A Lagrangian model of Copepod dynamics in turbulent flows

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Université de Lille 1 Sciences et Technologies

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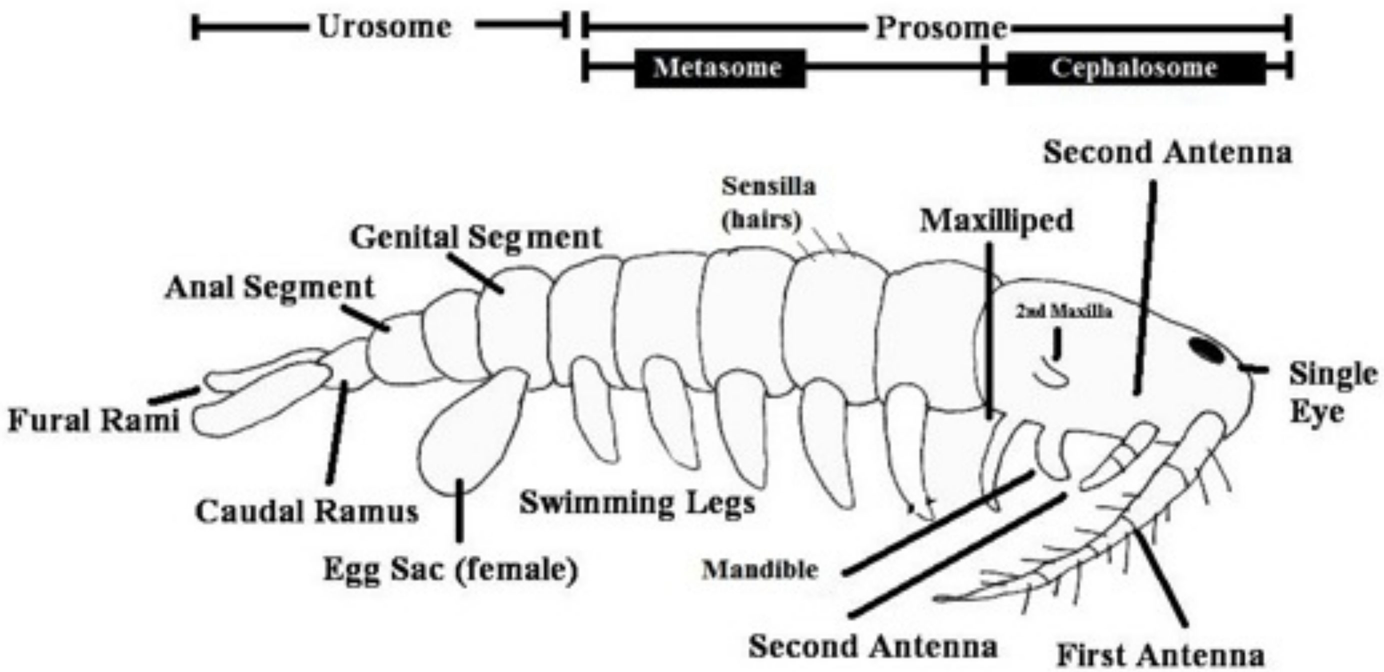
Outline

- Single Copepod dynamics (the essential)
- Results from a Simple Experiment
- A Lagrangian Copepod Model
- Numerical experiment with Turbulence
- Analysis and Main lessons
- Perspectives

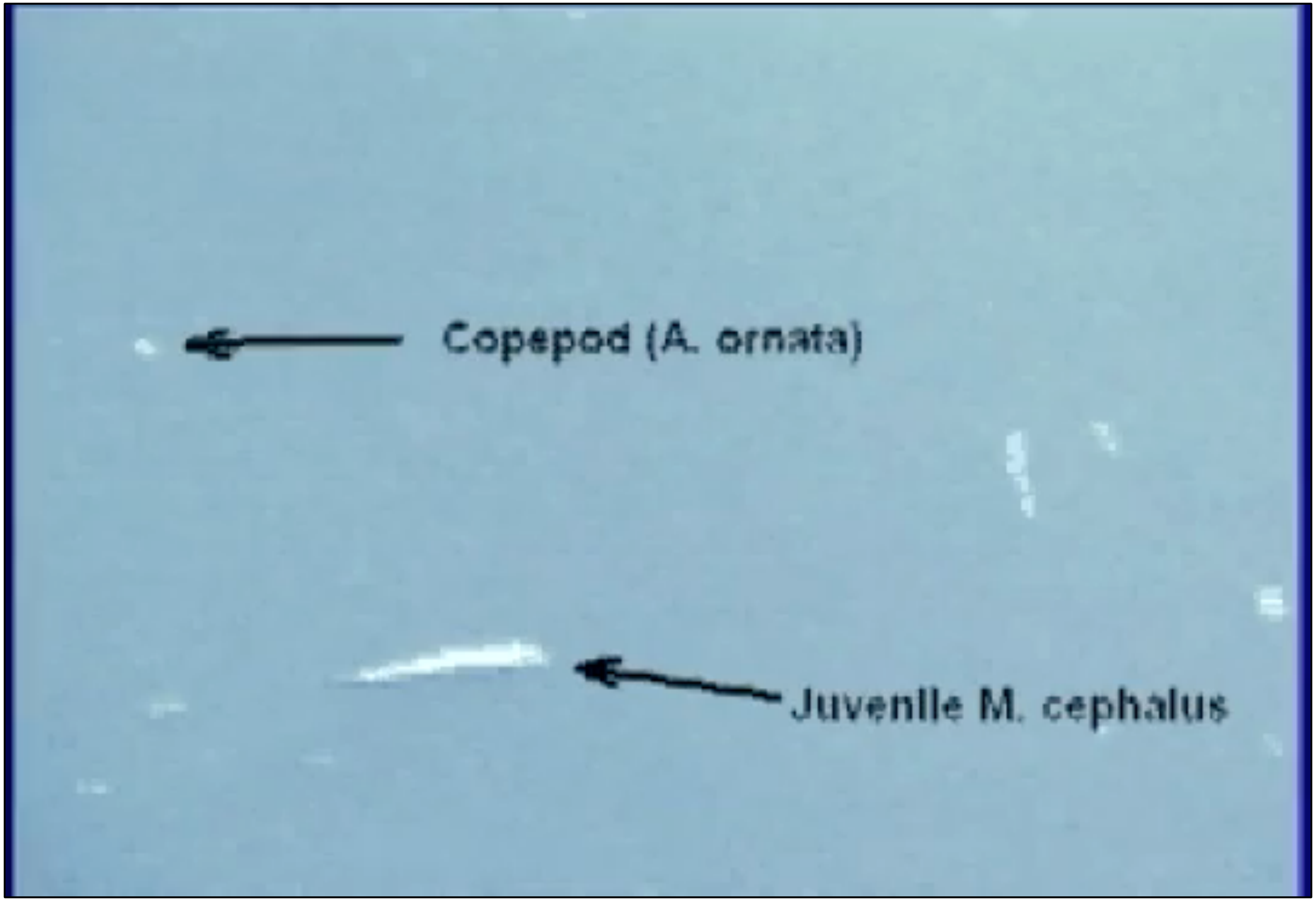
Copepods size and shape

- Group of crustaceans
- 9000 species
- **size 0.5mm to 15 mm**
- aspect ratio 3-5



Copepods locomotion

Main feature: **Swimming by jumps**



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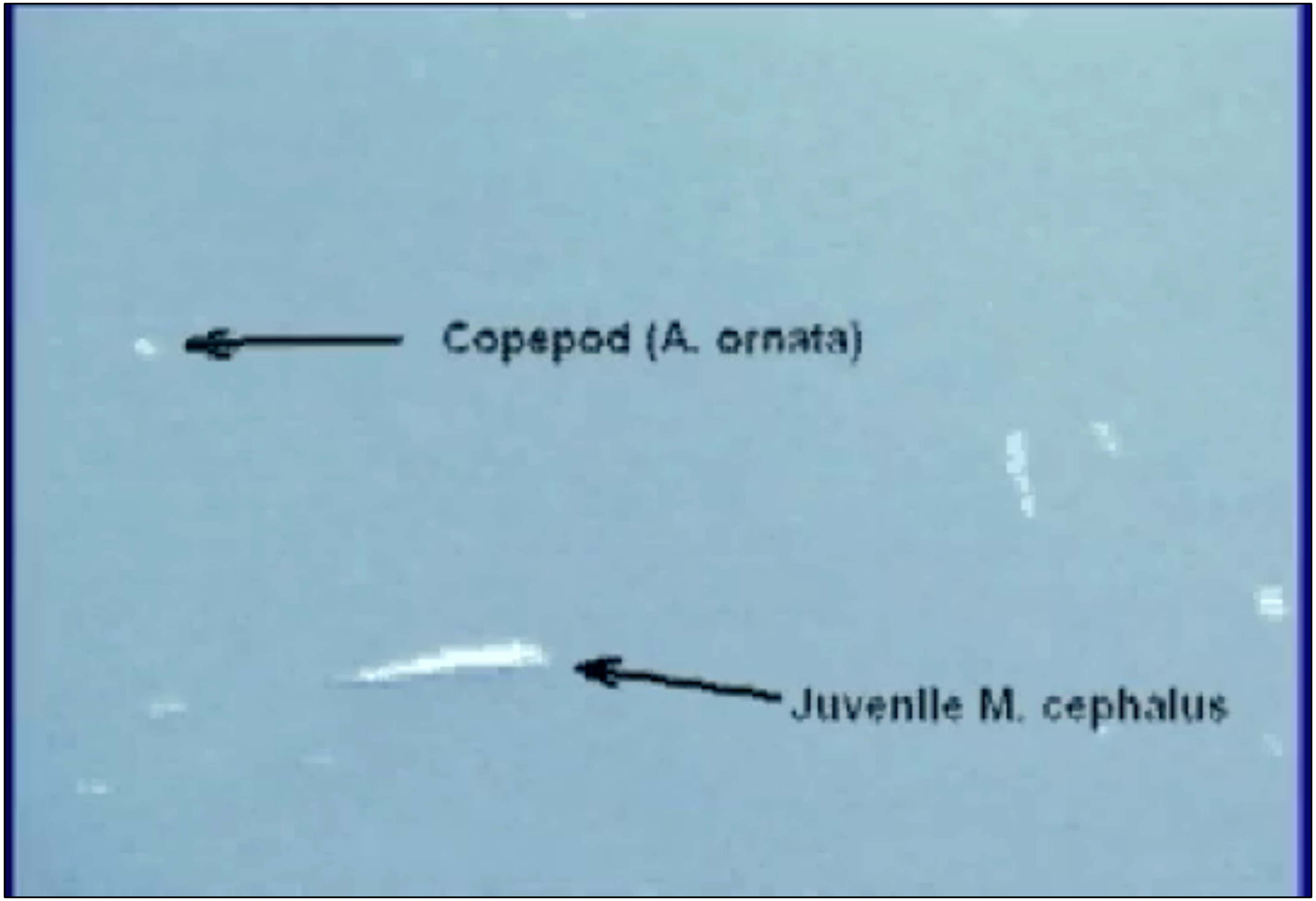

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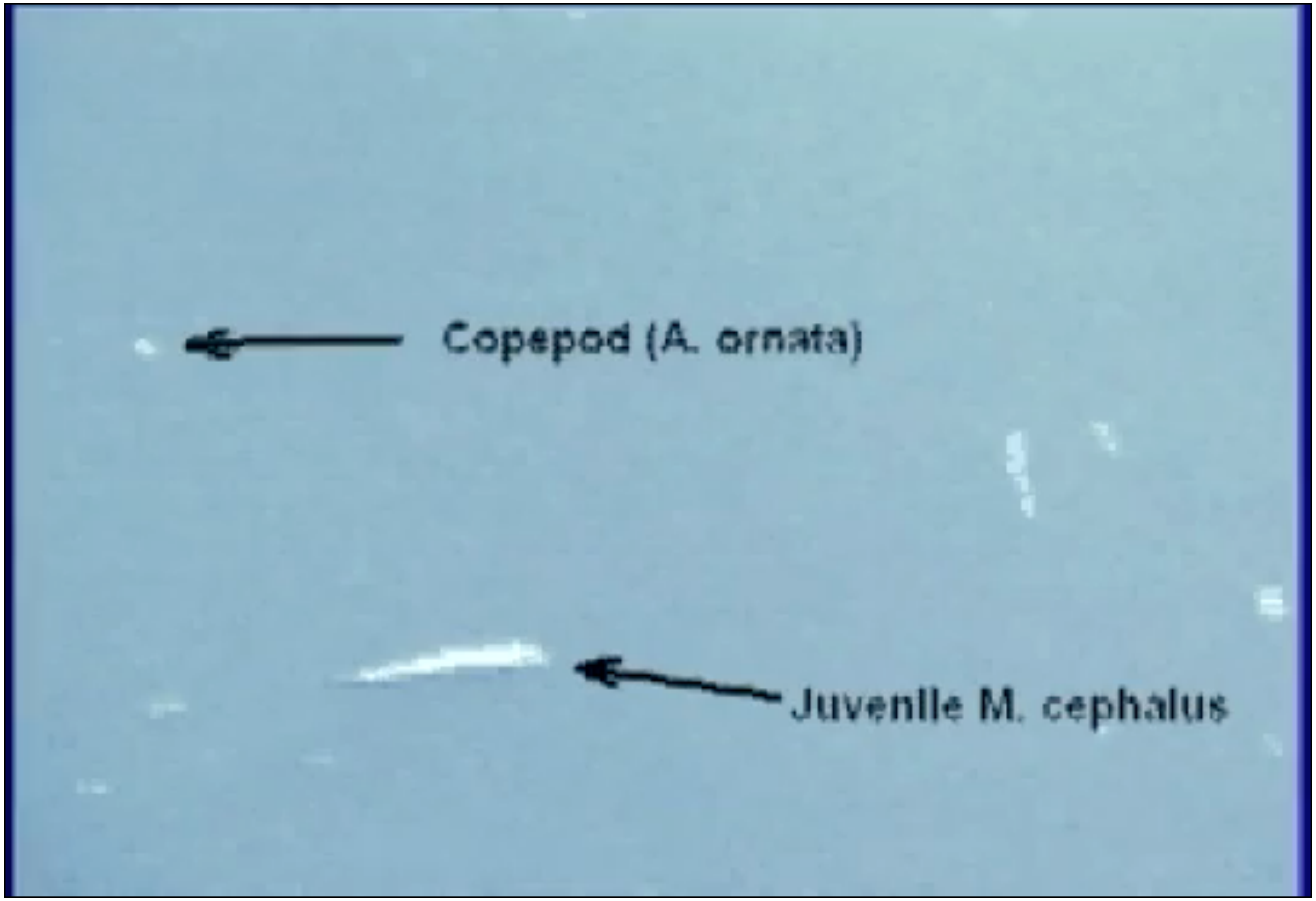
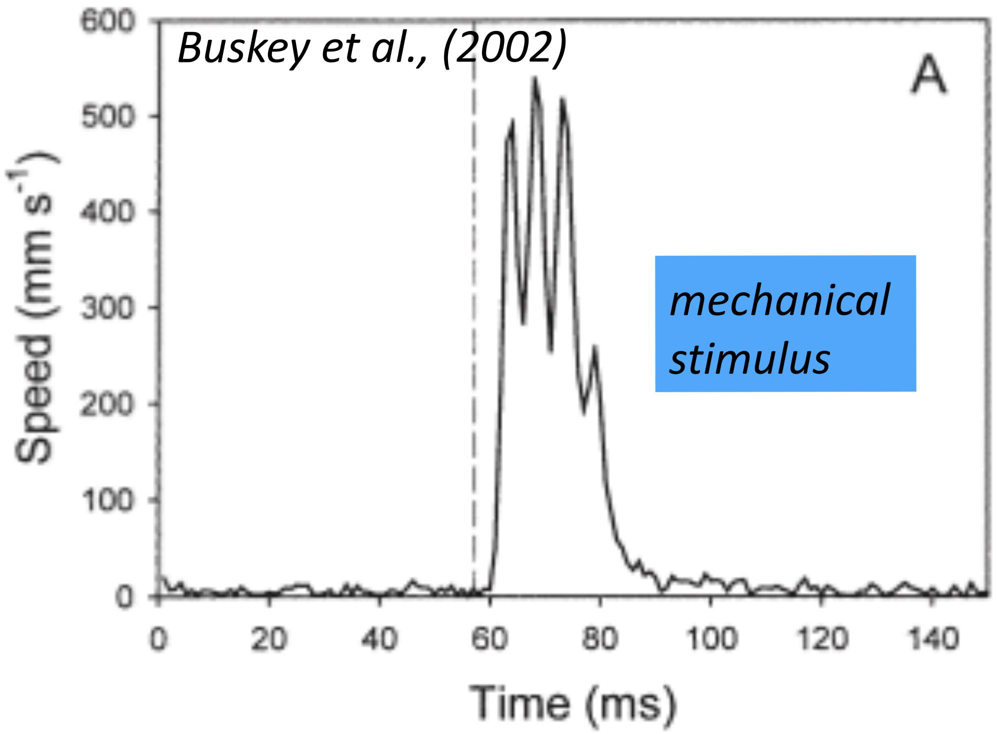
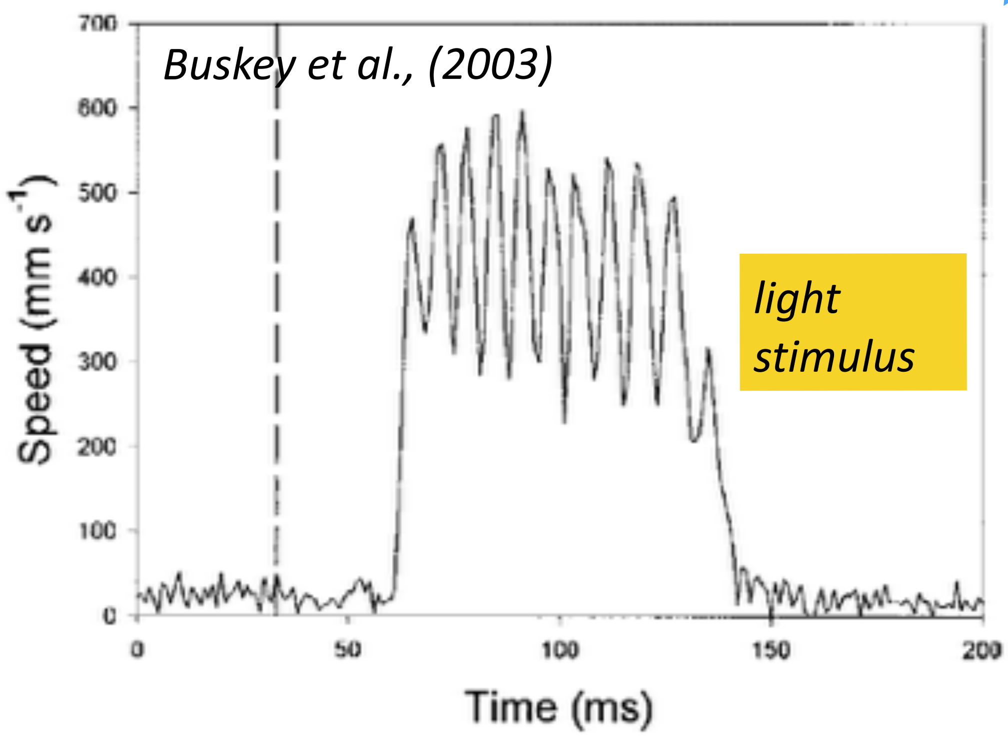


Copepods locomotion

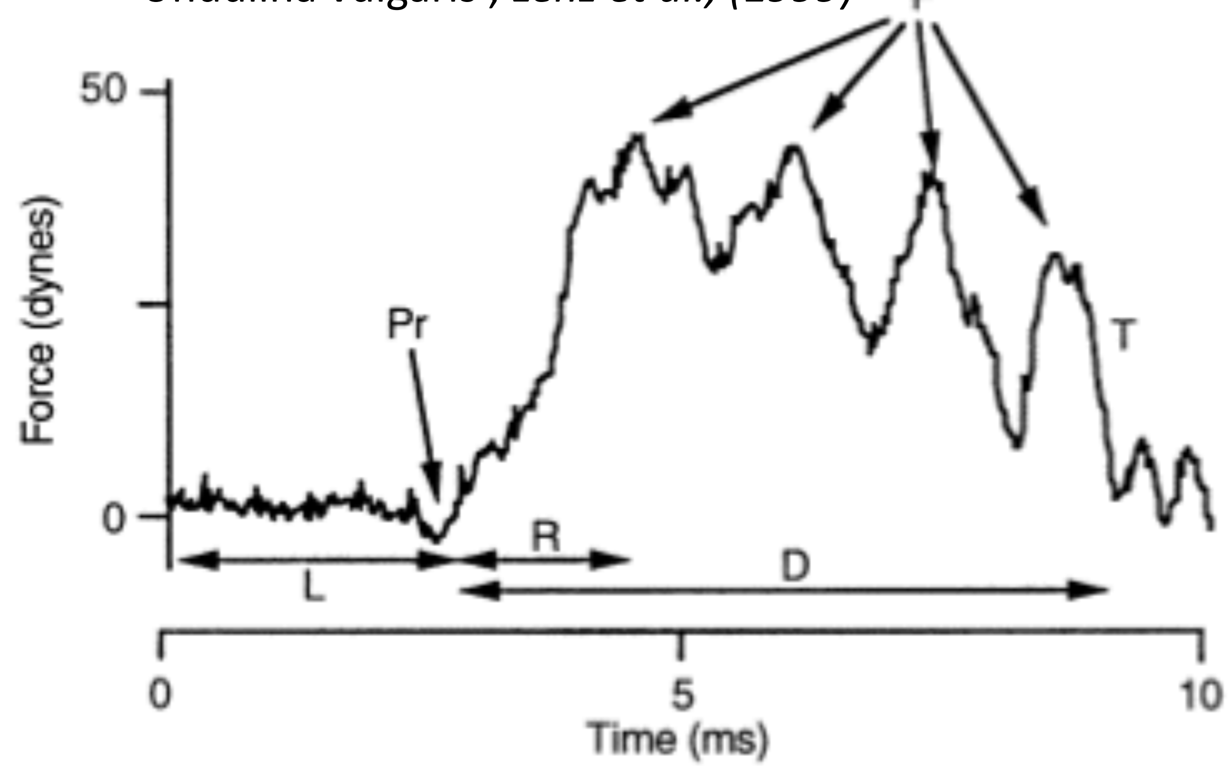
Jump response to light and mechanical signals



Acartia tonsa:
- Stimulus (dashed line)
- Initiation of the escape response (3ms delay for mechanical)
- Acceleration during initial escape ~10 g!



