Heavy particle transport in 2D elastic turbulence

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Motivation

Recent experimental [1] and numerical [2] investigations have shown the possibility to destabilize the laminar flow of a dilute polymer solution, due to elastic forces, in the limit of vanishing fluid inertia. The resulting "turbulent" motions could be used to enhance mixing efficiency at low Reynolds numbers, such as in microchannels. Understanding how solid impurities distribute in space in such flows is essential in view of applications, as well as from a fundamental perspective. We study the transport of inertial particles in elastic turbulent flows by means of direct numerical simulations of the Oldroyd-B model and Lagrangian tracking of particles. Our preliminary results show that the clustering properties of particles, both at large and small scales, are tightly related to the statistical features of the advecting flow.

What is Elastic turbulence ?

- Spectacular effect arising in the flow of dilute polymer solutions in the limit of vanishing fluid inertia (Re) and large polymer elasticity (Wi). Stretching of polymer molecules results in an irregular, turbulent-like behavior in both space and time.
- Statistical characterization of this turbulent like state is given by the spectrum of velocity fluctuations in the wavenumber (k) domain. Energy spectrum develops a power-law behaviour $E(k) \simeq k^{-\alpha}$ and $\alpha > 3$.
- This implies that elastic turbulence corresponds to a temporally random, spatially smooth flow, dominated by strongly nonlinear interactions between few large scale modes.

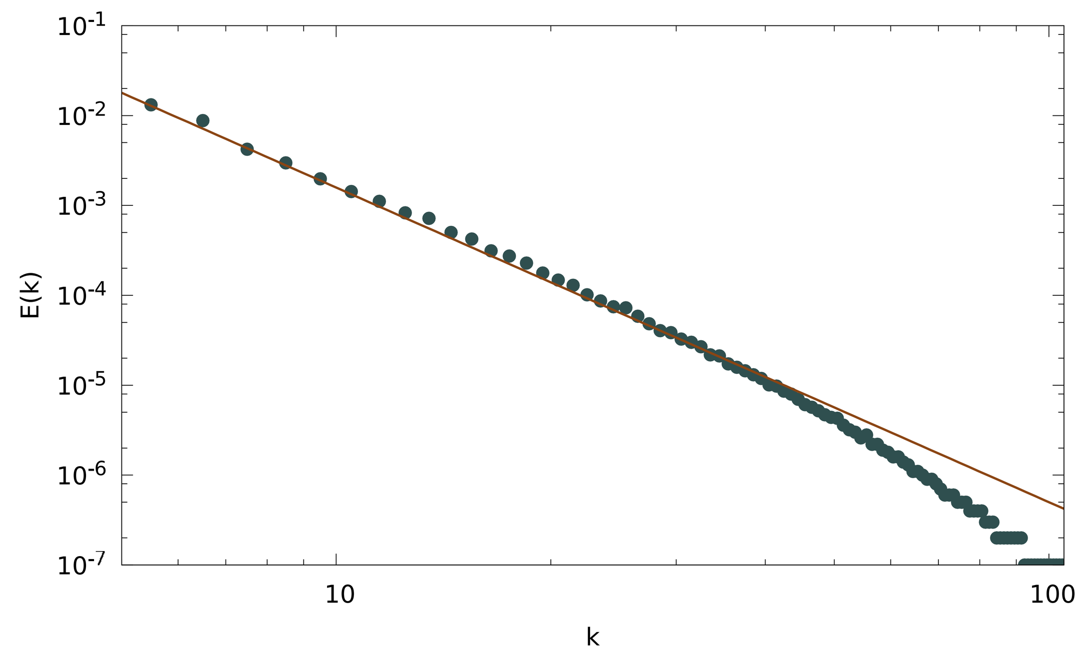

Figure 1: Spectrum of velocity fluctuations E(k) from a direct numerical simulation of elastic turbulence for Re $\simeq$ 1 and Wi $\simeq$ 24; the line corresponds to the best fit $k^{-3.8}$

