Basal Melting Driven by Turbulent Thermal Convection

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Convective flows and phase change

Lava lakes

Persistent lava lake of Mount Nyiragongo

Magma cambers

Magma ocean

Ice melt ponds

Arctic ICESCAPE mission, NASA, July 12, 2011

Ice-albedo positive feedback mechanism

Melt ponds formation

increased Earth temperature

reduced albedo
Thermal convection in Arctic ice melt ponds

Thermal convection in ponds is turbulent

\[ \Delta T = 0.2 \, K \]
\[ \frac{H}{10 \, cm} \]

\[ Ra = 10^6 \]

up to \( Ra \sim 10^9 \) with \( Pr \sim 10 \)

How does the heat-flux scale in a pond?

\[ Nu \approx 0.04 \, Ra^{1/3} \]

Malkus (1954) scaling


A model system

Basal melting driven by natural thermal convection (CM)

$T_s = T_m$

$T_0 > T_m$

$H_{\text{max}}$

$T_o > T_m$

$T_o$

$T_0$

$H(t)$

$L$

time
Equations for the model system

Boussinesq equations

$$\rho_0 \left( \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla p + \mu \Delta \mathbf{u} + \rho_L g,$$

$$\nabla \cdot \mathbf{u} = 0,$$

$$\rho_L = \rho_0 \left( 1 - \beta (T - T_0) \right),$$

$$\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \kappa \Delta T,$$

+ Boundary conditions

$$-\kappa \left. \nabla T \right|_{x=x_m(t)} = \frac{L}{c_p} \dot{x}_m(t)$$

$$\left. \mathbf{u} \right|_{x=x_m(t)} = 0$$

$$T\left|_{x=(x,y,0)} = T_0 \right.$$  
$$\forall x, y \in [0, L][0, L]$$

liquid

phase-change interface

bottom wall

lateral walls

periodic bc
Control parameters

Reference scales

**length** \( H_{\text{max}} \)

**time** \( \frac{H_{\text{max}}^2}{\kappa} \)

**temperature** \( \Delta T = T_0 - T_m \)

**Dimensionless form**

\[
\frac{\partial \tilde{u}}{\partial \tilde{t}} + \tilde{u} \cdot \nabla \tilde{u} = -\nabla \tilde{p} + \text{Pr} \Delta \tilde{u} + \text{Ra}_{\text{max}} \text{Pr} \tilde{z},
\]

\[
\nabla \cdot \tilde{u} = 0,
\]

\[
\frac{\partial \tilde{T}}{\partial \tilde{t}} + \tilde{u} \cdot \nabla \tilde{T} = \kappa \Delta \tilde{T} - \frac{1}{\text{St}} \frac{\partial \phi_l}{\partial \tilde{t}}.
\]

**Dimensionless scales**

\[
\Gamma_{\text{min}} = \frac{L}{H_{\text{max}}}
\]

**Effective Rayleigh**

\[
\text{Ra}_{\text{eff}} = \frac{\beta \rho_0 g \Delta T H(t)^3}{\mu \kappa}
\]

\[
\Gamma_{\text{eff}} = \frac{L}{H(t)}
\]

Global heat budget

\[ N u_{eff}(t) = \frac{Q(t)}{\rho_0 c_p \kappa \frac{\Delta T}{H(t)}} \]

\[ N u_{in}^{eff} = N u_{out}^{eff} + \langle \phi_i \rangle^2 \langle \partial_t \tilde{T} \rangle v_i > 0 \]

**Effective Nusselt number**

**Conductive regime:**

\[ H(t) = 2\lambda \sqrt{\kappa t} \]

\[ \lambda \exp(\lambda^2) \text{ erf}(\lambda) = \frac{St}{\sqrt{\pi}} \]

\[ N u_{in}^{eff} = \frac{2\lambda^2}{St} e^{\lambda^2} \]

\[ N u_{out}^{eff} = \frac{2\lambda^2}{St} \]

independent of time and > 1

Global heat flux: convection

\[ \textit{Nu}_{\text{eff}}(\textit{Ra}_{\text{eff}}, \textit{Pr}, \textit{St}) = ? \]

Conductive case and \( \textit{St} \) small

\[ \alpha = \delta = \gamma = 0 \quad \Rightarrow \quad \langle \phi_t \rangle \sim \tilde{t}^{1/2} \textit{St}^{1/2} \]