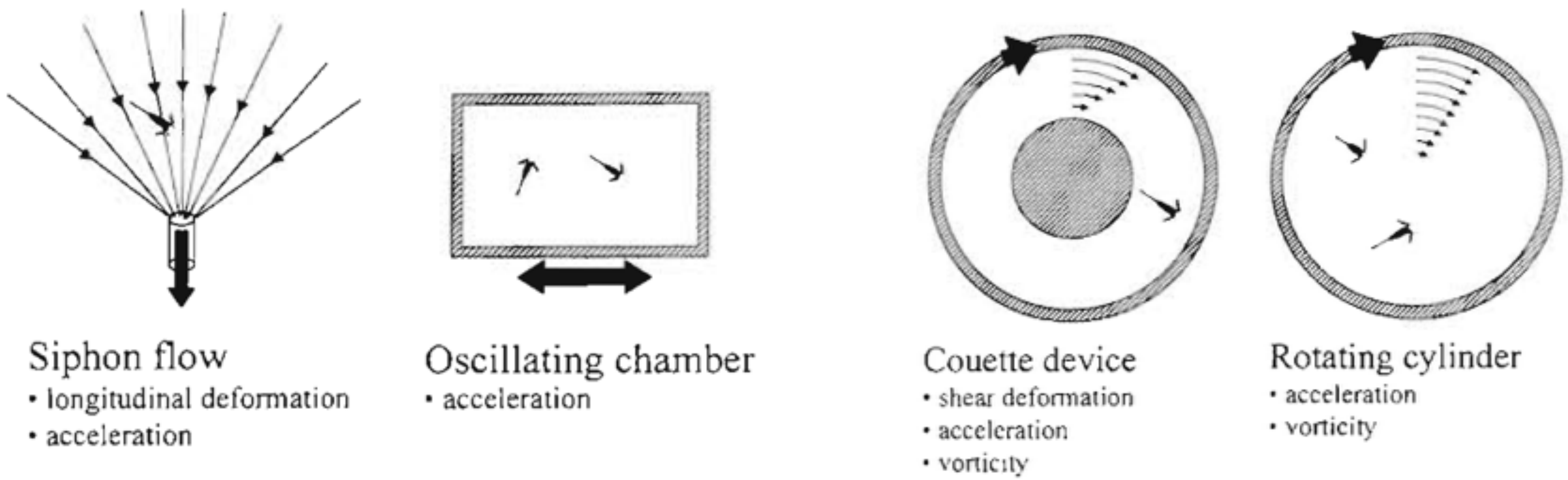
Undulina vulgaris , *Lenz et al., (1999)*



Copepods reaction to disturbances

Copepods react to deformation rate

Kiørboe & Visser (1999)



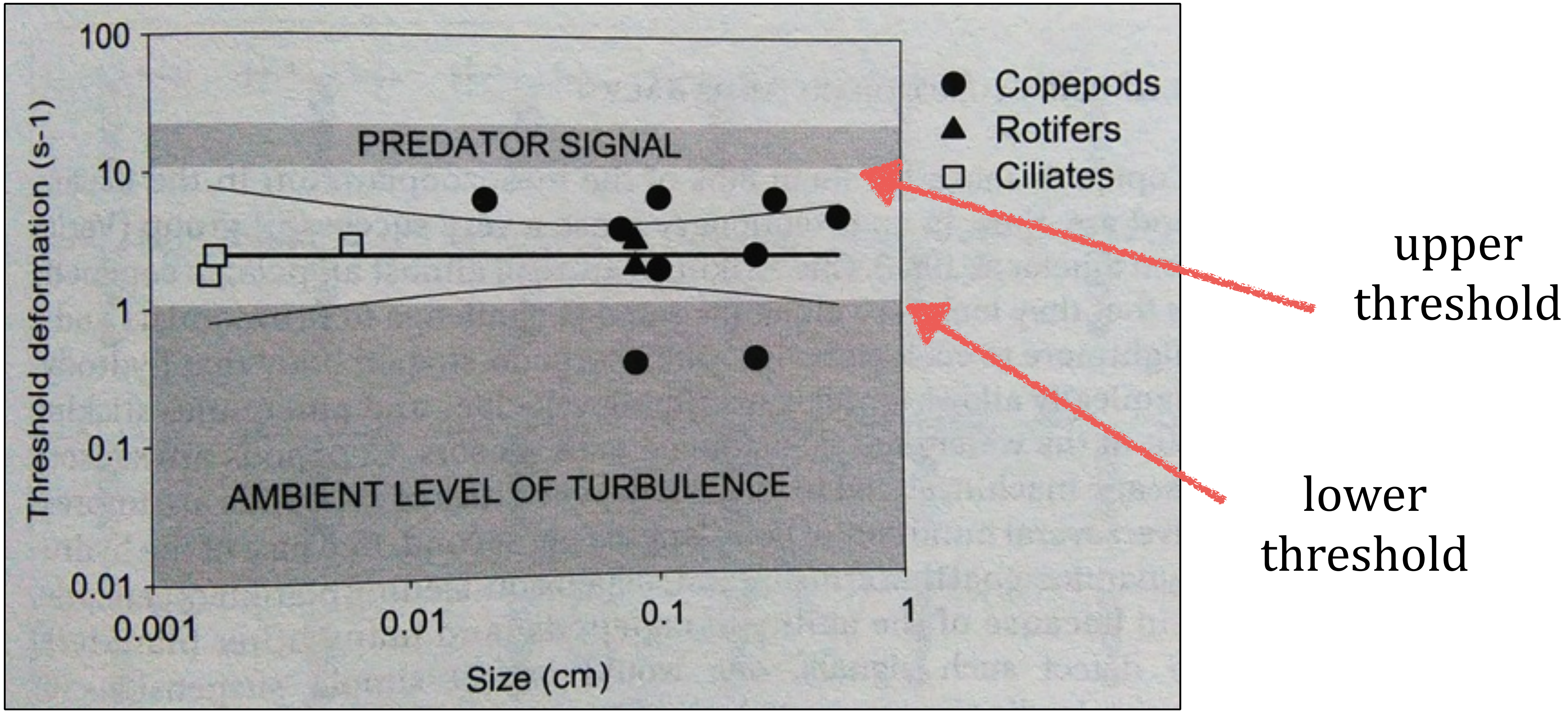
Acartia tonsa estimated threshold ~ 0.4 s^{-1}

Threshold is copepod size dependent

Sensitivity to strain rate ~ 0.025 s^{-1} Woodson et al. (2005,2007)

Copepods reaction to disturbances

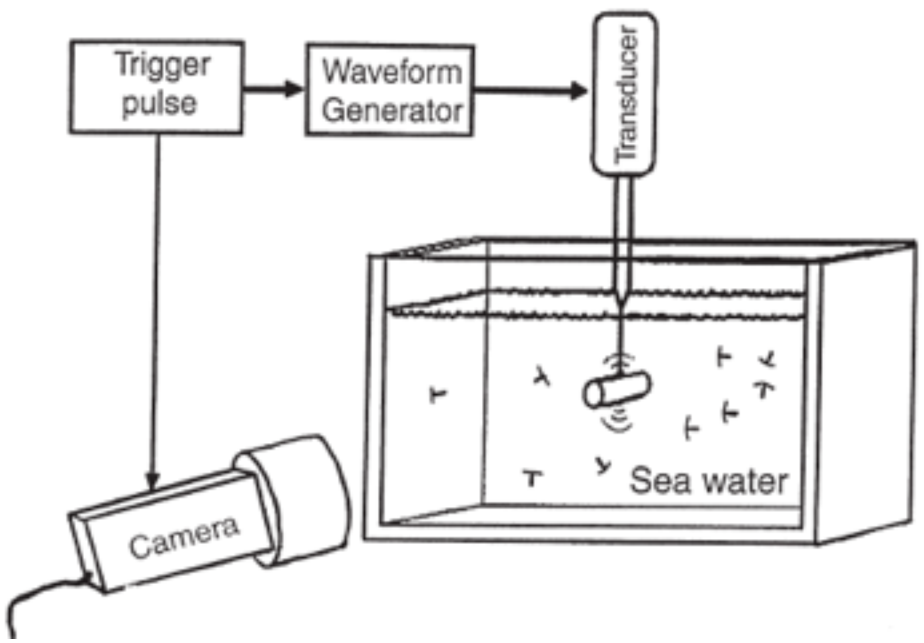
Copepods comfort region

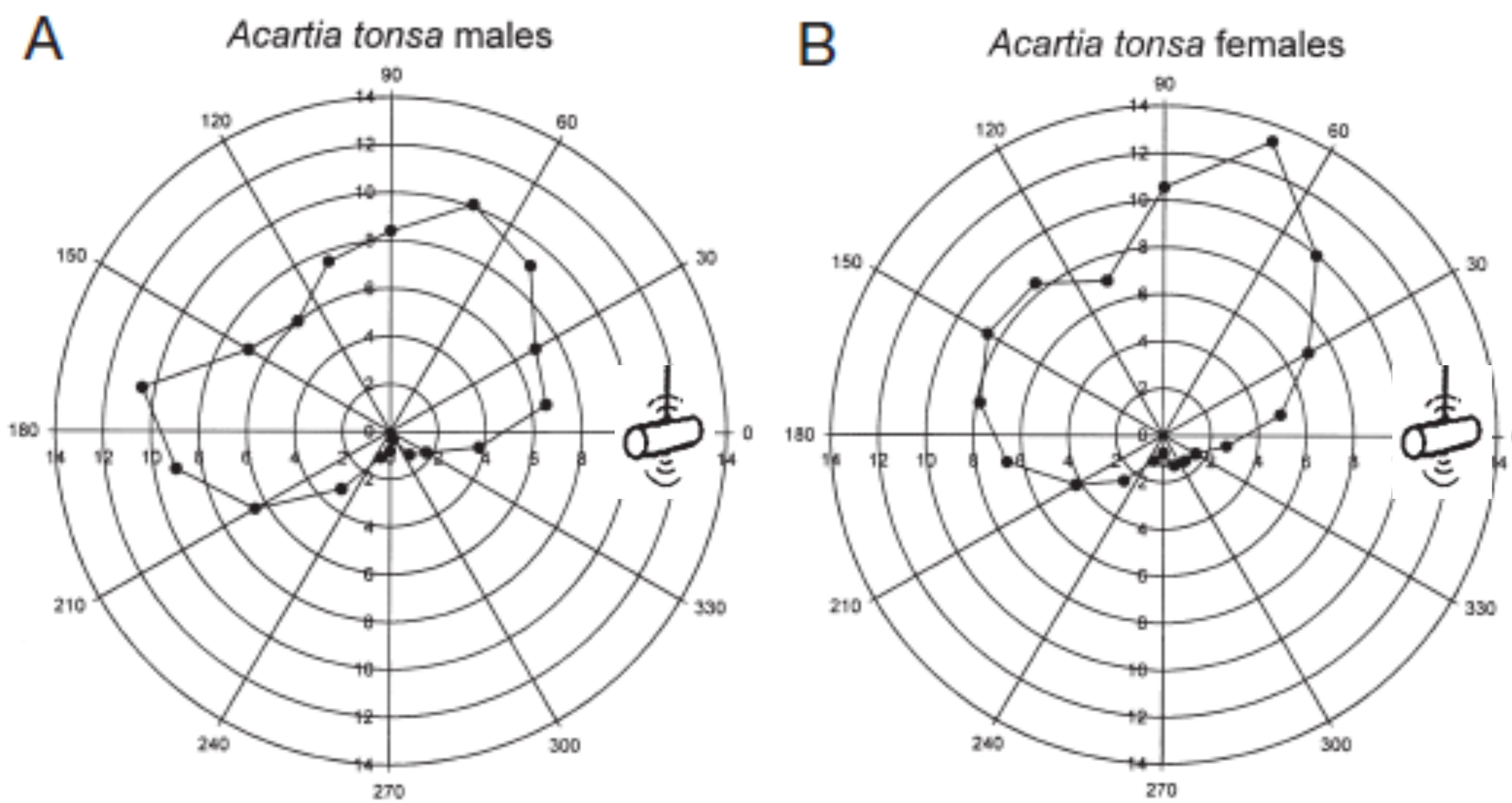


From T. *Kiørboe, A Mechanistic Approach to Plankton Ecology, (2008)*

Copepods reaction to disturbances

Direction of Escape





Buskey et all., (2002)

Escapes often begin with rapid reorientation away from the source of the disturbance, with maximum turning rates of about 30° ms^{-1}

Simple experiment

Copepods cultures at **LOG Lab** in Wimereux (France)

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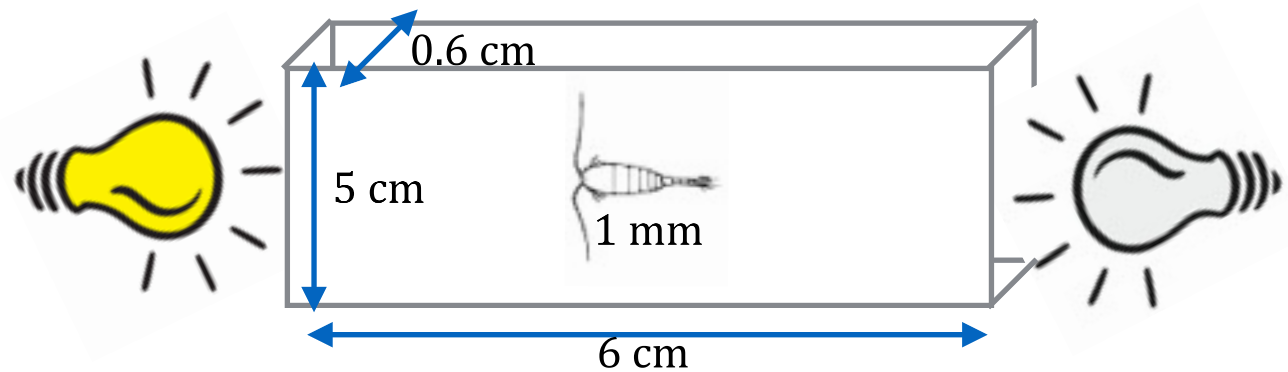
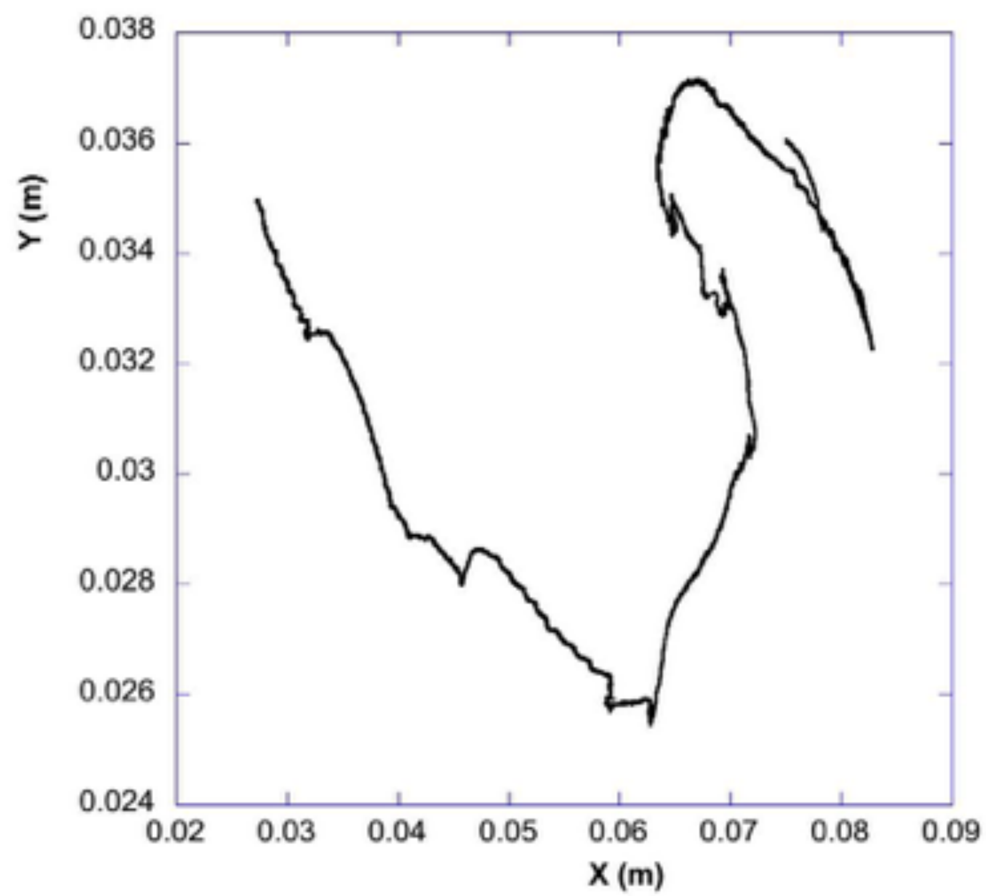
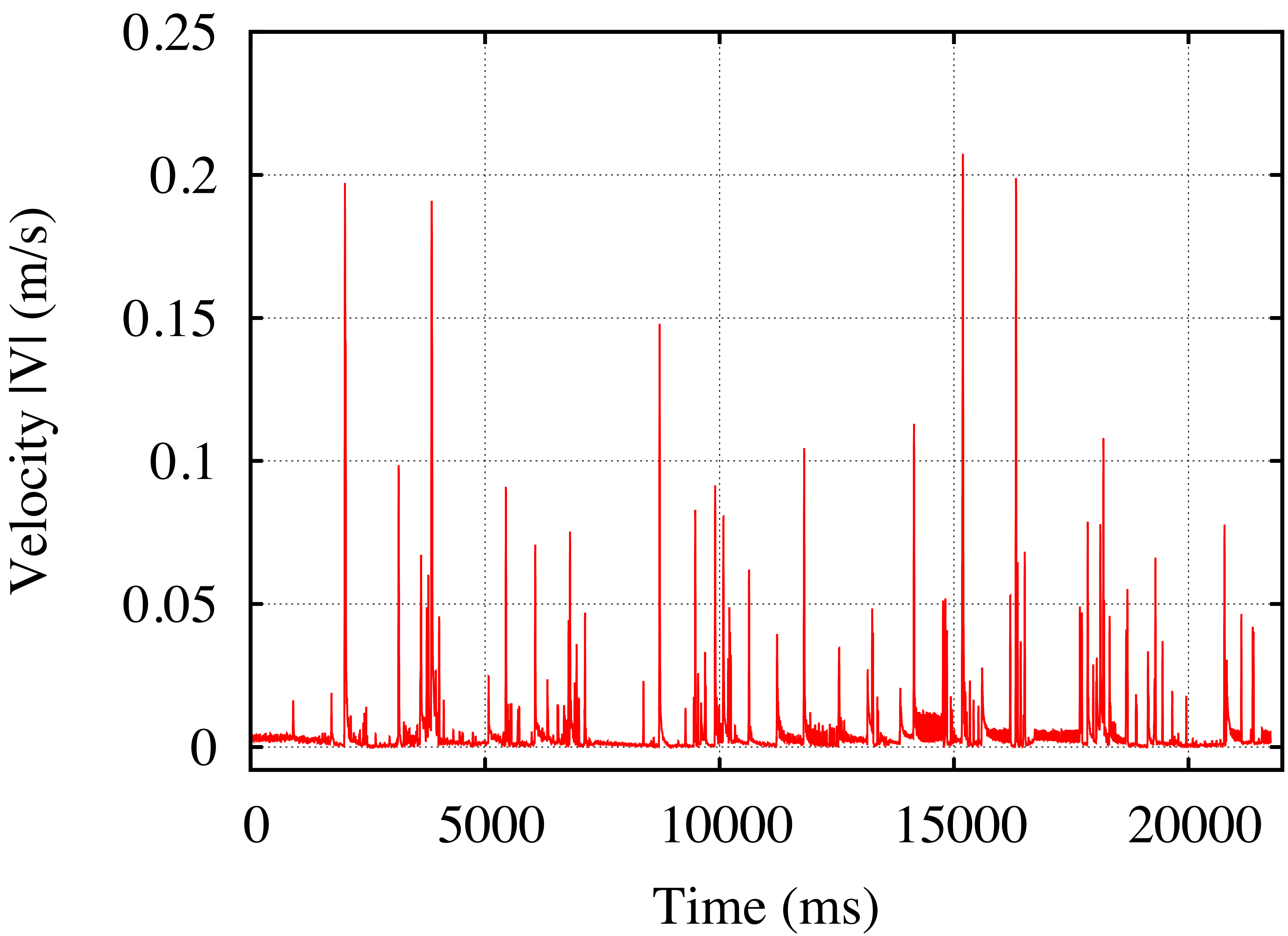


Response to **light stimuli**
of copepod "*Eurytemora affinis*"
in **still water**



Jump Data Analysis (1)