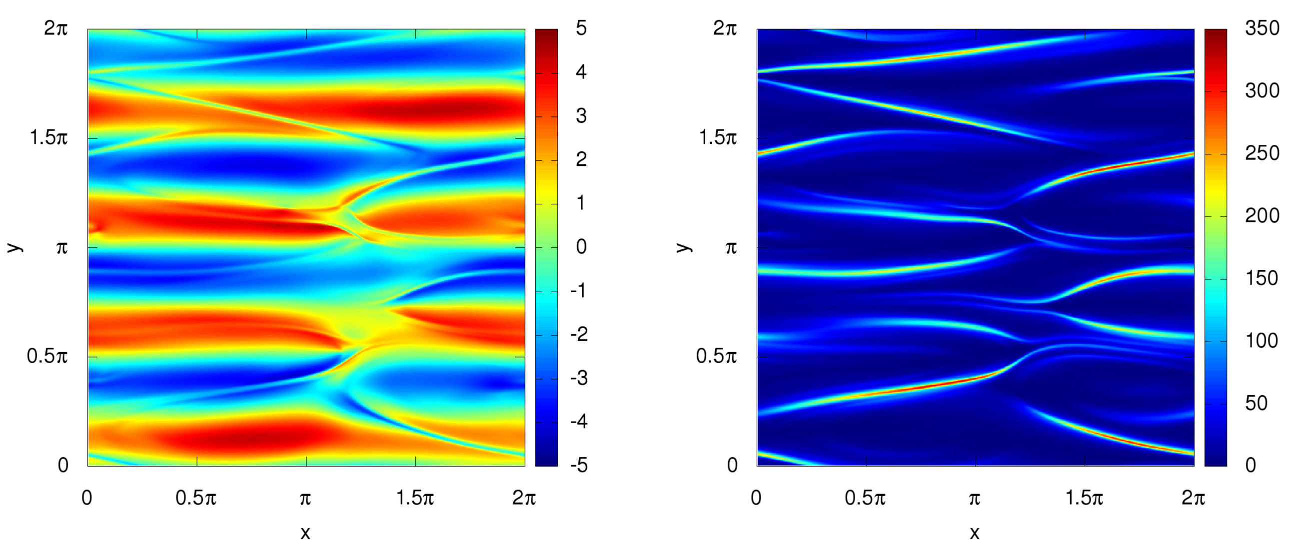

Figure 2: Snapshot of vorticity $\zeta(x, y)$ and component $\sigma_{11}(x, y)$ of conformation tensor at a fixed time for Re $\simeq$ 1 and Wi $\simeq$ 24

Mass impurities of finite size

- Density different from that of the fluid $\rho_p \neq \rho_f$.
- Velocity mismatch with that of the fluid,passive particles; no feedback on the fluid.

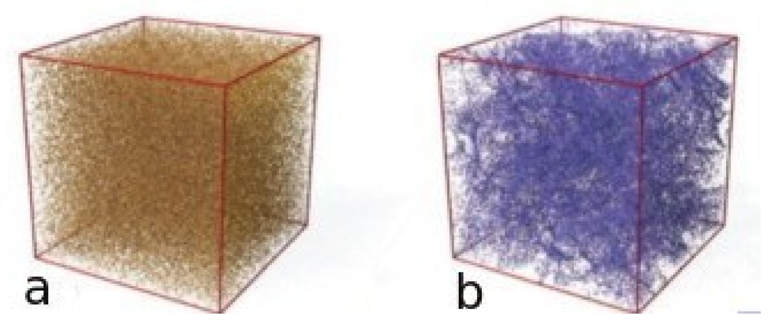

Figure 3: Snapshots of particle distributions in a turbulent flow field [3] (a) Tracers and (b) Heavy particles

Materials and Methods

- DNS of Oldroyd-B model by means of pseudospectral method with periodic bounday conditions to obtain dynamics of dilute polymer solution at resolution 512^2.
- Positive definiteness of conformation tensor is ensured by Cholesky decomposition algorithm.
- Mechanical forcing acts to maintain a mean velocity field corresponding to Kolmogorov periodic shear flow.
- Dynamics of inertial particles heavier than the fluid are tracked via Lagrangian approach.
- For inertial particles only the effect of Stokes drag is taken into account.

$$\partial_t \boldsymbol{u} + (\boldsymbol{u}.\bigtriangledown)\boldsymbol{u} = -\bigtriangledown p + \nu \triangle \boldsymbol{u} + \frac{2\eta\nu}{\tau} \bigtriangledown .\boldsymbol{\sigma} + \boldsymbol{f} \quad (1)$$

$$\partial_t \boldsymbol{\sigma} + (\boldsymbol{u}.\bigtriangledown)\boldsymbol{\sigma} = (\bigtriangledown \boldsymbol{u})^T.\boldsymbol{\sigma} + \boldsymbol{\sigma}.(\bigtriangledown \boldsymbol{u}) - \frac{2(\boldsymbol{\sigma} - \boldsymbol{1})}{\tau} \quad (2)$$

$$\dot{\boldsymbol{x}} = \boldsymbol{V}(\boldsymbol{x}, t) \quad (3)$$

$$\dot{\boldsymbol{V}} = \frac{1}{\tau_s}(\boldsymbol{u} - \boldsymbol{V}) \quad (4)$$

τ= Polymer relaxation time
η = Zero shear contribution of polymers to total viscosity of solution
ν= solvent viscosity
τ_s = particle relaxation time

Results

- Visualization of large scale and small scale inhomogeneities in the particle distribution for different value of particle inertia.