RB Malkus scaling

\[ \alpha = 1/3 \]

RB Ultimate regime

\[ \alpha = 1/2 \text{ and } \delta = 1/2 \]

constant melt front speed

\[ \tilde{\nu}_m = \frac{d}{dt} \langle \phi_t \rangle = \text{const} \]

constant melt front acceleration

\[ \tilde{a}_m = \frac{d^2}{dt^2} \langle \phi_t \rangle = \text{const} \]
DNS results: convective melting in 2D

\[ \text{St}=1 \ , \ \text{Pr}=10 \ , \ \Gamma_{\text{min}}=2 \]
Nusselt vs Rayleigh (2D)

\[ \text{Nu}^{\text{in}}_{\text{eff}} \text{ inflowing heat flux} \]

- Delayed onset compared to RB and rapid growth
  (Kim, Lee, Choi 2008)
  (Vasil & Proctor JFM 2011)

- \( \text{Nu}_{\text{eff}} > \text{Nu} @ \text{RB} \) (~20%)
  but vanish at large Ra

- Consistent with
  Ulvrova et al. (2012)
  (although different conditions:
  Pr = 7, St = 1.1, free-slip walls + adiabatic lateral walls)
Reynolds vs Rayleigh (2D)

Why convective melting is so similar to RB?

\[ Re_{\text{eff}} = \frac{u_{\text{rms}} H(t)}{\nu} \]

Similar scaling as in RB system
Convective melting in 3D

\( Ra_{\text{max}} = 8 \times 10^6 \)

\( St = 1 \)

\( Pr = 10 \)

\( \Gamma_{\text{min}} = 1 \)
Nusselt vs Rayleigh (3D)

• $\text{Nu}_{\text{eff}}^{3D} > \text{Nu}_{\text{eff}}^{2D}$
  
  same trend as in 2D-3D RB system (E.P. van der Poel et al. JFM 2013)

• $\text{Nu}_{\text{eff}}^{3D} > \text{Nu}_{\text{RB}}^{3D}$ (max increase 47% but transient effect)

• Ra exponent $< 1/3$
Interface shape analysis (1)

\[ Ra_{\text{max}} = 1.5 \times 10^7, \ P_r = 10, \ S_t = 1 \]
Interface shape analysis (2)

Longitudinal correlation length $L_c$ vs $R_{a_{\text{eff}}}$

Height standard deviation vs $R_{a_{\text{eff}}}$

Asymptotic aspect ratio of flows patterns = 1

Higher roughness in 3D
Very small roughness, ineffective on Nu modulation

(Zhu, Stevens, Verzicco, Lohse, PRL 119, 154501 (2017))
Effect of Stefan number

Stefan

\[ St = \frac{c_p \Delta T}{L} \]

(Note that in ice melt ponds \( St \sim 0.01 \))

- At small-\( Ra \) \( St \) affects the convection onset: goes to RB (\( Ra_c = 1708 \)) for \( St \to 0 \)
- At high-\( Ra \) only \( St \) weakly increase the heat flux: \( Nu \sim St^{0.05} \)
1. **RB phenomenology is valid**: RB heat flux captures the correct order of magnitude and asymptotic scaling for $\text{Nu}_{\text{eff}}$ in Convective Melting

   Why? $u_{\text{rms}} \gg v_m$

   $\text{Nu}_{\text{eff}} > \text{Nu}_{RB}$ for moderate Ra -> not yet fully understood

   $\text{Nu}_{\text{eff}} \sim \text{Ra}_{\text{eff}}^\alpha$ with $\alpha < 1/3$ -> front speed $v_m(t) \sim t^{<0}$ weakly decreases with time

2. **Small roughness**, and interface shape controlled by large-scale structures Ar=1

3. Weak Nu dependence on Stefan for St=[0.1,100]

   $\text{Nu} \sim \text{Ra}_{\text{eff}}^\alpha \text{St}^0$ -> implies front speed $v_m \sim \text{St}$

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*Basal melting driven by turbulent thermal convection*

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For more realistic modelling of melt ponds:

- Radiative heating (non-homogeneous volume term)
- Effect of wind stress
- Convection in a cavity and measure of vertical/lateral heat fluxes
- Simulating the merging of multiple melting cavities