Velocity track



Jump Data Analysis (2)

Zooming in…

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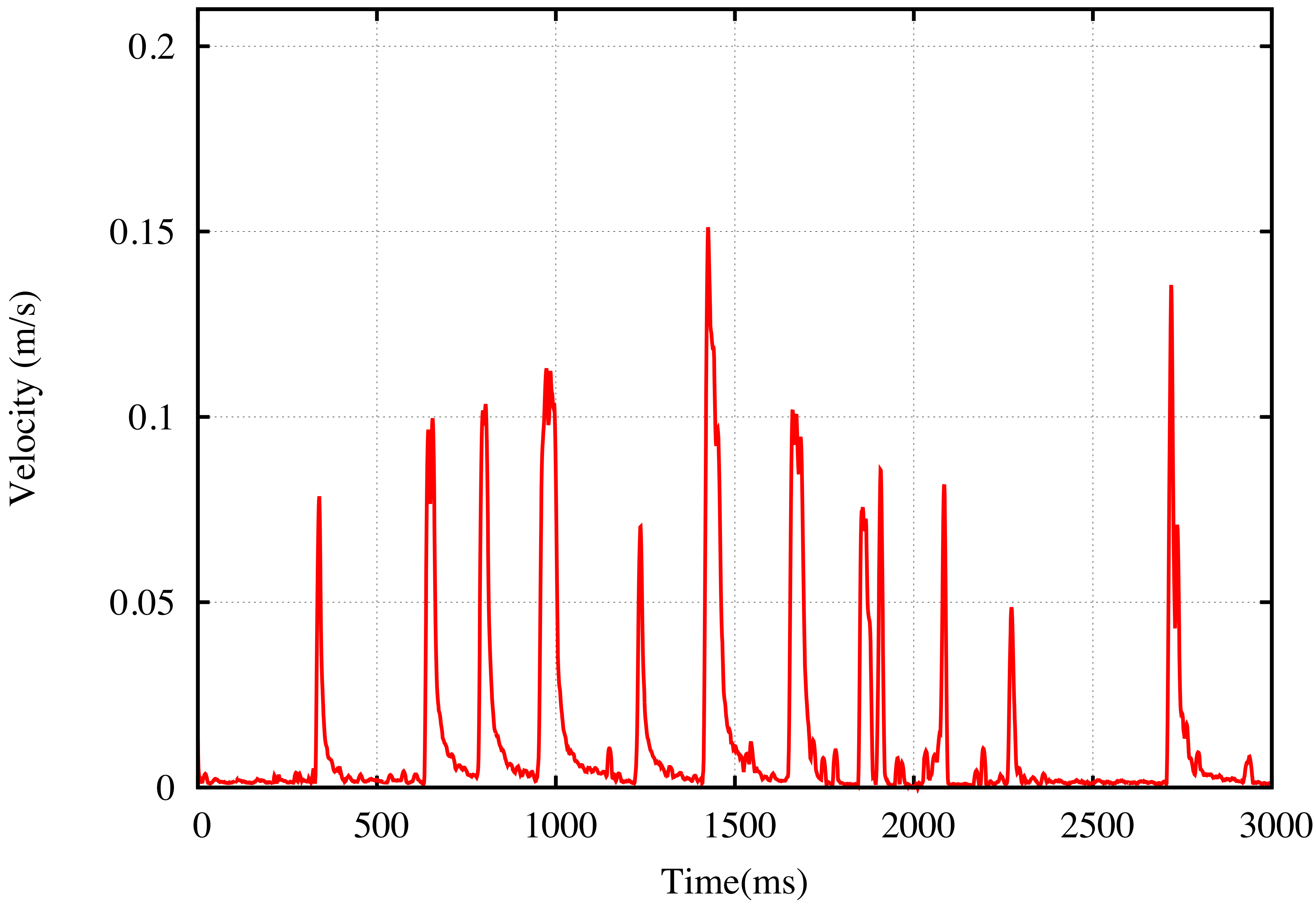
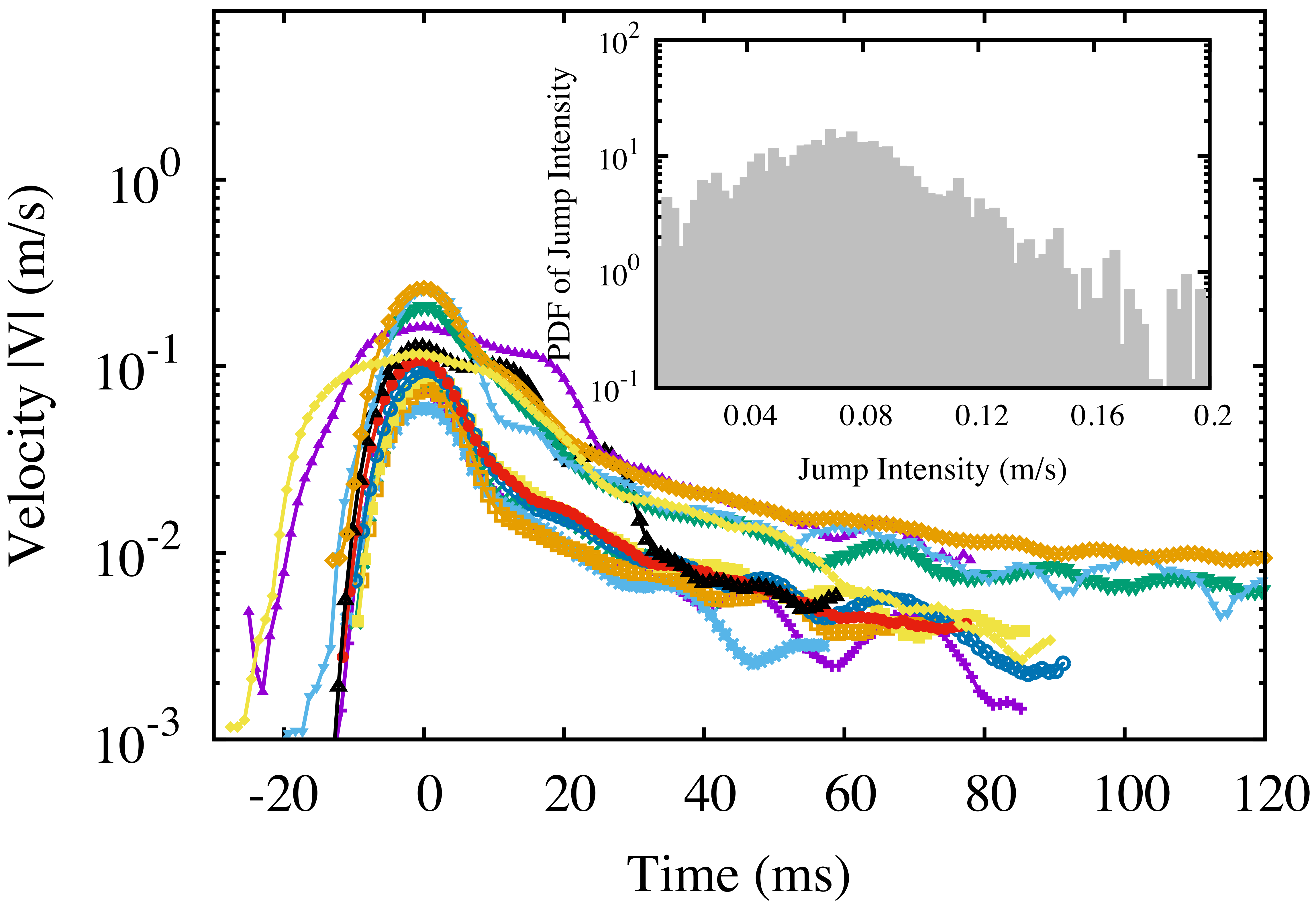
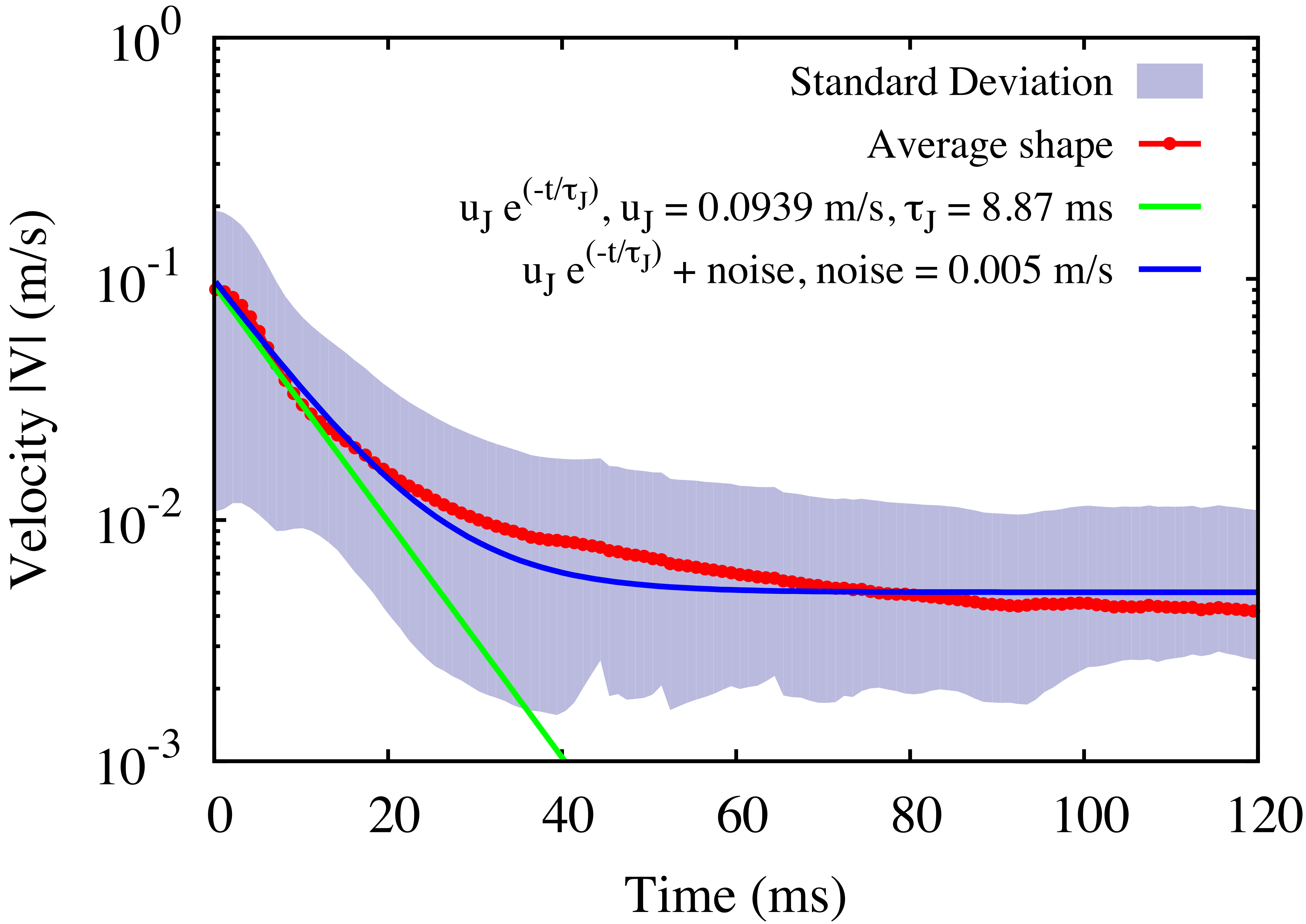


Jump Data Analysis (3)



Jump Data Analysis (4)



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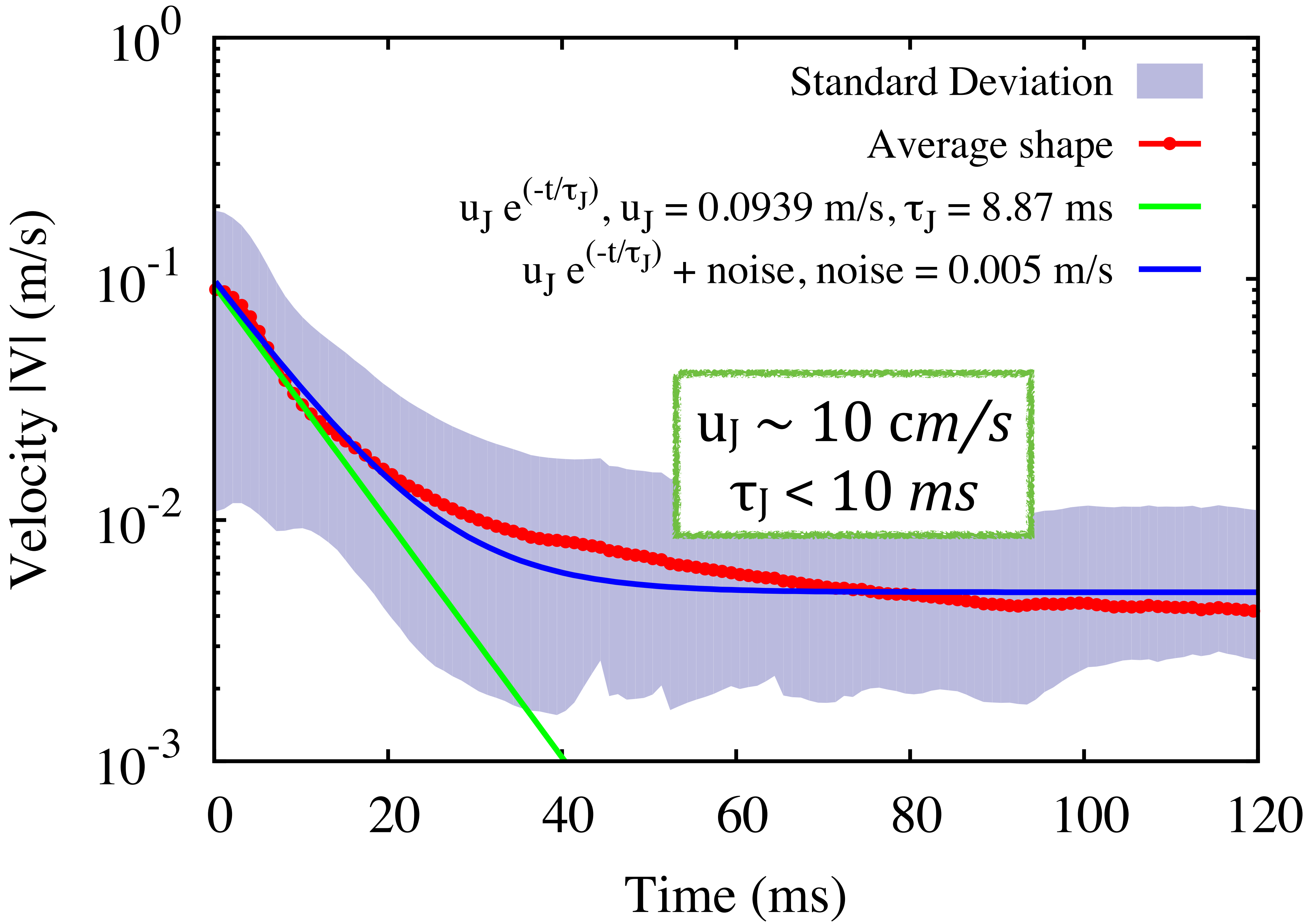


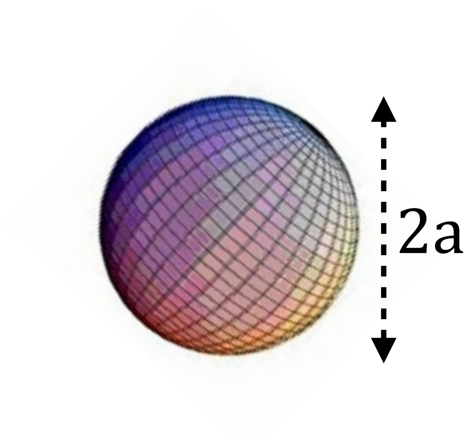
Hydrodynamical forces on a particle in a flow

Point-like model

$$\mathcal{V}\ \rho_p\ \ddot{\mathbf{X}} = \mathbf{F_P} + \mathbf{F_{AM}} + \mathbf{F_D} + \mathbf{F_H} + \mathbf{F_L} + \mathbf{F_B} + \mathbf{F_{FX}}$$

pressure gradient | added mass | Stokes drag | History drag | Lift | Buoyancy | Faxen

$$\begin{aligned}
\mathbf{F_P} &= \mathcal{V}\ \rho_f \frac{D\mathbf{U}}{D\mathcal{T}} \\
\mathbf{F_{AM}} &= \mathcal{V}\ \rho_f C_M \left(\frac{D\mathbf{U}}{D\mathcal{T}} - \ddot{\mathbf{X}} \right) \\
\mathbf{F_D} &= 6\ a\ \pi\ \mu\ (\mathbf{U} - \dot{\mathbf{X}}) \\
\mathbf{F_H} &= 6\ a^2 \sqrt{\pi\nu} \int_0^{\mathcal{T}} \frac{1}{\sqrt{\mathcal{T} - \tau}} \frac{d(\mathbf{U} - \dot{\mathbf{X}})}{d\tau} d\tau \\
\mathbf{F_L} &= \mathcal{V}\ \rho_f C_L (\mathbf{U} - \dot{\mathbf{X}}) \times \mathbf{\Omega} \\
\mathbf{F_B} &= \mathcal{V}\ (\rho_p - \rho_f)\ g\ \hat{\mathbf{e}}_z \\
\mathbf{F_{Fx}} &: \left(\frac{D\mathbf{U}}{D\mathcal{T}}, \mathbf{U} \right) \rightarrow \left(\langle \frac{D\mathbf{U}}{D\mathcal{T}} \rangle_\mathcal{V}, \langle \mathbf{U} \rangle_\mathcal{S} \right)
\end{aligned}$$



Hydrodynamical forces on a particle in a flow (2)

Point-like model

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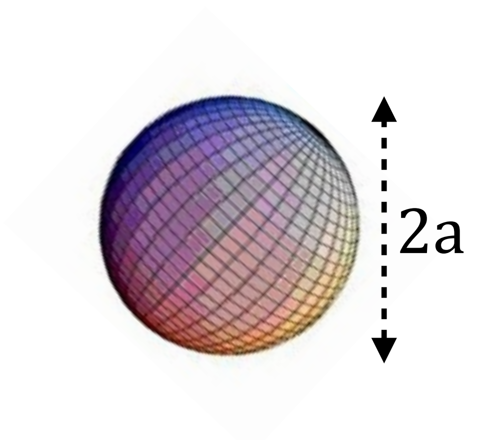

Hydrodynamical forces on a particle in a flow (2)

Major approximations

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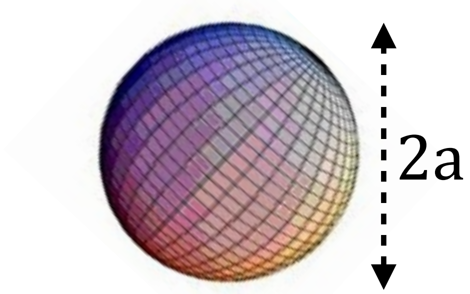

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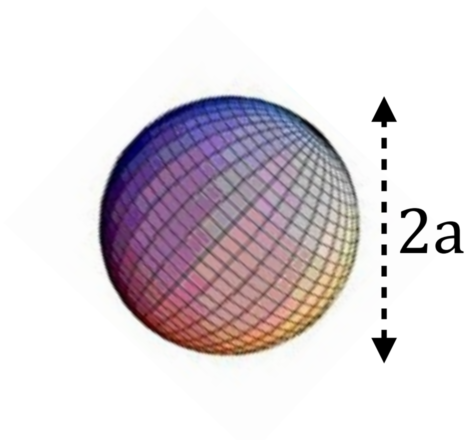
$$\ddot{\mathbf{X}} = \frac{1}{\tau_p}(\mathbf{U} - \dot{\mathbf{X}}) \qquad \tau_p = \frac{2}{9}\frac{\rho_p}{\rho_f}\frac{a^2}{\nu}$$



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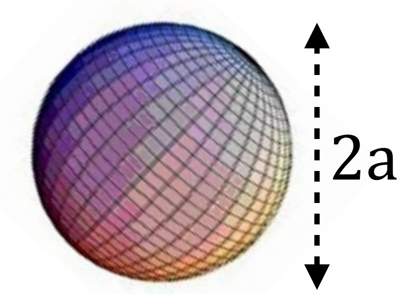
formal solution

$$\dot{\mathbf{X}}(\mathcal{T}) = e^{-\mathcal{T}/\tau_p}\int_0^{\mathcal{T}} \frac{e^{t/\tau_p}}{\tau_p}\mathbf{U}(t)dt + \dot{\mathbf{X}}(0)\; e^{-\mathcal{T}/\tau_p}$$



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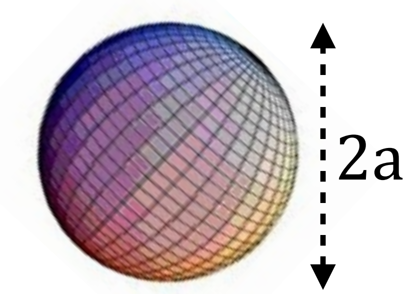

Initial condition $\dot{\mathbf{X}}(0) = \mathbf{U}(0) + \mathbf{U}_J$

Slowly varying $\mathbf{U}(t)$

$$\dot{\mathbf{X}}(\mathcal{T}) \simeq \mathbf{U}(0) + \mathbf{U}_J\ e^{-\mathcal{T}/\tau_p}$$