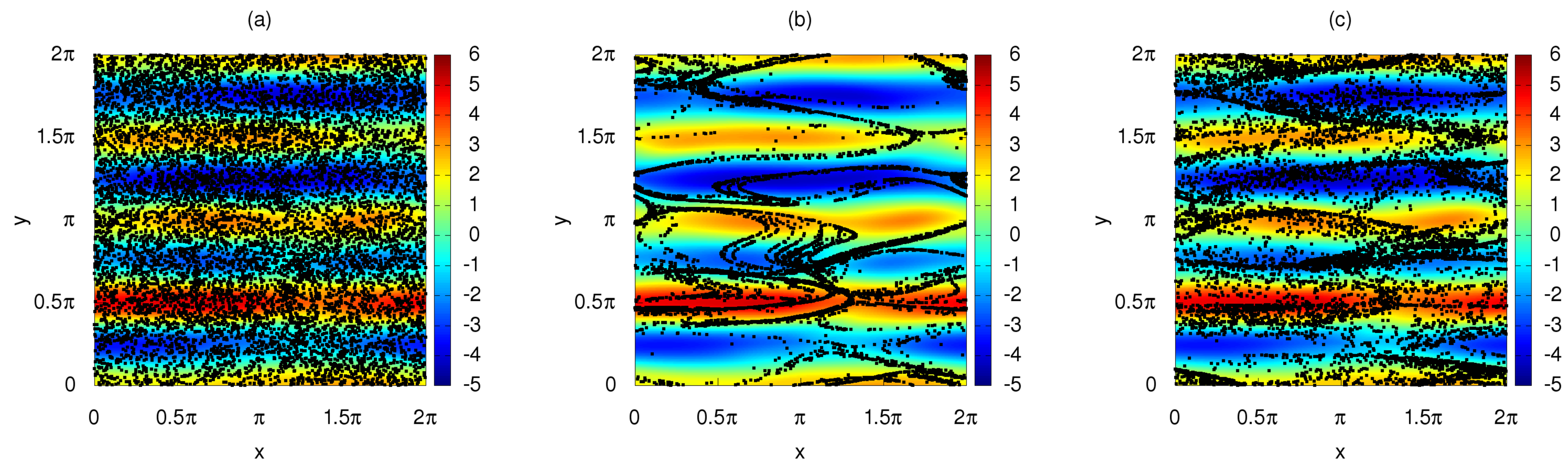

Figure 4: Spatial distribution of particles (black dots) with relaxation times $\tau_s = 0.01, 0.2, 1$, from left to right panel, and horizontal component of velocity (in color); here the number of particles is $N_p = 10^4$ and initially particles were seeded homogeneously in space.

