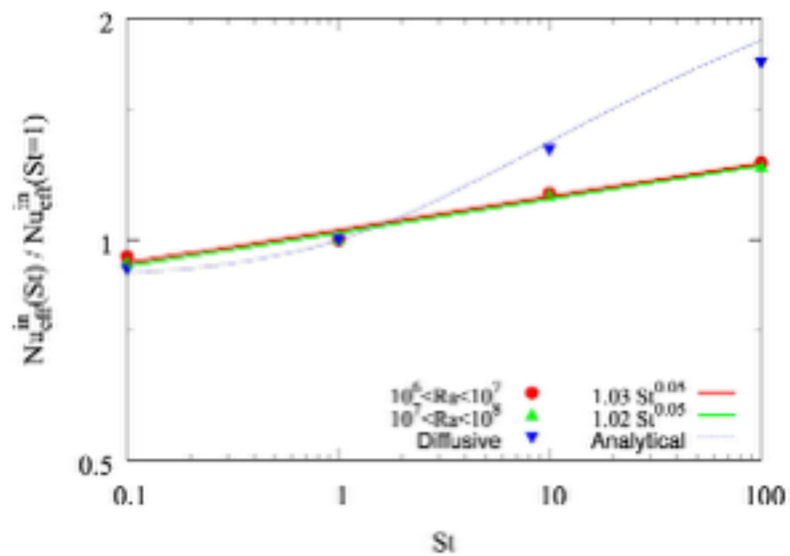
ineffective on Nu modulation

(Zhu, Stevens, Verzicco, Lohse, PRL 119, 154501 (2017)

# Effect of Stefan number

**Stefan** $St = \frac{c_p \Delta T}{L}$ ( Note that in ice melt ponds St ~ 0.01 )




- At small-*Ra St* affects the convection onset: goes to RB ($Ra_c$ = 1708) for *St -> 0*
- At high-*Ra* only St weakly increase the heat flux: $Nu \sim St^{0.05}$

# Summary

1. **RB phenomenology is valid :** RB heat flux captures the correct order of magnitude and asymptotic scaling for $Nu_{eff}$ in Convective Melting

   Why? $\mathbf{u_{rms} >> v_m}$

   $\mathbf{Nu_{eff} > Nu_{RB}}$ for moderate Ra -> not yet fully understood

   $\mathbf{Nu_{eff} \sim Ra_{eff}^{\alpha}}$ with $\boldsymbol{\alpha < 1/3}$ -> front speed $\mathbf{v_m(t) \sim t^{<0}}$ weakly decreases with time

2. **Small roughness**, and interface shape controlled by large-scale structures Ar=1

3. Weak Nu dependence on Stefan for St=[0.1,100]

   $\mathbf{Nu \sim Ra_{eff}^{\alpha}\, St^{0}}$ -> implies front speed $\mathbf{v_m \sim St}$

*Basal melting driven by turbulent thermal convection*
B. Rabbanipour Esfahani, S. C. Hirata, S. Berti and E. Calzavarini
arXiv:1801.03694 - in press on Phys. Rev. Fluids (2018)

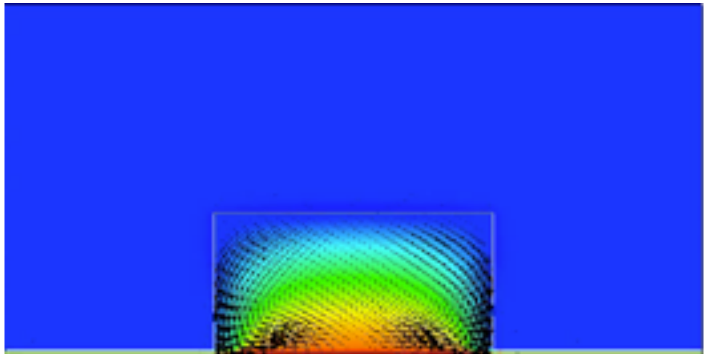
# Perspectives

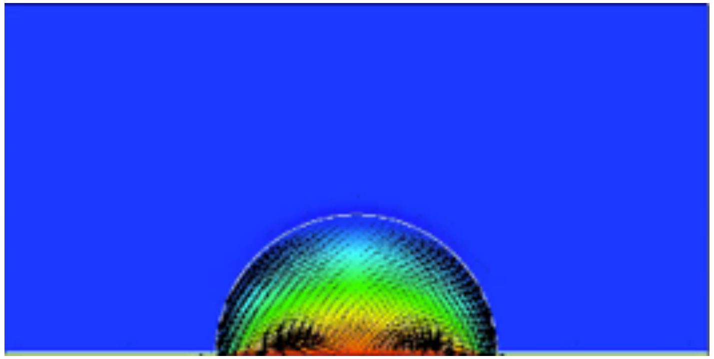
For more realistic modelling of melt ponds:

- Radiative heating (non-homogeneous volume term)

- Effect of wind stress

- Convection in a cavity and measure of vertical/lateral heat fluxes





- Simulating the merging of multiple melting cavities