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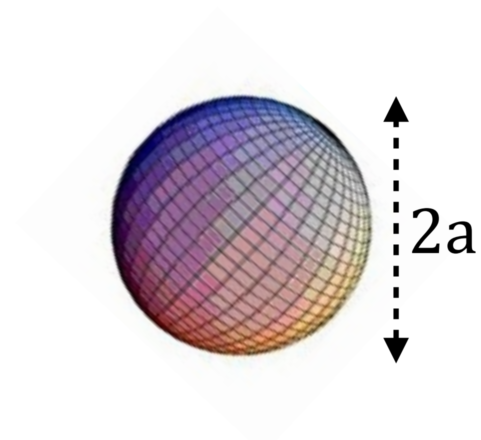

Initial condition $\dot{\mathbf{X}}(0) = \mathbf{U}(0) + \mathbf{U}_J$

Slowly varying $\mathbf{U}(t)$

$$\dot{\mathbf{X}}(\mathcal{T}) \cong \mathbf{U}(0) + \mathbf{U}_J\, e^{-\mathcal{T}/\tau_p}$$

Our guess
$\tau_p \sim \tau_J$

A minimal model

$$\dot{\mathbf{x}}(t) = \mathbf{u}(\mathbf{x}(t), t) + \boldsymbol{J}(t, t_i, t_e, \dot{\gamma}, \mathbf{p})$$

copepod velocity

fluid velocity

Jump velocity term

Modified Chlamydomonas Model

A minimal model

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Modified Chlamydomonas Model

copepod velocity

fluid velocity

Jump velocity term

$$\boldsymbol{J}(t, t_i, t_e, \dot{\gamma}, \mathbf{p}) = H[\dot{\gamma}(t_i) - \dot{\gamma}_T]\, H[t_e - t] u_J\, e^{\frac{t_i - t}{\tau_J}}\, \mathbf{p}(t_i)$$

shear-rate trigger

inhibition time

exponential decay

orientation

jump initial time

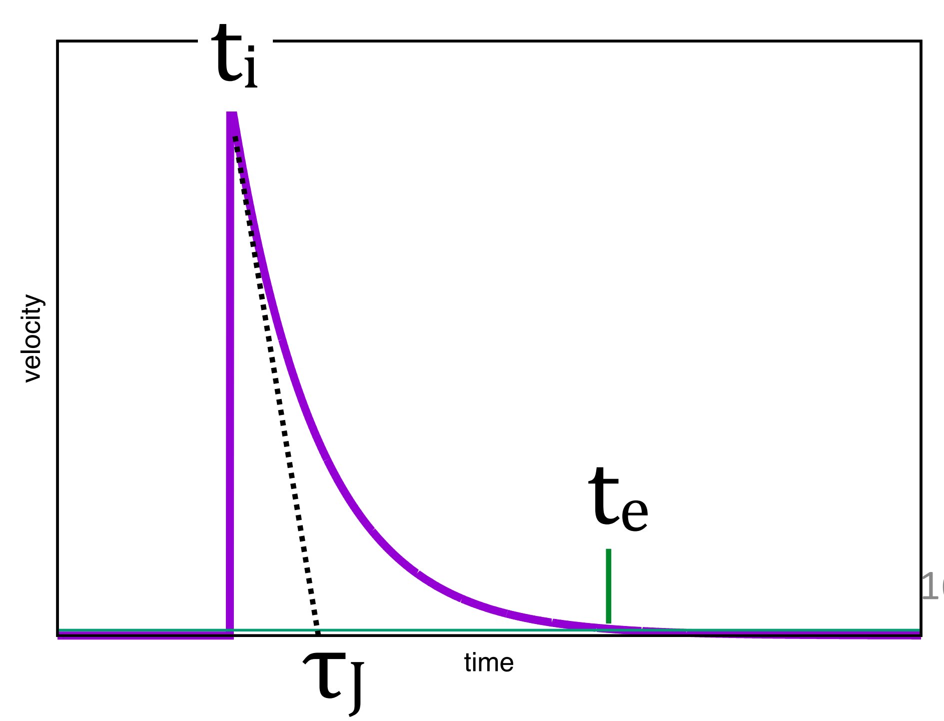
$$t_i = t \quad \text{if} \quad (\dot{\gamma}(t) > \dot{\gamma}_T) \cap (t > t_e)$$

jump end time

$$t_e = t_i + c\,\tau_J = t_i + \log(10^2)\,\tau_J$$

$\dot{\gamma}_T$ upper shear-rate threshold value

$$\dot{\gamma} = \sqrt{2\mathcal{S} : \mathcal{S}}$$



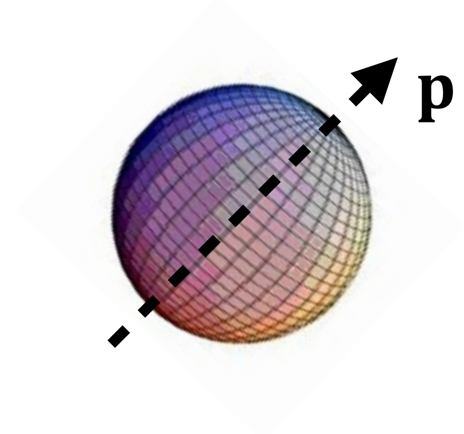
A minimal model (orientation)

Sphere

$$\dot{\mathbf{p}}(t) = \boldsymbol{\Omega} \cdot \mathbf{p}(t)$$

$$\Omega_{ij} = 1/2(\partial_i u_j - \partial_j u_i)$$

antisymmetric gradient tensor

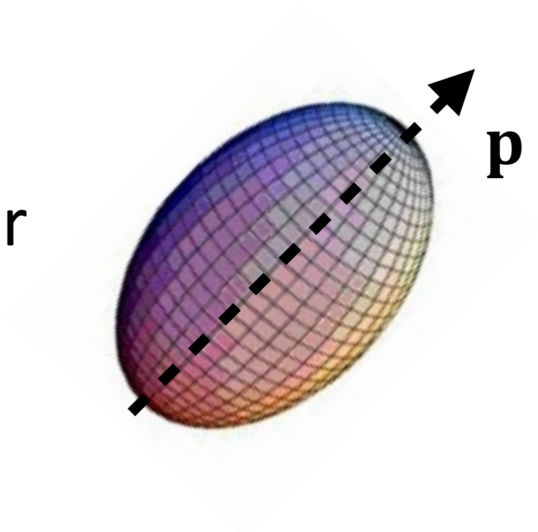


Ellipsoid: Jeffery equation

$$\dot{\mathbf{p}}(t) = \left(\boldsymbol{\Omega} + \frac{\alpha^2-1}{\alpha^2+1}\left(\mathcal{S} - \mathbf{p}^T(t)\cdot\mathcal{S}\cdot\mathbf{p}(t)\right)\right)\cdot\mathbf{p}(t)$$

$$S_{ij} = 1/2(\partial_i u_j + \partial_j u_i)$$

symmetric gradient tensor



$$\alpha \equiv l/d$$

Aspect ratio

A minimal model (summary)

Model assumptions:

- Rigid particle (spherical or ellipsoidal)
- neutral, homogeneous density
- Hydrodynamical forces: only Stokes Drag
- passive orientation

- Fixed response to external flow disturbances
- React to high shear-rate intensity (i.e. scalar single threshold)
- no-memory of previous jumps

- Model parameters: u_J τ_J, $\dot{\gamma}_T$, (t_e-t_i)

Lagrangian Copepods (LC) in a turbulent flow

In turbulent ocean flows:

Parameter	Unit	Range	
ν	m^2s^{-1}	$\sim 10^{-6}$	
ϵ	m^2s^{-3}	10^{-8}	10^{-4}
η	m	3×10^{-3}	3×10^{-4}
τ_η	s	10	0.1
u_η	ms^{-1}	3×10^{-4}	3×10^{-3}
Re_λ	–	$\mathcal{O}(10^2)$	

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$$u_J / u_\eta \sim O(10^2)$$

$$\tau_J / \tau_\eta \sim O(10^{-2})$$

$$\tau_\eta \dot{\gamma}_T \sim O(1)$$

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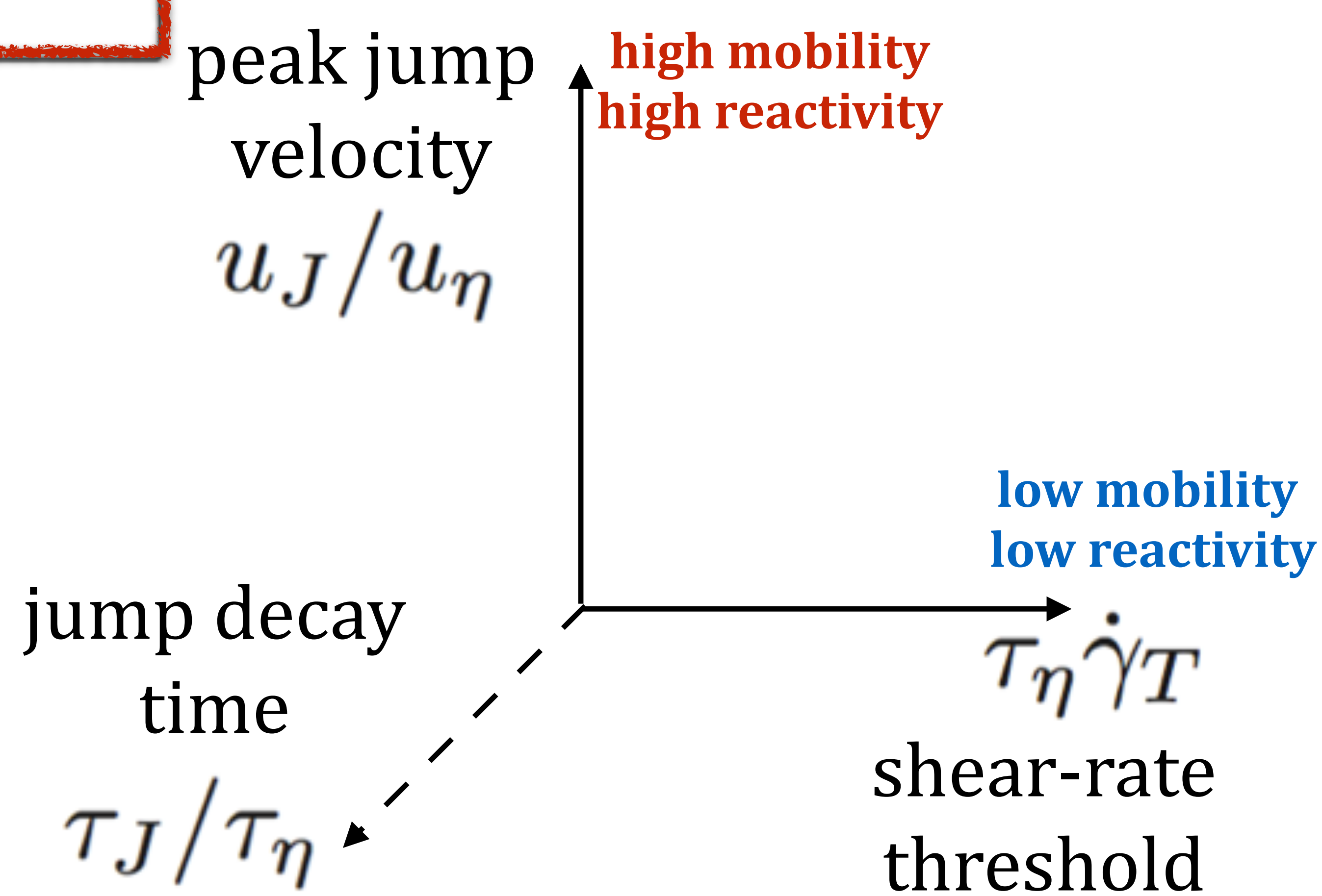
Model Parameter Space



Numerical Experiment

Lagrangian — Eulerian

LC model

$$\dot{\mathbf{x}}(t) = \mathbf{u}(\mathbf{x}(t), t) + \boldsymbol{J}(t, t_i, t_e, \dot{\gamma}, \mathbf{p})$$

Tracking of $\sim 10^6$ LC

families with different

u_J

$\dot{\gamma}_T$

$\tau_J \sim 10^{-2} \tau_\eta$

Homogeneous Isotropic Turbulence

$$\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p/\rho + \nu \Delta \mathbf{u} + \boldsymbol{f}$$

$$\nabla \cdot \mathbf{u} = 0$$

pseudo-spectral algorithm

$Re_\lambda \sim 80$

$N = 128^3$

3-periodic cube

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Will we see clusters?

Numerical Experiment

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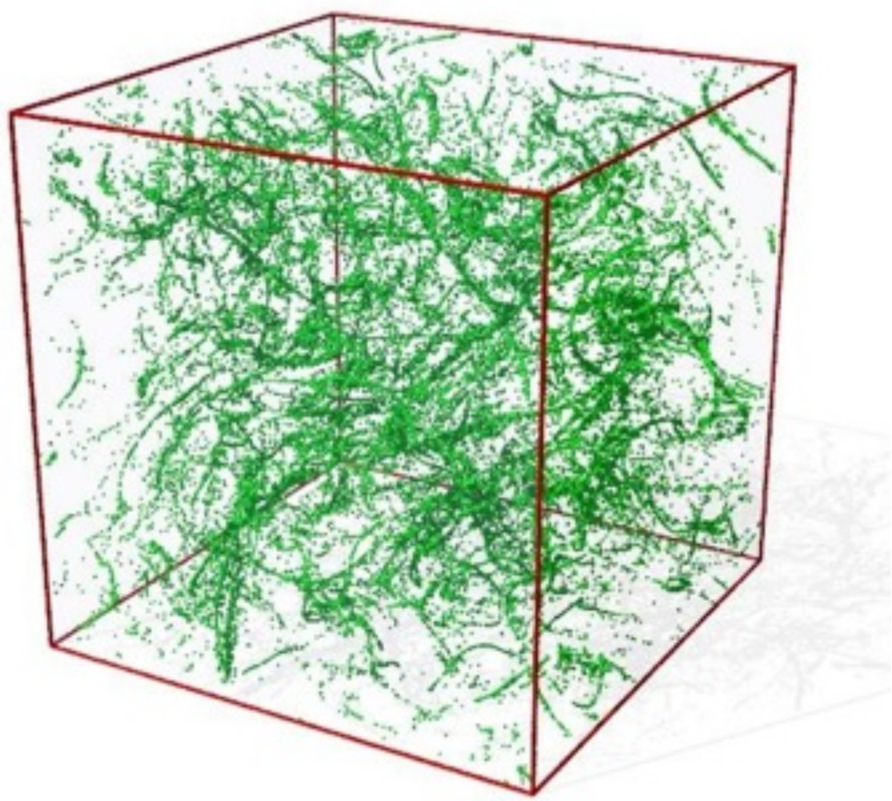

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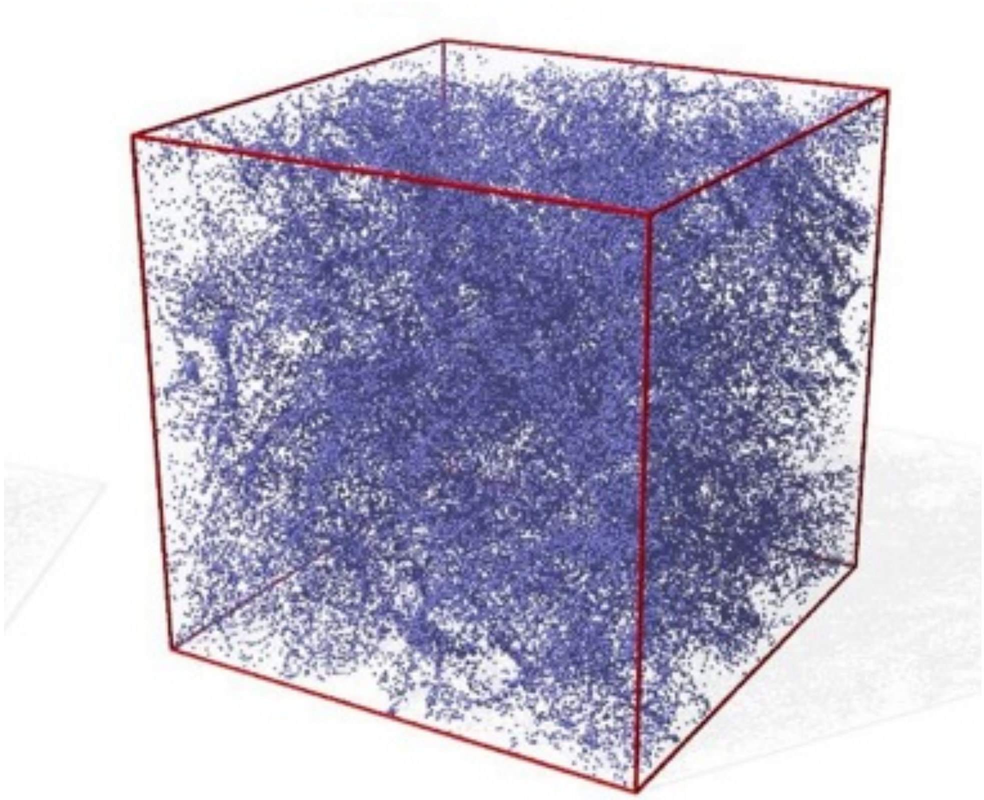


pseudo-spectral algorithm

$Re_\lambda \sim 80$

$N = 128^3$

3-periodic cube

Numerical Experiment

Lagrangian — Eulerian

LC model

$$\dot{\mathbf{x}}(t) = \mathbf{u}(\mathbf{x}(t), t) + \boldsymbol{J}(t, t_i, t_e, \dot{\gamma}, \mathbf{p})$$

Tracking of $\sim 10^6$ LC

families with different

u_J

$\dot{\gamma}_T$

$\tau_J \sim 10^{-2}\, \tau_\eta$

Homogeneous Isotropic Turbulence

$$\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p/\rho + \nu \Delta \mathbf{u} + \boldsymbol{f}$$

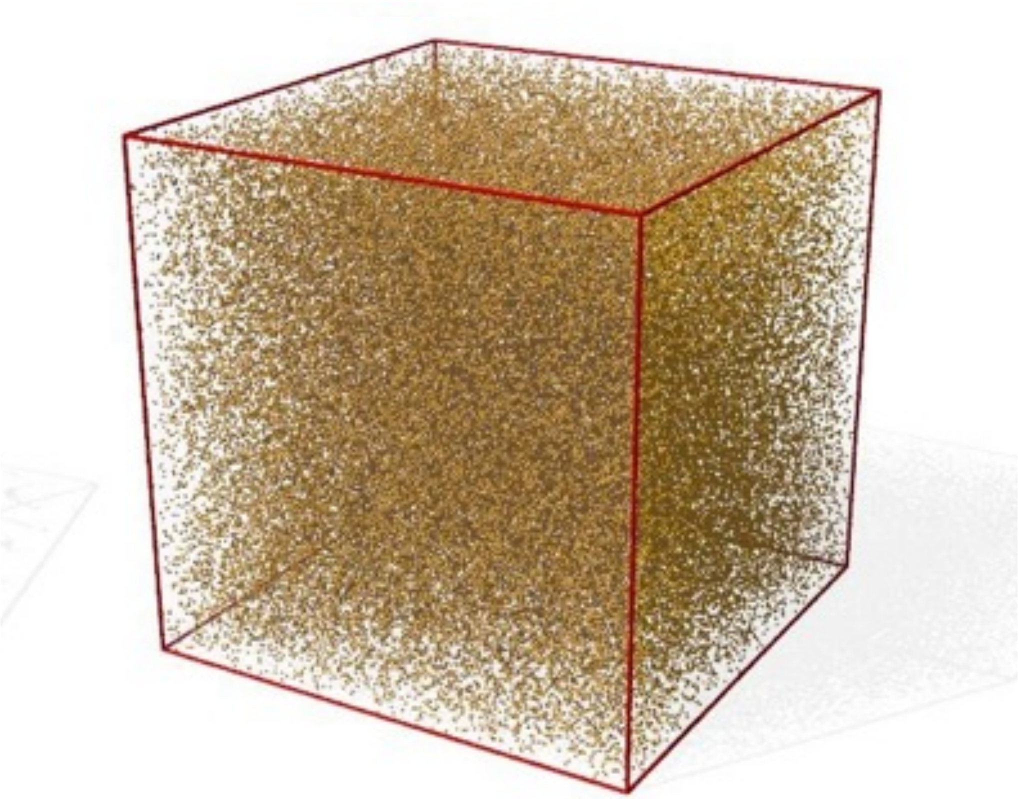
$$\nabla \cdot \mathbf{u} = 0$$

pseudo-spectral algorithm

$Re_\lambda \sim 80$

$N = 128^3$

3-periodic cube

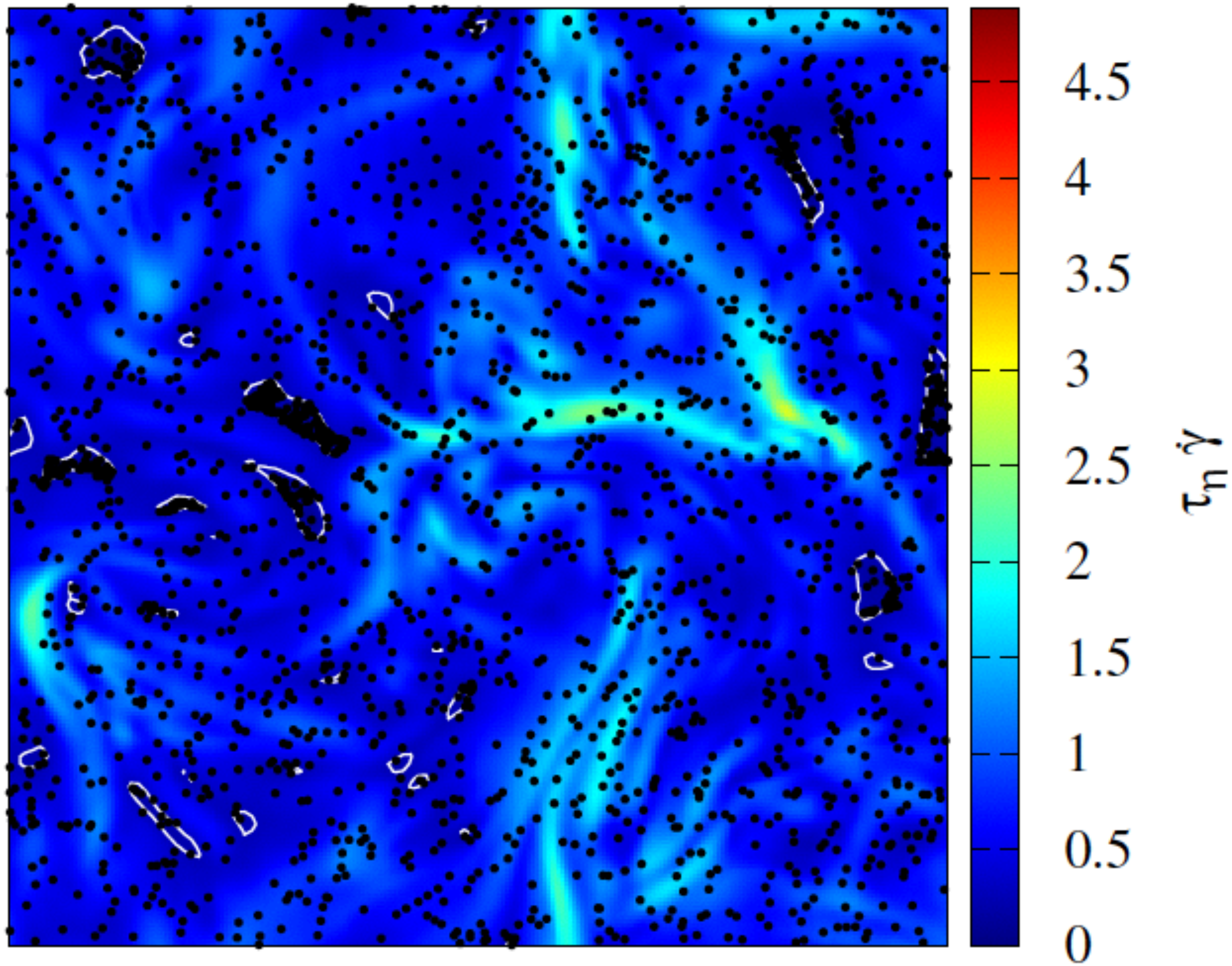


Spatial distribution

2D slice of thickness η , Jump intensity $u_J/u_\eta = 250$, $\tau_J = 10^{-2}\,\tau_\eta$