Large Scale Clustering

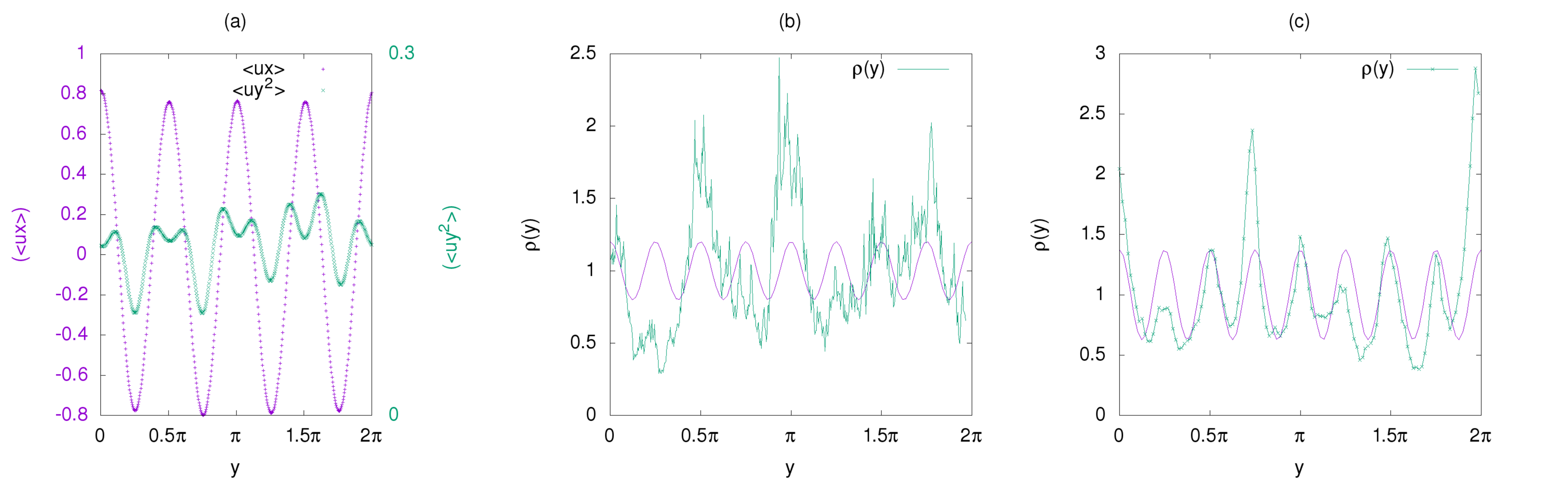

Figure 5: Profiles of longitudinal velocity $\langle u_x \rangle$ and shear-normal velocity fluctuations $\langle u_y^2 \rangle$ (a); number density profile $\rho(y)$ of particles for $\tau_s = 0.2$ (b) and $\tau_s = 1$ (c); in (b) and (c) the purple solid line corresponds to the function [4] : $\rho(y) = \frac{1}{\rho_0}(1 + acos(\frac{2y}{L}))$, where $L = \frac{1}{4}$ and a is a free-parameter

Small Scale Clustering

- Chaotic trajectories obtained from Eqs.(3,4) evolve to a fractal attractor
- Quantitative measure of clustering at small scales is obtained by measuring the correlation dimension D_2 of the attractor [5].
- D_2 is related to the probability that two particles are separated by a distance less than r.
- Polymer changes substantially the clustering properties of the particles, maximal clustering (i.e. minimum of D_2) is found for $\tau_s = 0.2$.

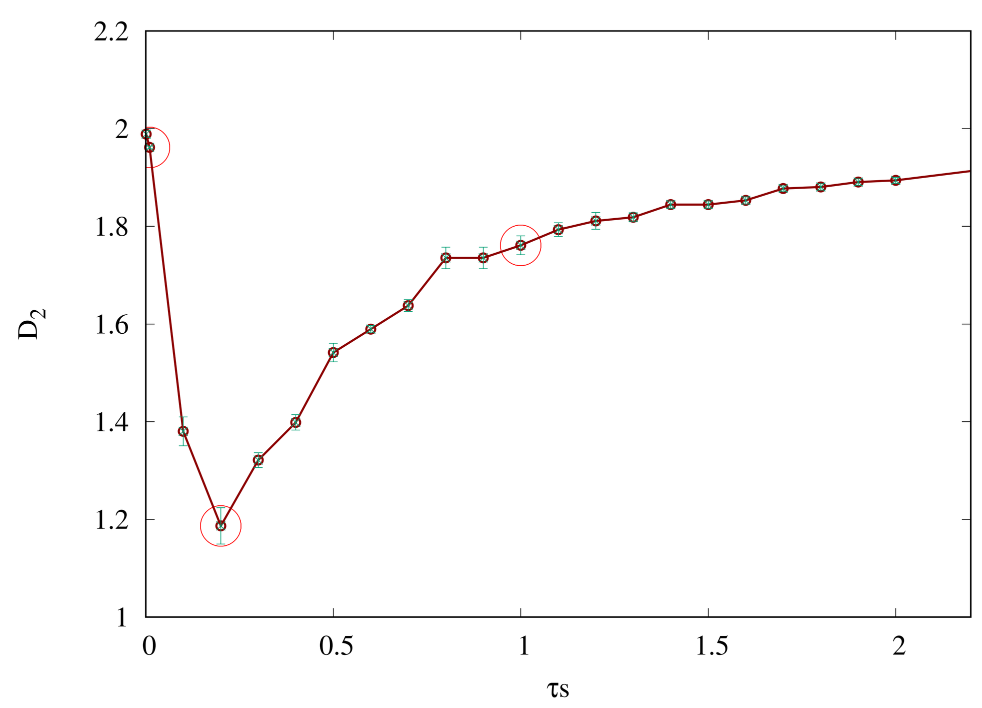

Figure 6: Correlation dimension D_2 as a function of particle relaxation time τ_s. Circled points correspond to particle distributions as in Figure 4

Forthcoming Research

- Complete understanding of distribution of particles over scales.
- Dependence on τ_s? Comparison with flow time scales. Other relevant parameters?
- Particle clustering and mixing efficiency at different Wi.

References

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