Jump shear-rate threshold

1) **Low**, many jumps, few dense islands



$\tau_\eta\,\dot\gamma_T = 0.35$

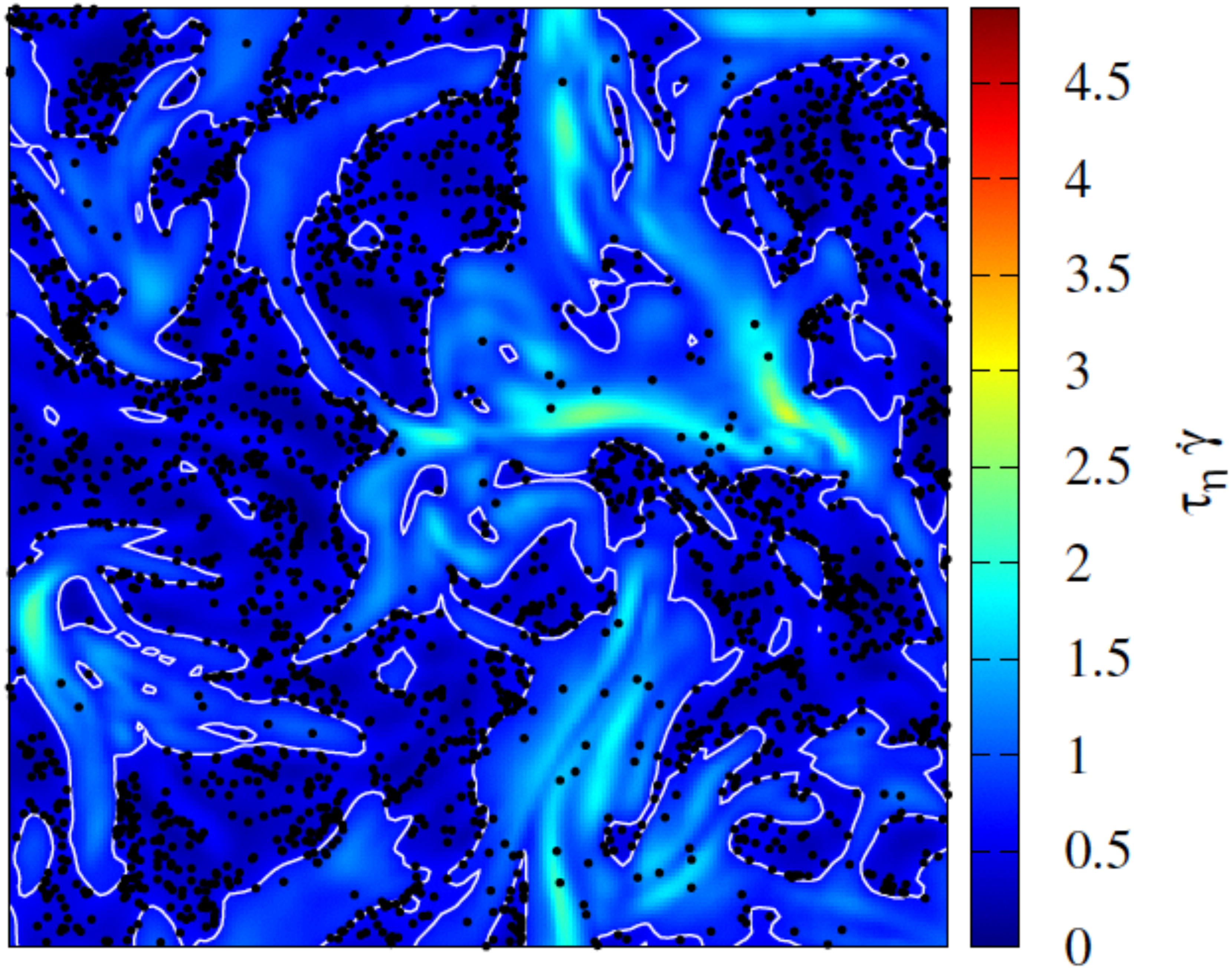
Spatial distribution

2D slice of thickness η , Jump intensity $u_J/u_\eta = 250$, $\tau_J = 10^{-2}\,\tau_\eta$

Jump shear-rate threshold

1) **Low**, many jumps, few dense islands

2) **Intermediate,** efficient escape, sheet-like clusters



$\tau_\eta \dot{\gamma}_T = 0.92$

Spatial distribution

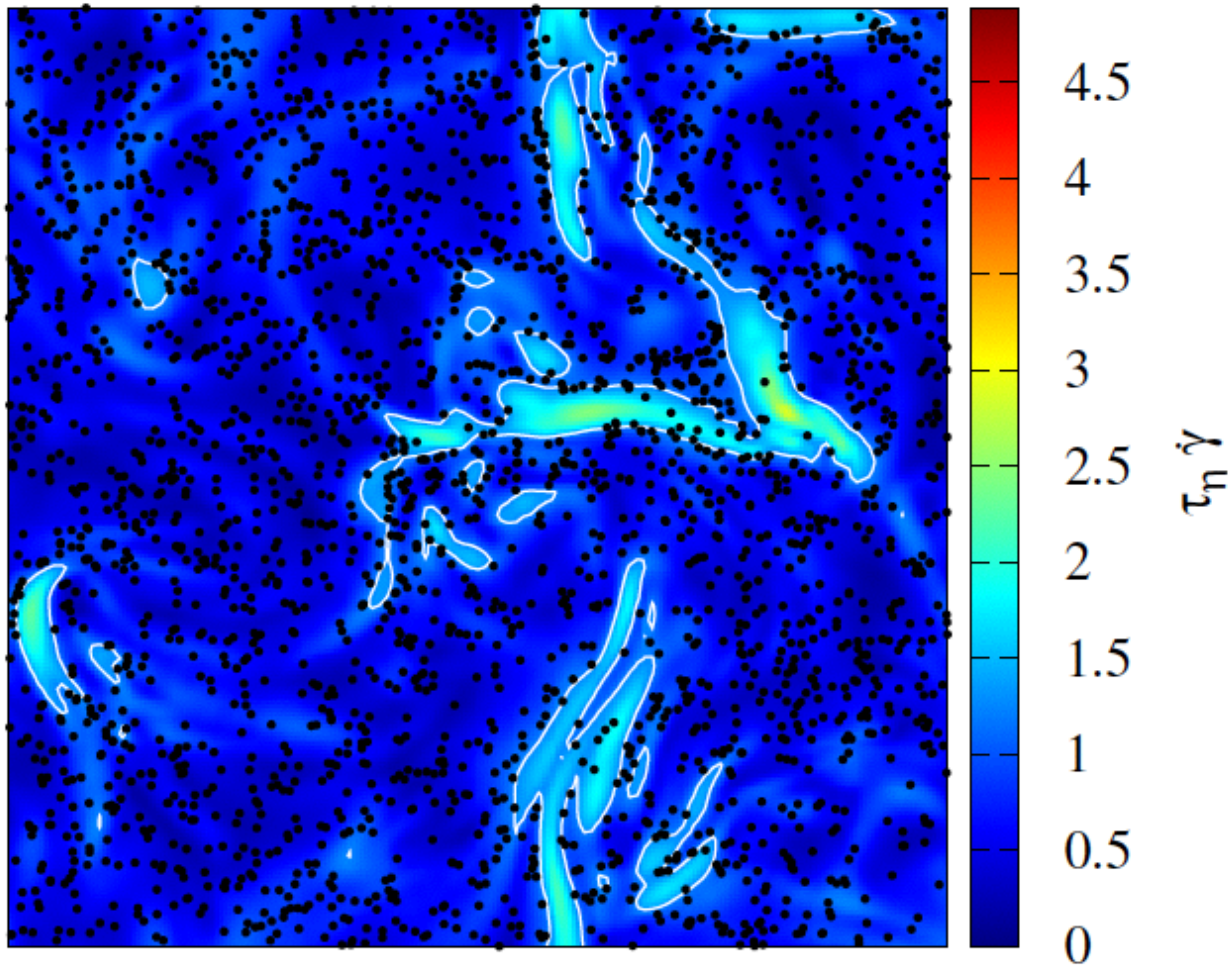
2D slice of thickness η , Jump intensity $u_J/u_\eta = 250$, $\tau_J = 10^{-2}\,\tau_\eta$

Jump shear-rate threshold

1) **Low**, many jumps, few dense islands

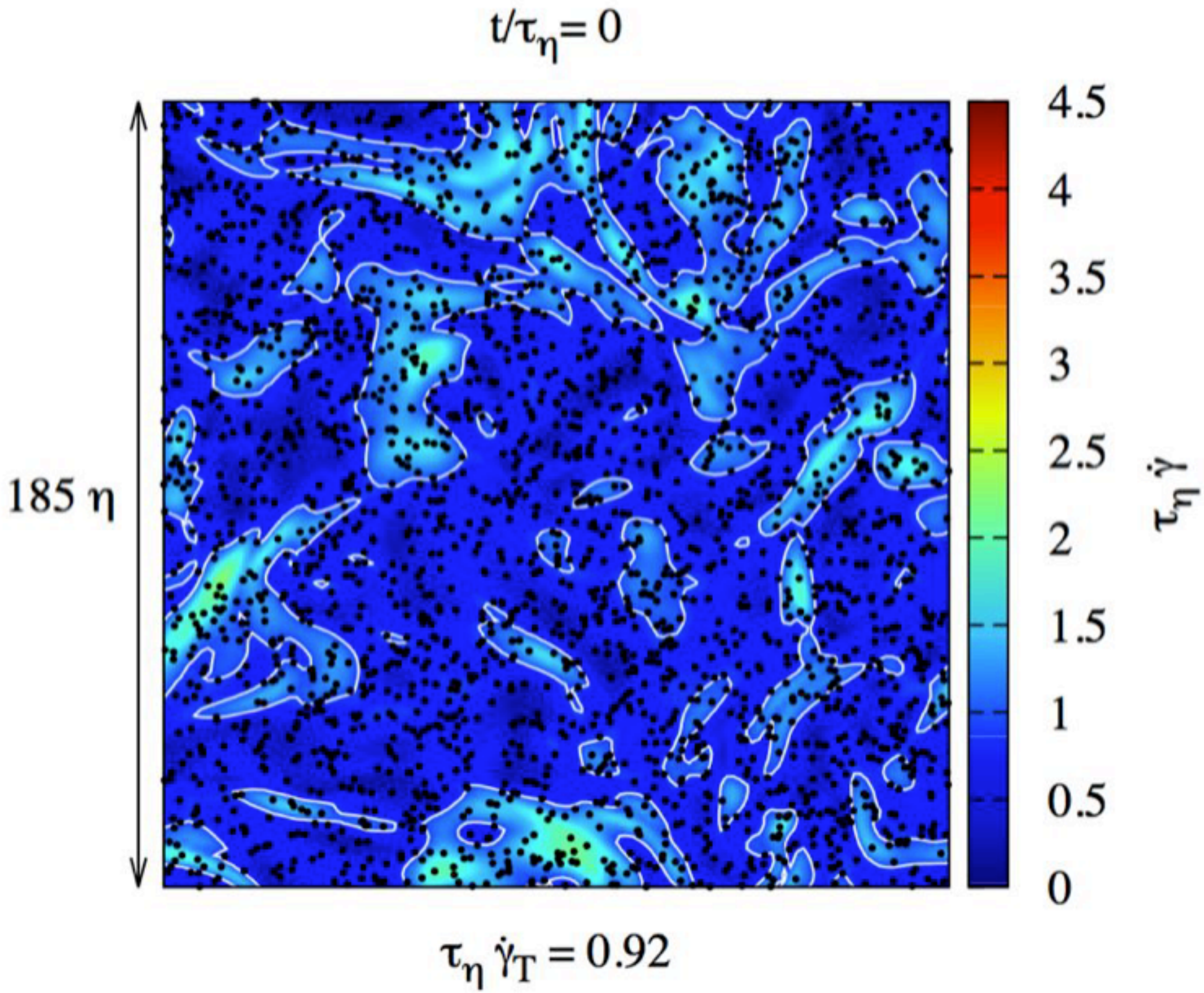
2) **Intermediate,** efficient escape, sheet-like clusters

3) **High,** efficient avoiding of extreme events, fading clusters



$\tau_\eta\, \dot{\gamma}_T = 1.77$

Movie



$t/\tau_\eta = 0$
185 η
4.5
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3.5
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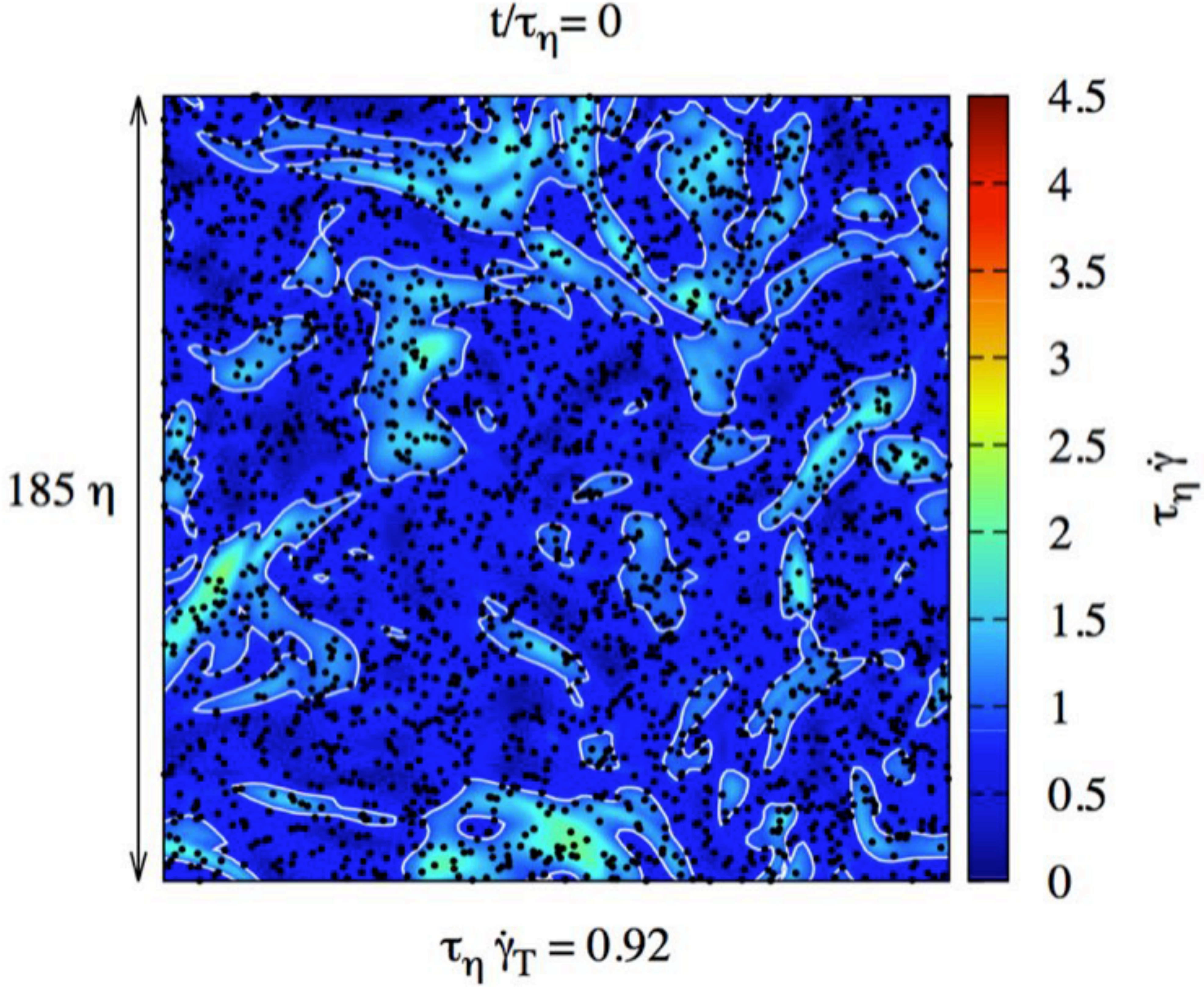
$\tau_\eta \dot{\gamma}$
$\tau_\eta \dot{\gamma}_T = 0.92$

Movie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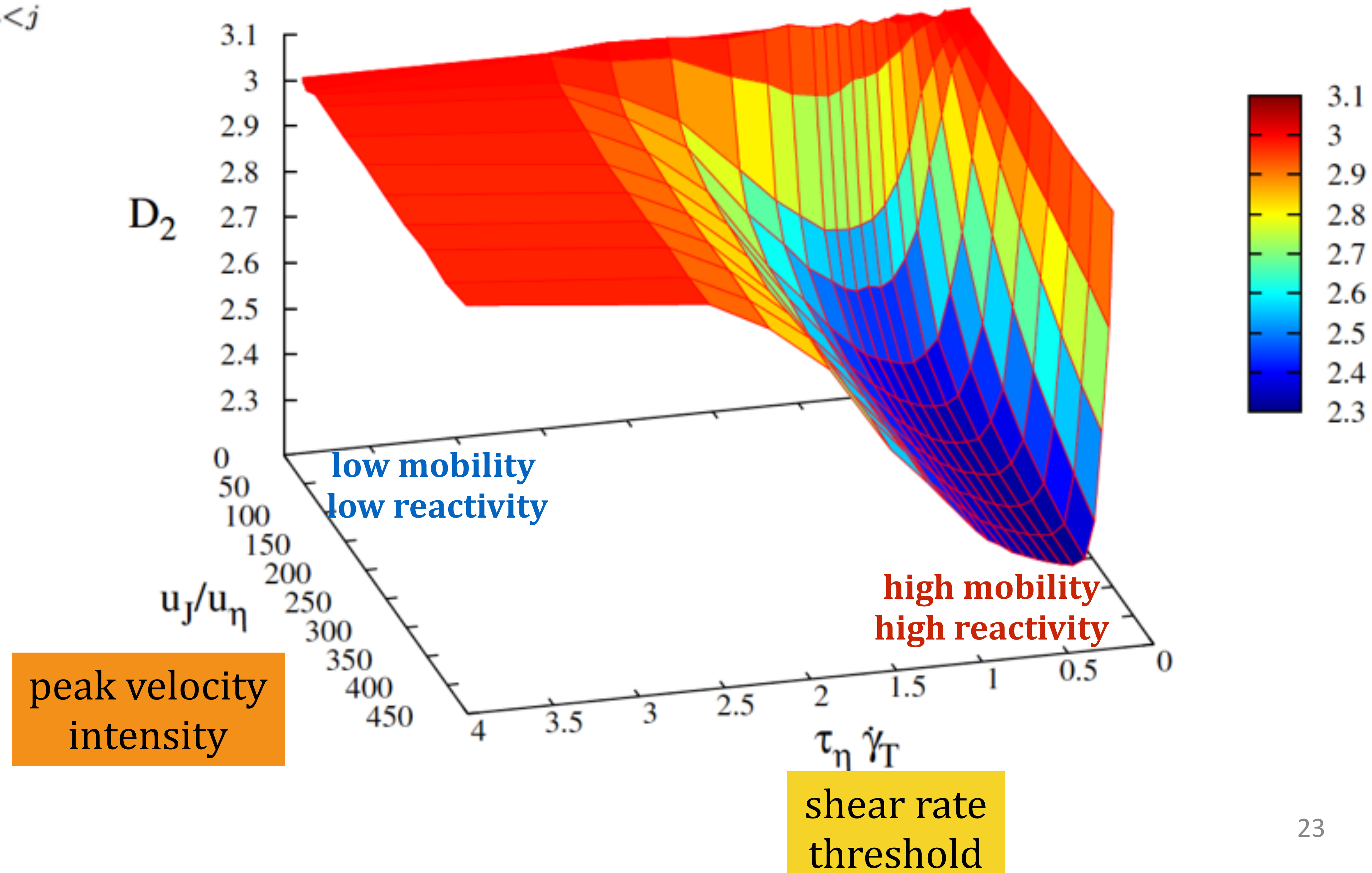
$\tau_\eta \dot{\gamma}$
$\tau_\eta \dot{\gamma}_T = 0.92$

Correlation Dimension

Grassberger-Procaccia D_2

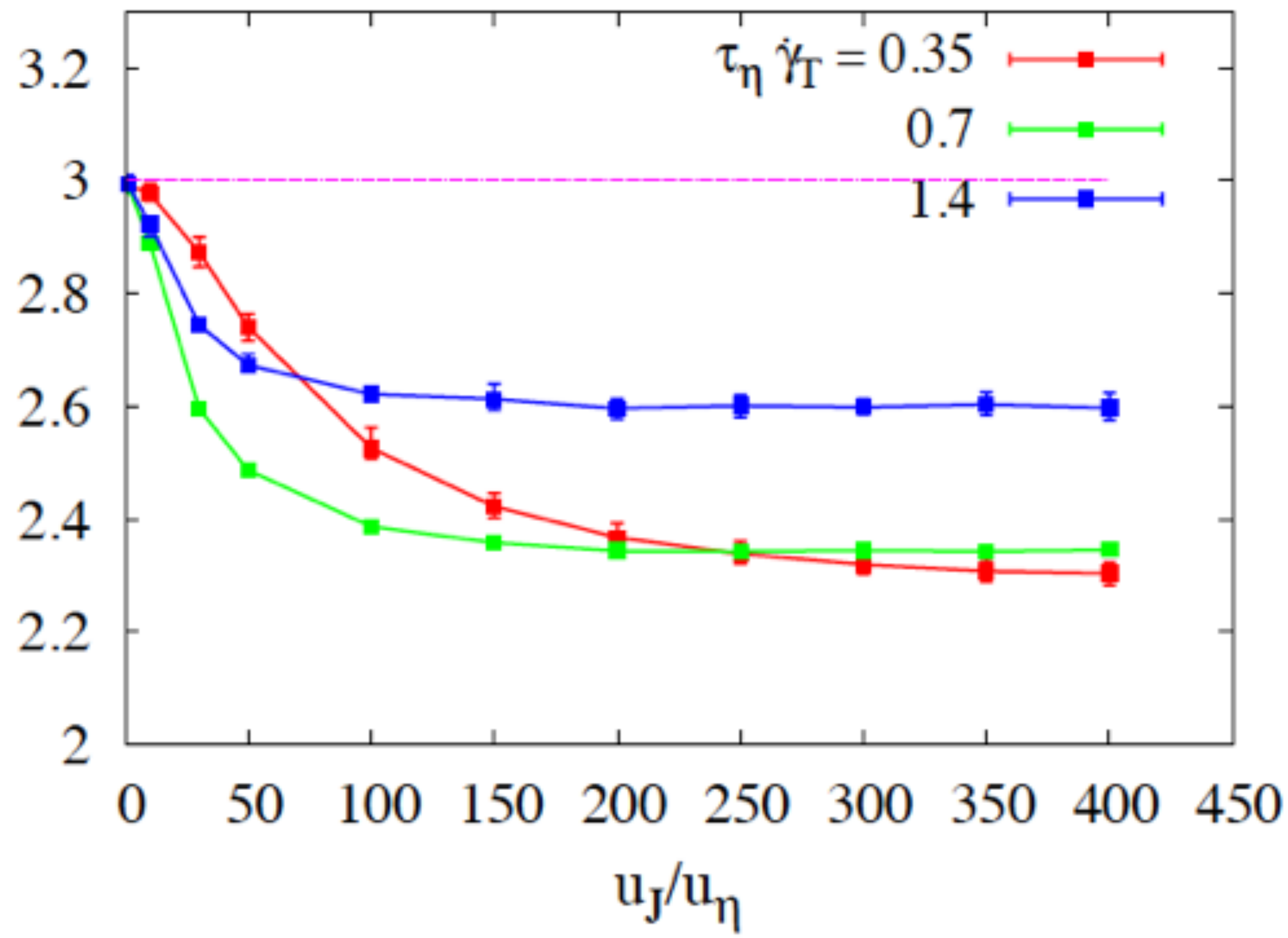
$$C(r) = \frac{2}{N(N-1)} \sum_{i<j} H(r - |X_i - X_j|)$$

$$D_2 = \lim_{r \to 0} \frac{\log C(r)}{\log r}$$



Correlation Dimension

D_2 *vs.* peak velocity intensity

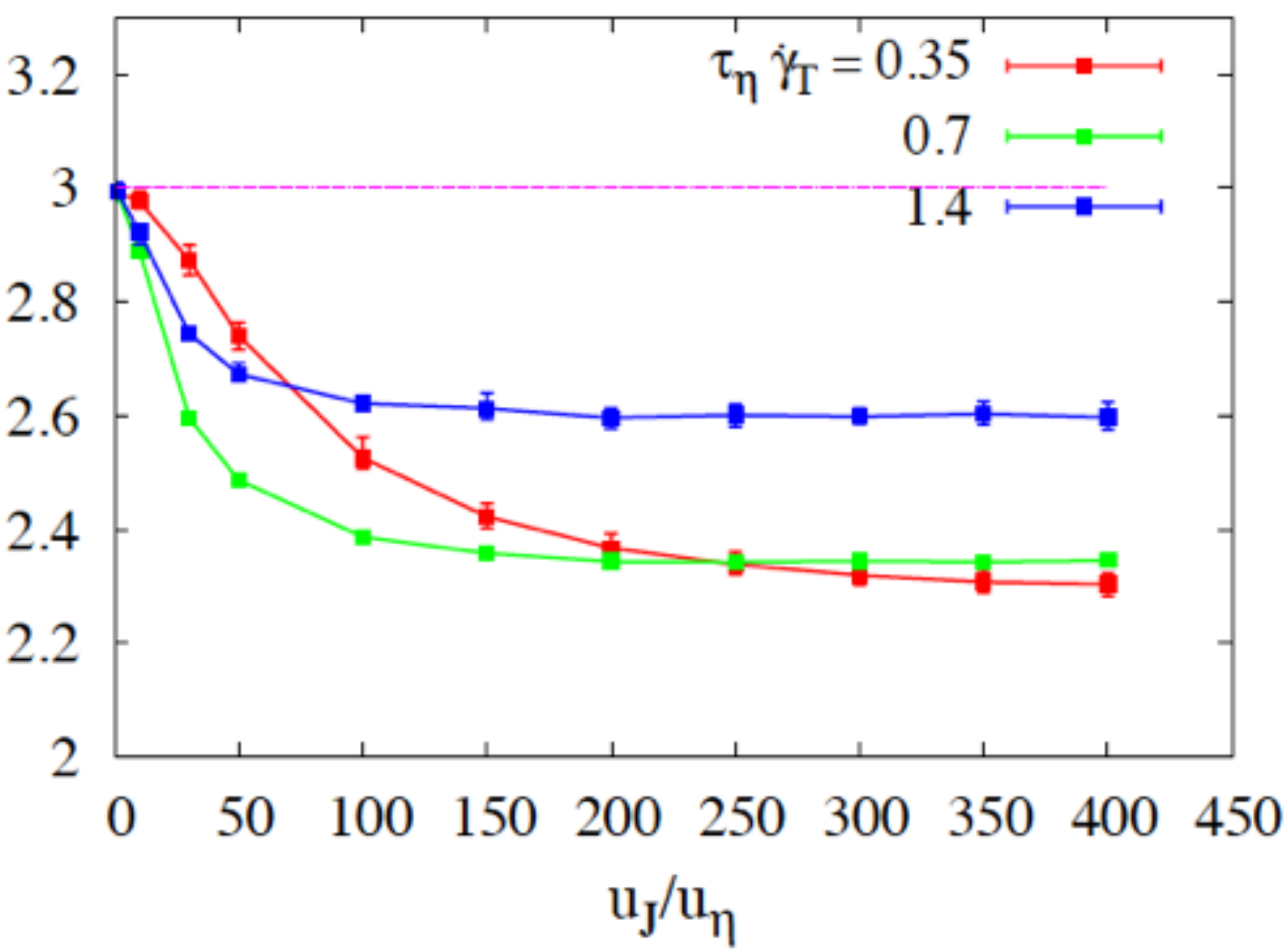


Saturation for huge values of peak jump intensity

$u_J > u_{rms} \sim 30\ u_\eta$

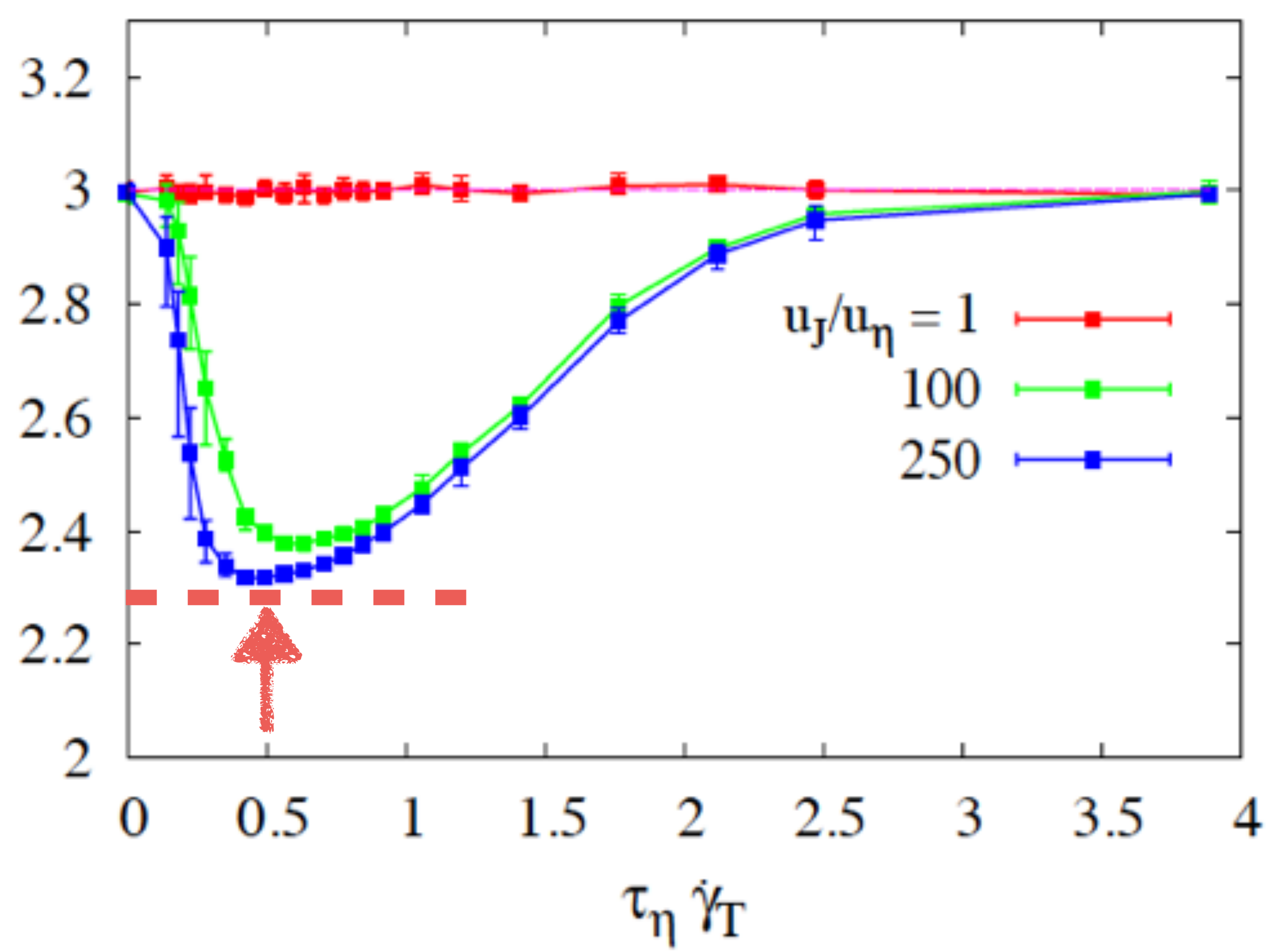
Correlation Dimension

D_2 *vs.* peak velocity intensity



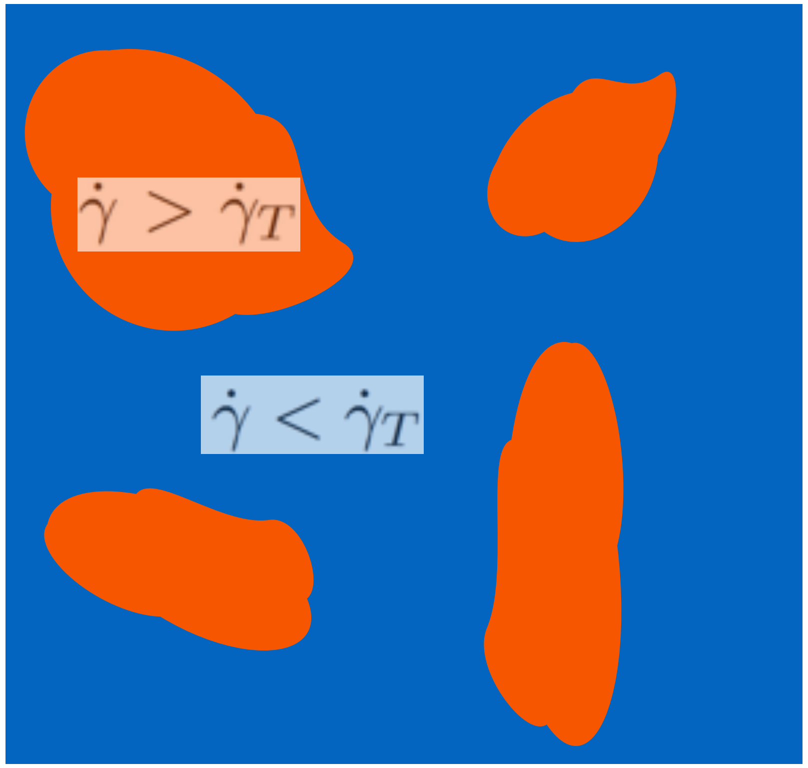
Saturation for huge values of peak jump intensity
$u_J > u_{rms} \sim 30\ u_\eta$

D_2 *vs.* shear rate threshold

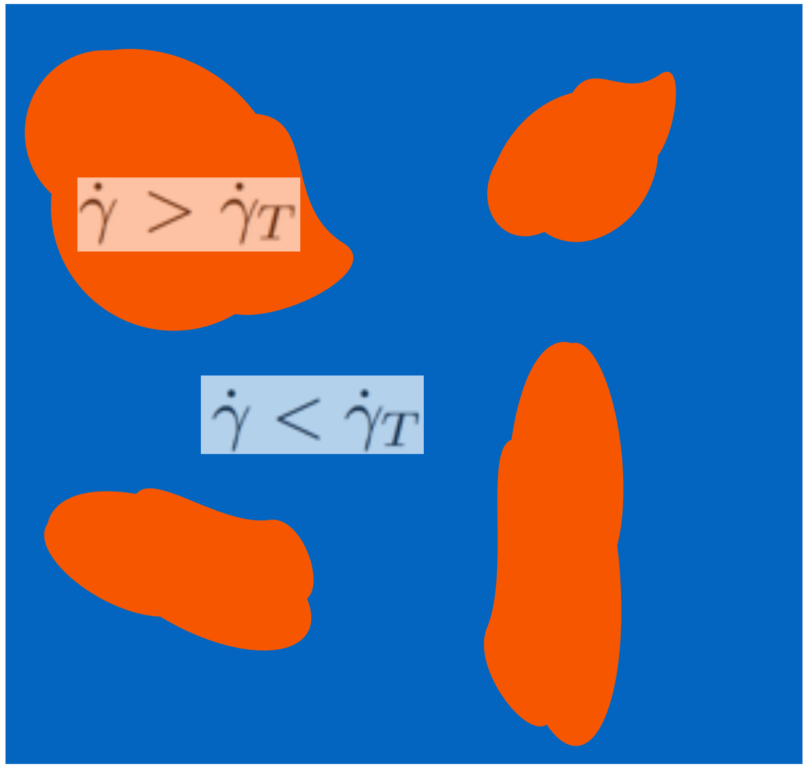


Optimal threshold value
why?

Clustering mechanism (1)



Clustering mechanism (1)

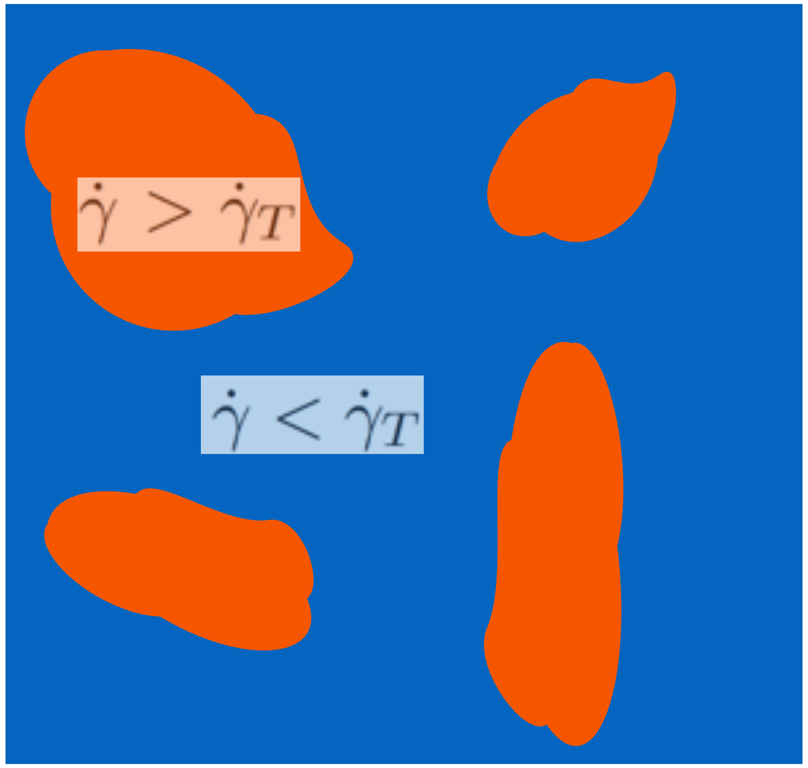


1) Probability of a successful jump

$$\sim \mathcal{V}_{\dot{\gamma}<\dot{\gamma}_T}$$

volume of comfort regions

Clustering mechanism (1)



1) Probability of a successful jump

$$\sim \mathcal{V}_{\dot{\gamma}<\dot{\gamma}_T}$$

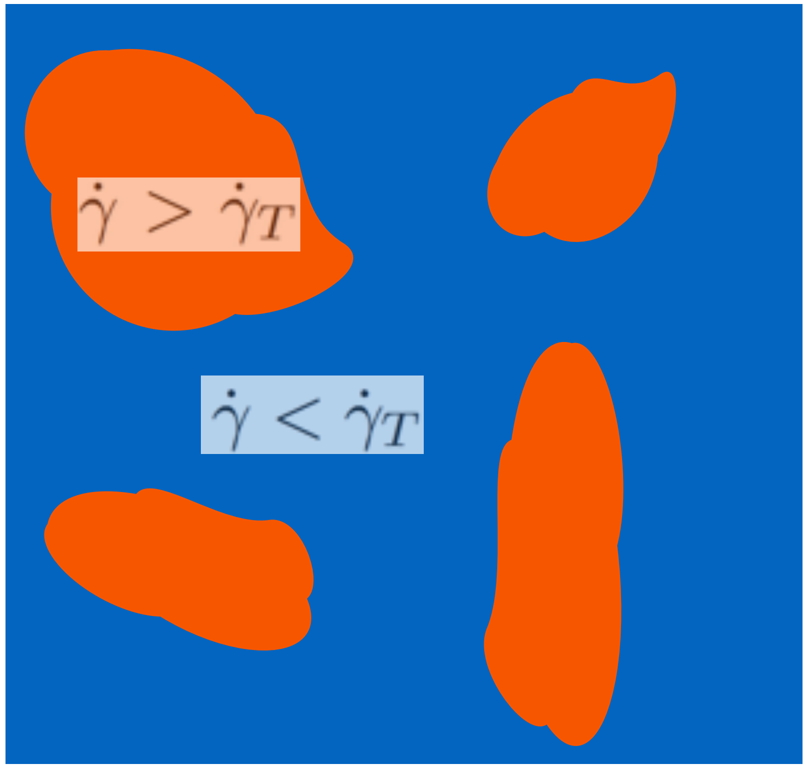
volume of comfort regions

2) Rate of jumps

$$\sim \mathcal{V}_{\dot{\gamma}>\dot{\gamma}_T}$$

volume of alert regions

Clustering mechanism (1)



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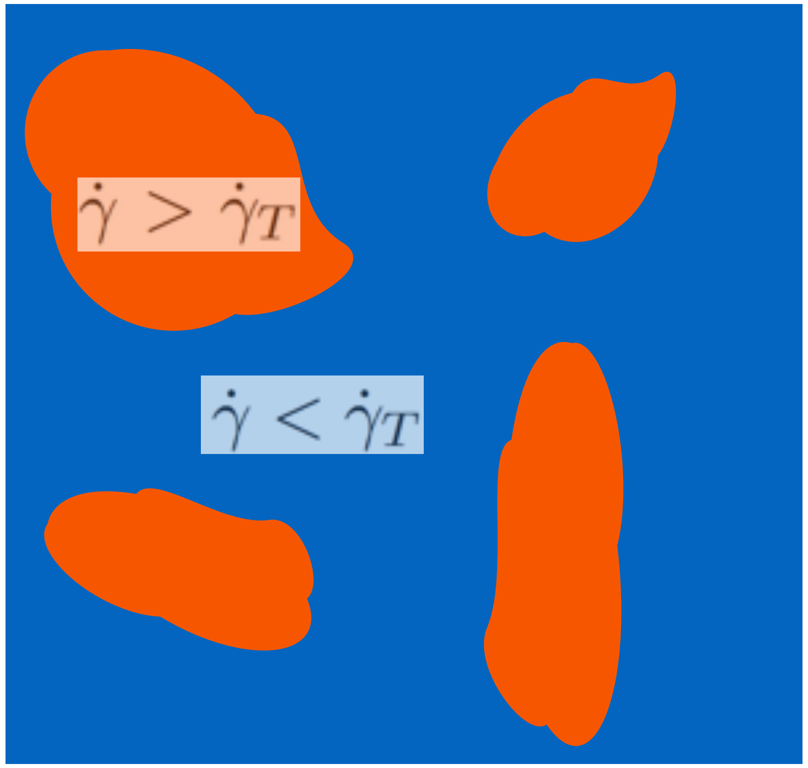
volume of alert regions

3) Clustering

$$\sim \mathcal{V}_{\dot{\gamma}<\dot{\gamma}_T} \cdot \mathcal{V}_{\dot{\gamma}>\dot{\gamma}_T}$$

$$= \mathcal{V}_{\dot{\gamma}>\dot{\gamma}_T} \cdot (\mathcal{V}_{tot} - \mathcal{V}_{\dot{\gamma}>\dot{\gamma}_T})$$

Clustering mechanism (1)



1) Probability of a successful jump

$$\sim \mathcal{V}_{\dot\gamma < \dot\gamma_T}$$

volume of comfort regions

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volume of alert regions

3) Clustering

$$\sim \mathcal{V}_{\dot\gamma < \dot\gamma_T} \cdot \mathcal{V}_{\dot\gamma > \dot\gamma_T}$$

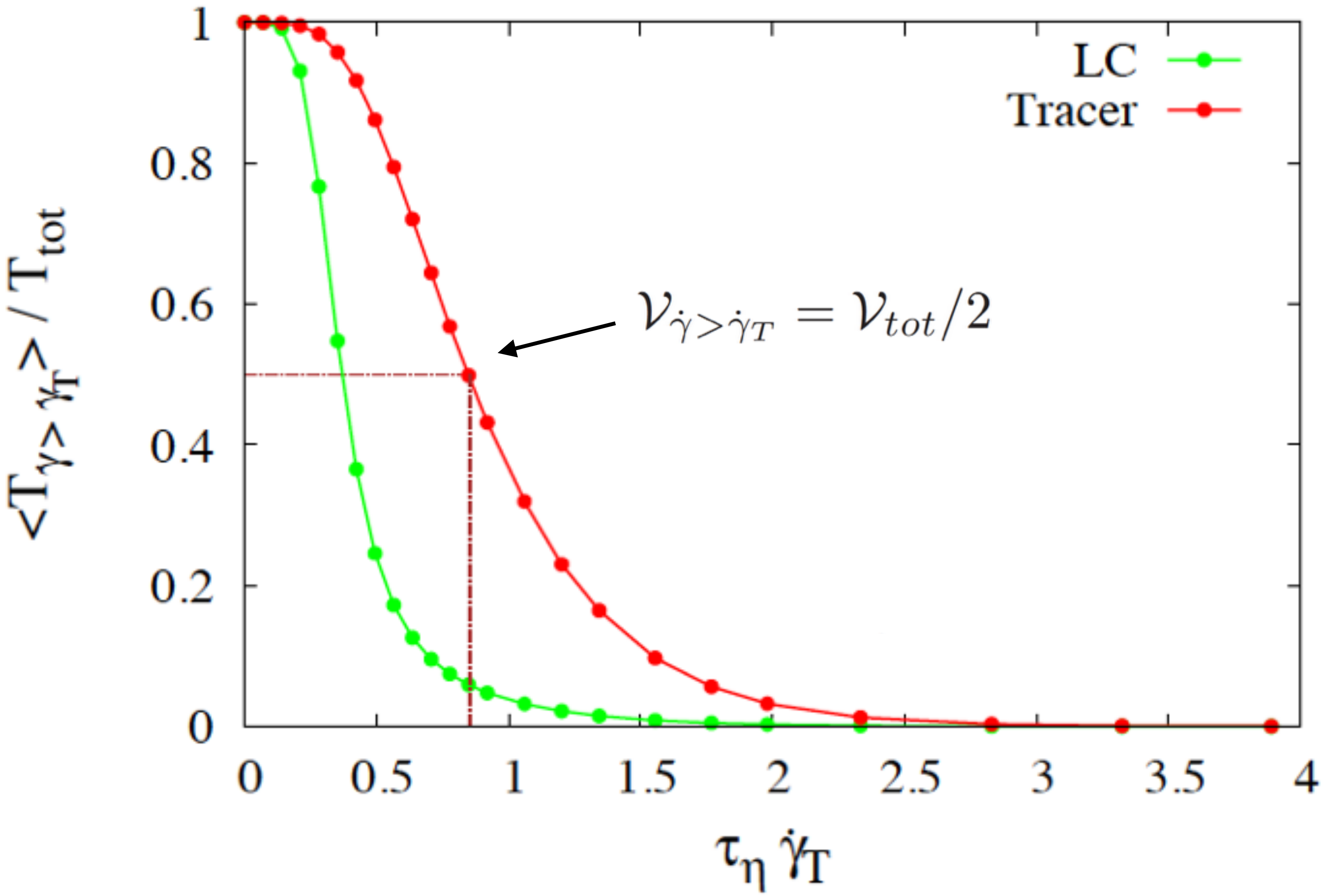
$$= \mathcal{V}_{\dot\gamma > \dot\gamma_T} \cdot (\mathcal{V}_{tot} - \mathcal{V}_{\dot\gamma > \dot\gamma_T})$$

Maximum for

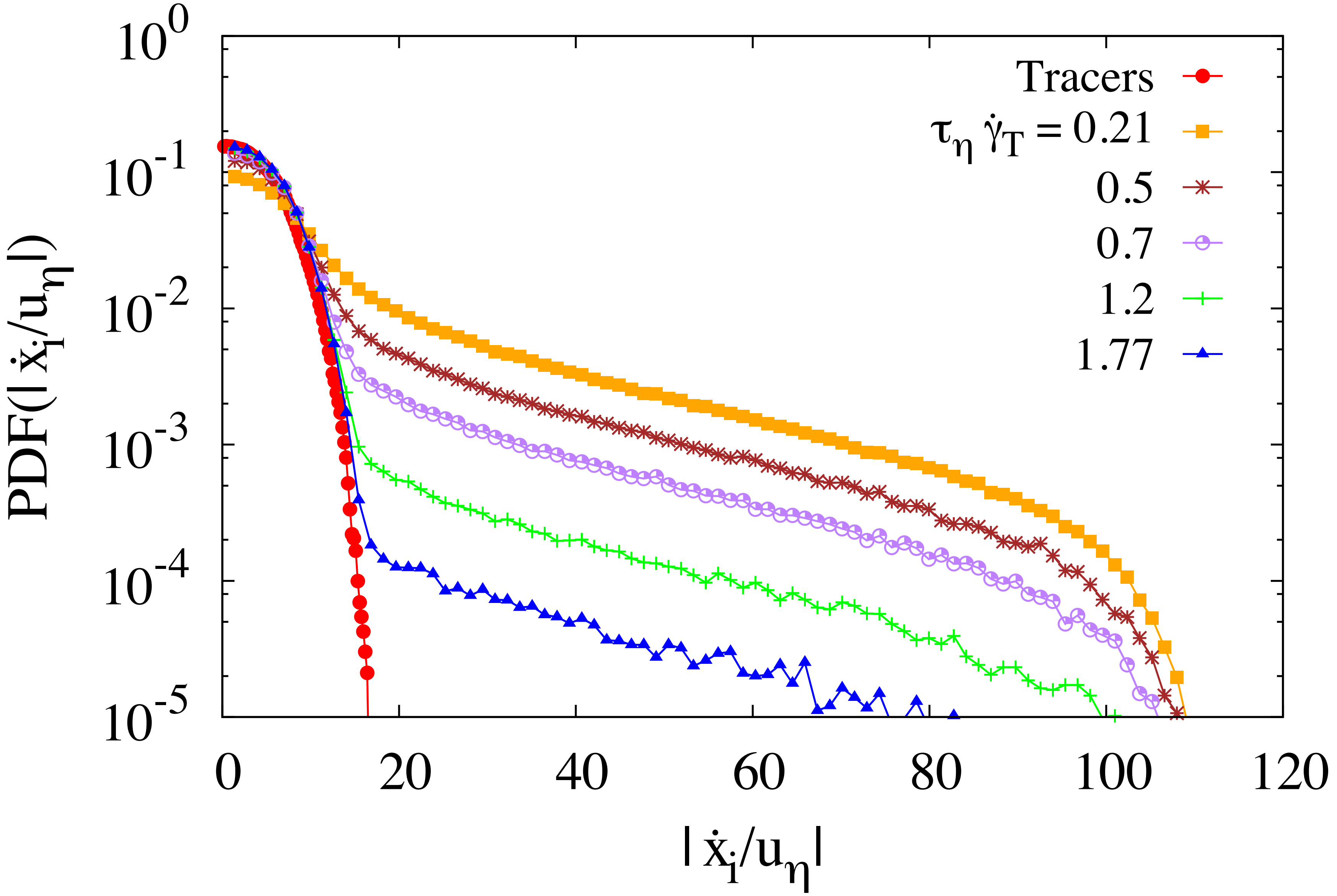
$$\mathcal{V}_{\dot\gamma > \dot\gamma_T} = \mathcal{V}_{tot}/2$$

Clustering mechanism (2)

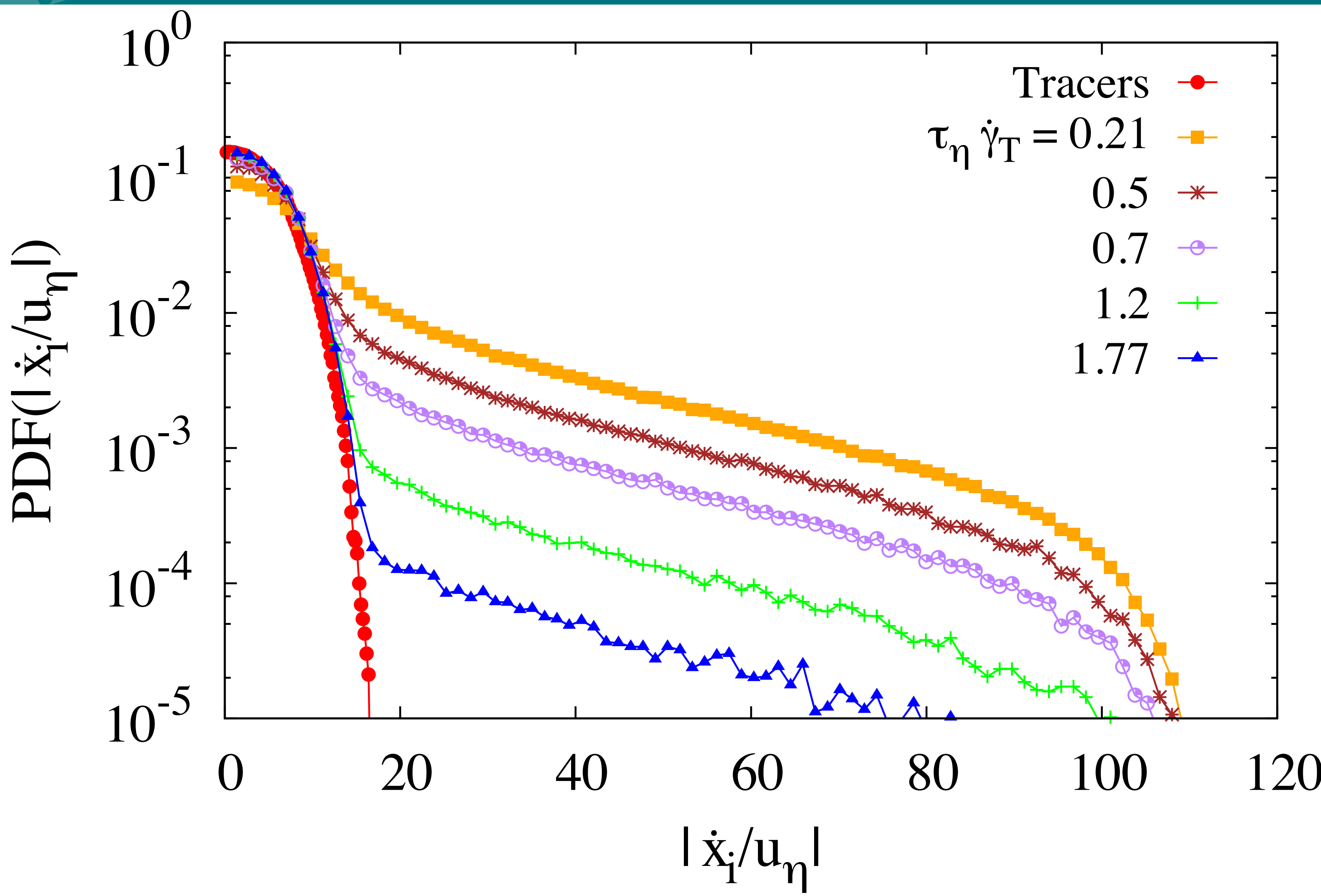
Fraction of time Time spent in **alert regions** $\dot{\gamma} > \dot{\gamma}_T$



PDF of Velocity



PDF of Velocity



$$\dot{\mathbf{x}} = \mathbf{U} + \alpha\, \boldsymbol{p}\, u_J e^{-t/\tau_J}$$

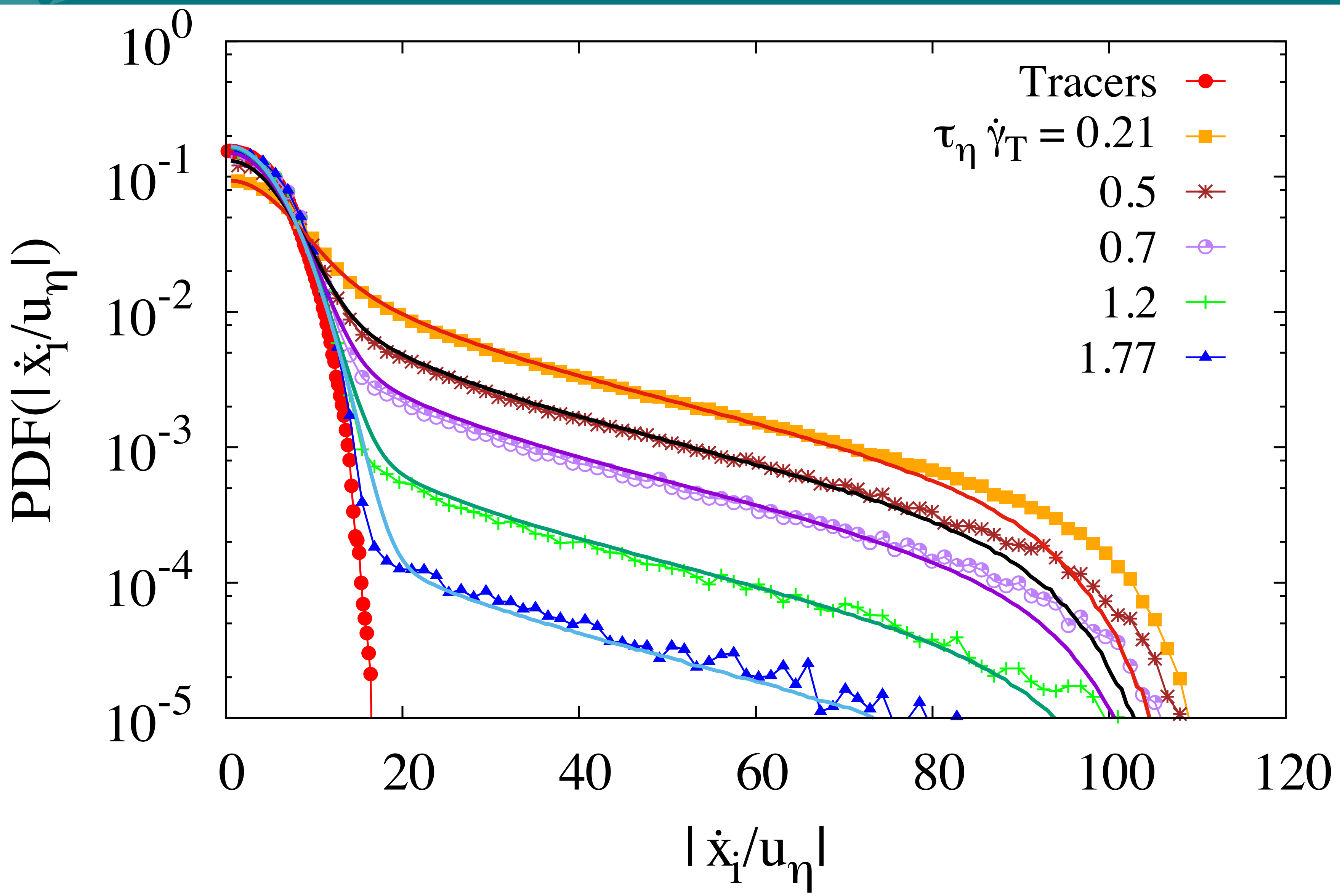
Gaussian random variable

jumping fraction?

random vector

flat random [0, t_e]

PDF of Velocity



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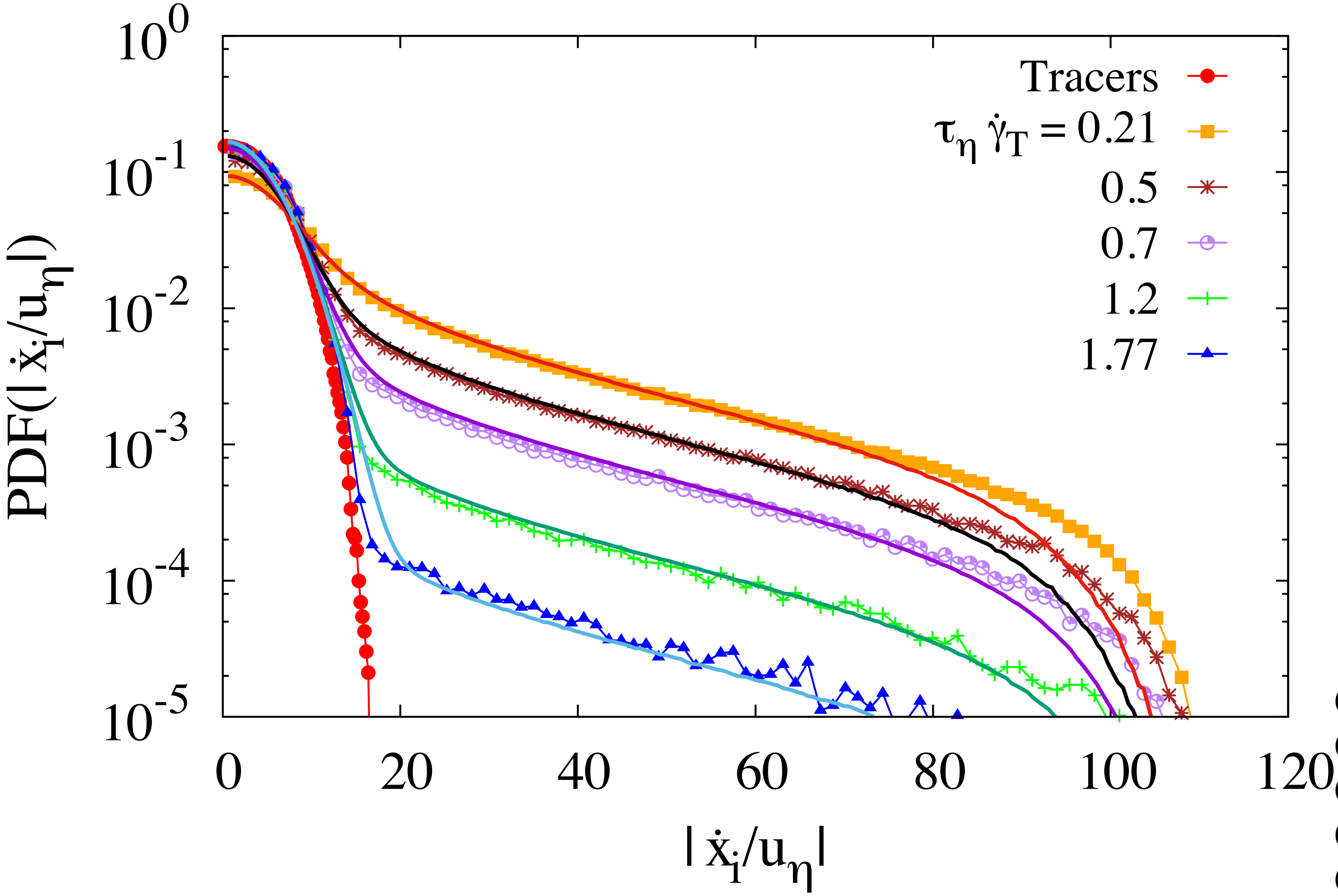
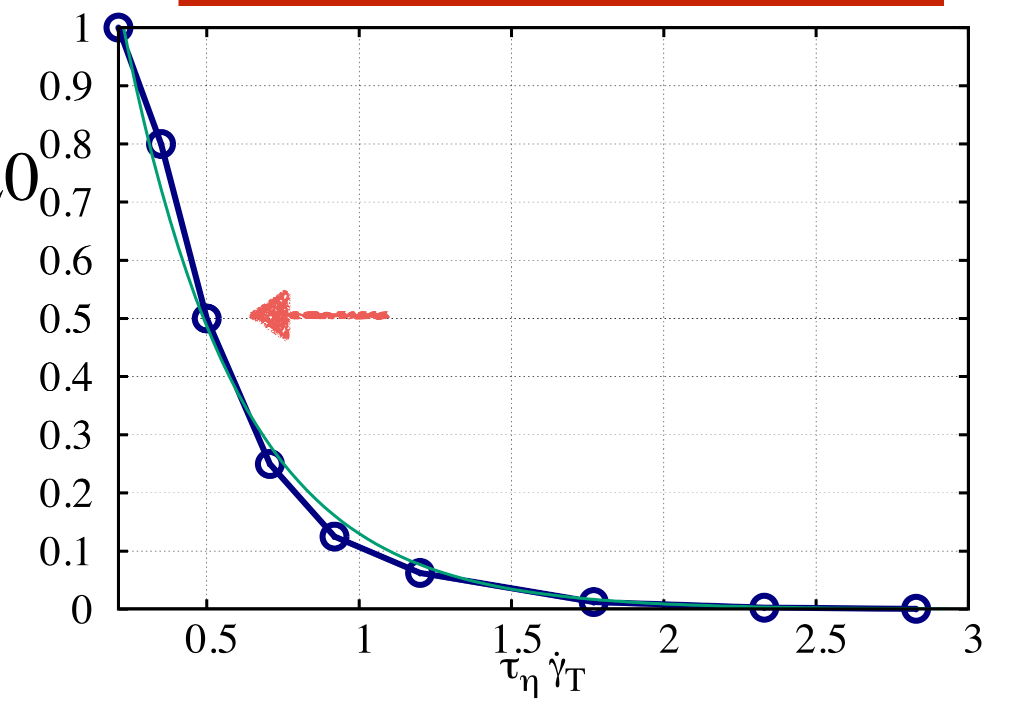
PDF of Velocity





Deduced α
fraction of jumping particles

$$\dot{\mathbf{x}} = \mathbf{U} + \alpha \, \boldsymbol{p} \, u_J e^{-t/\tau_J}$$

Gaussian random variable

jumping fraction?

random vector

flat random [0, tₑ]

Summary

- LC model exhibits clustering in turbulence $D_2 \sim 2.3$
- Necessary conditions for Clustering:

  1) high jump speed $u_J > u_{rms}$

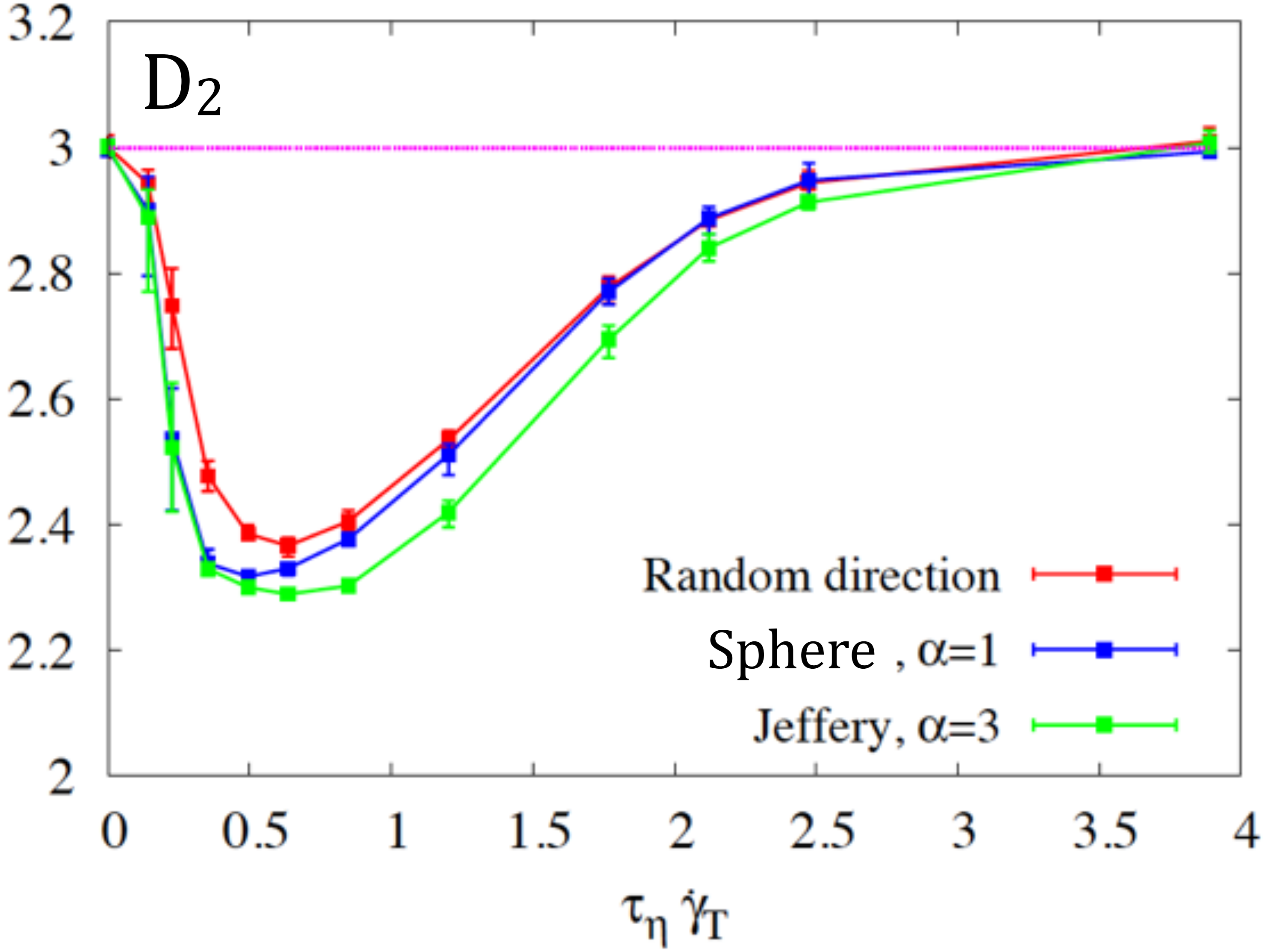
  2) sharp sensitivity to shear rate $O(\tau_\eta^{-1})$

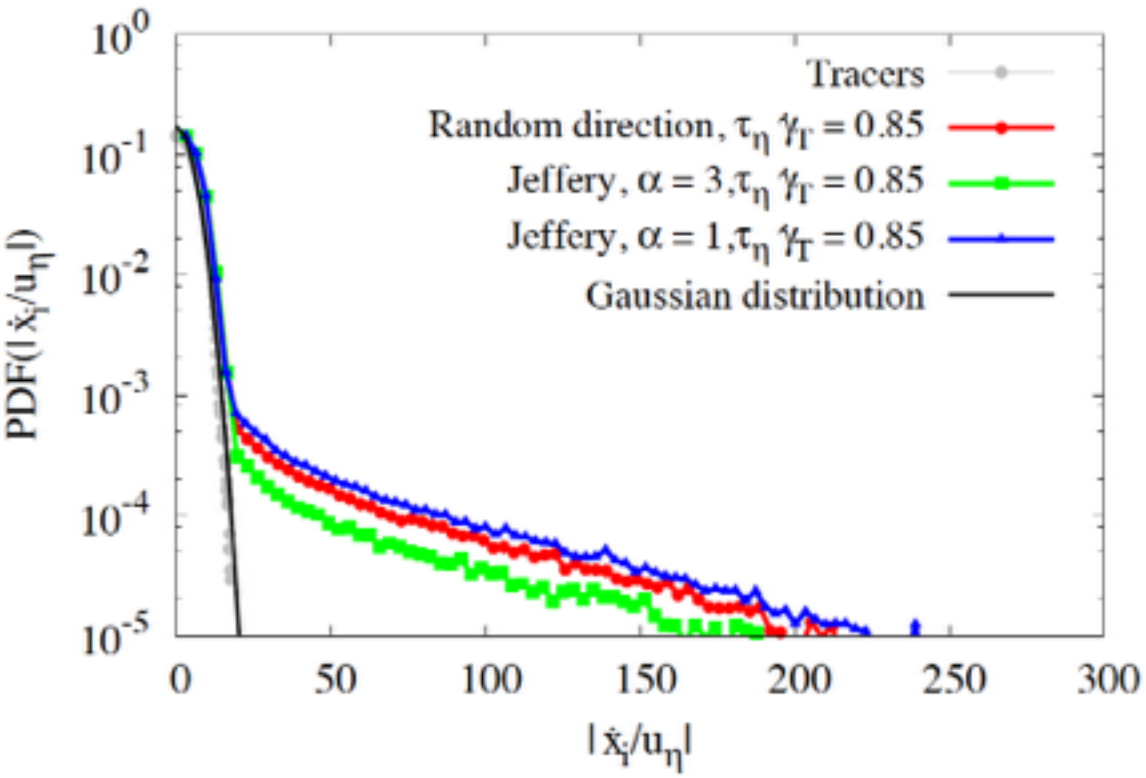
- Clustering comes from **inhomogeneity** rather than anisotropy of the model (**Excluded volume mechanism**)
- Different mechanism from the one identified for motile algae (gyrotaxis induced).

LC Model v1.0

Many possible extensions…

Effect of Aspect ratio

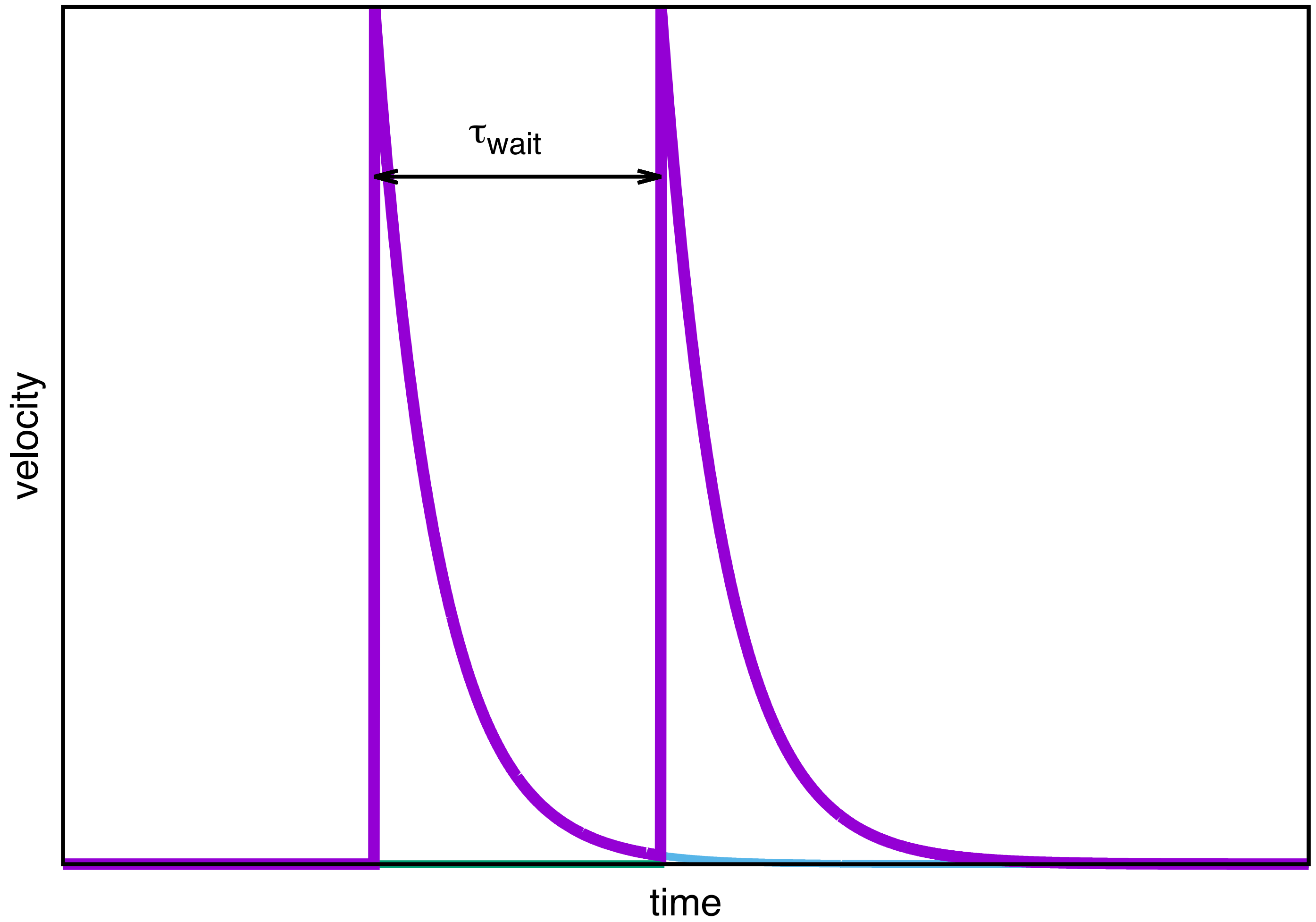




Particle orientation dynamics
no impact on clustering
or velocity distribution

Effect of jump time latency

What if one varies $t_e - t_i = \tau_{wait}$?



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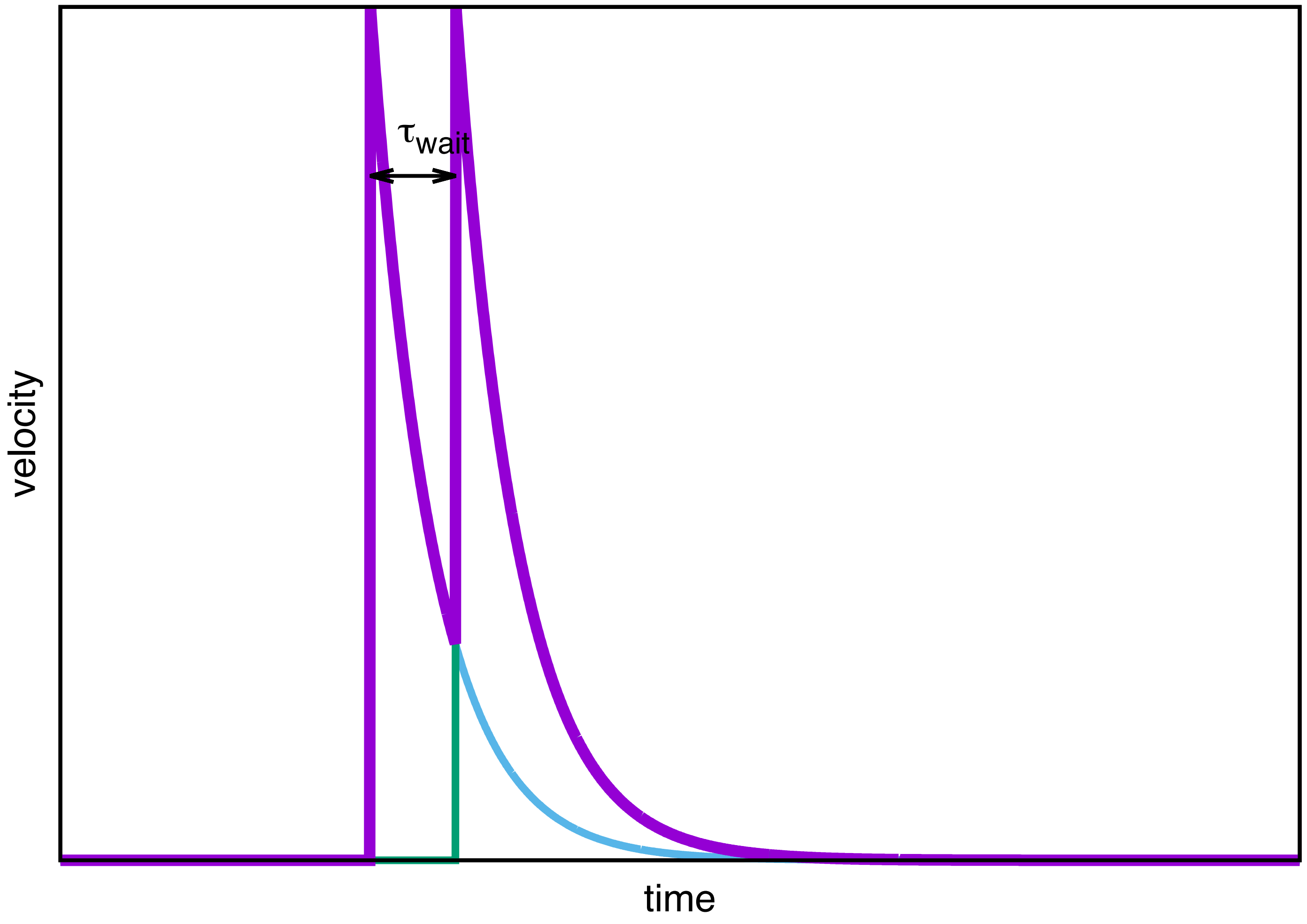

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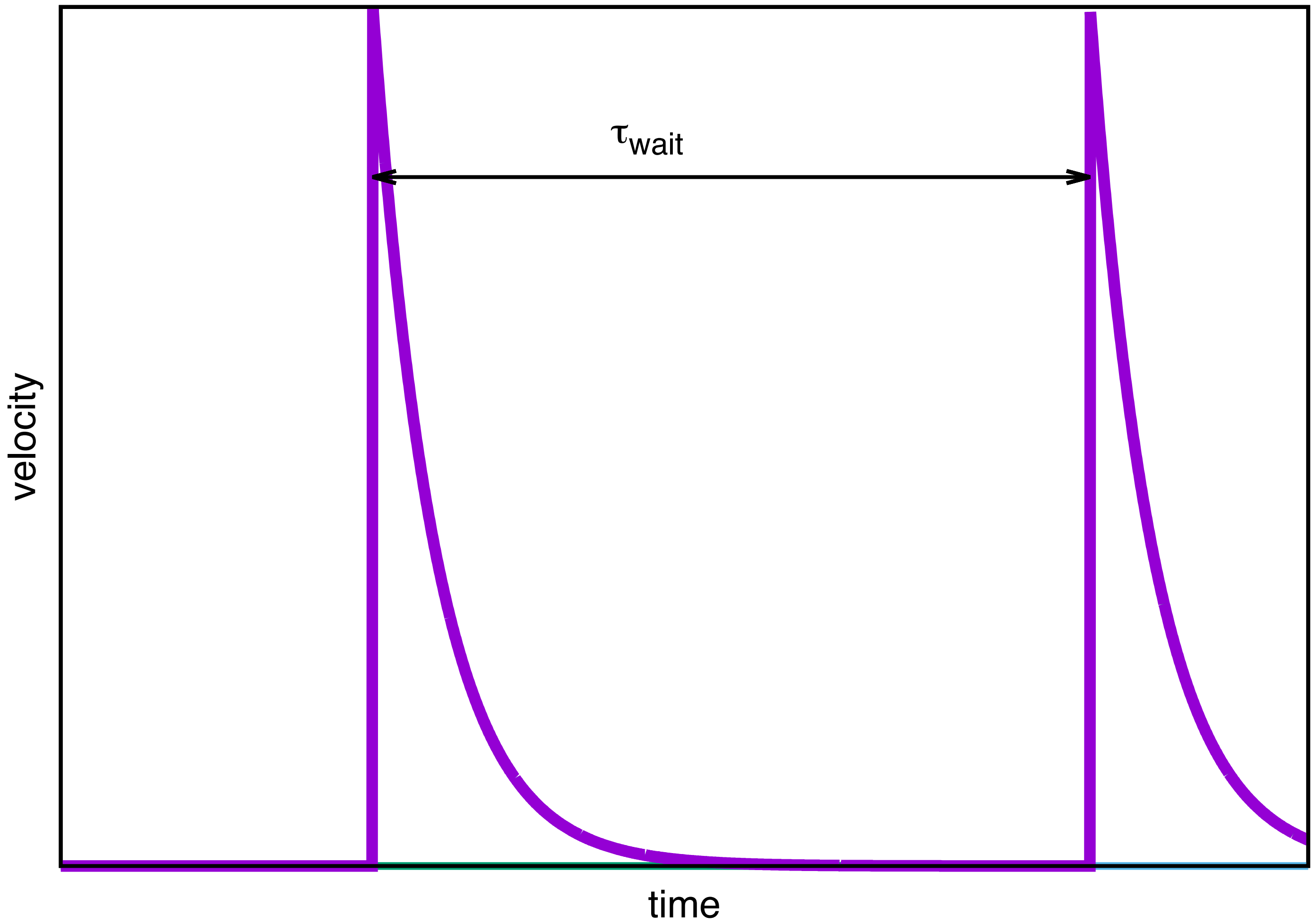

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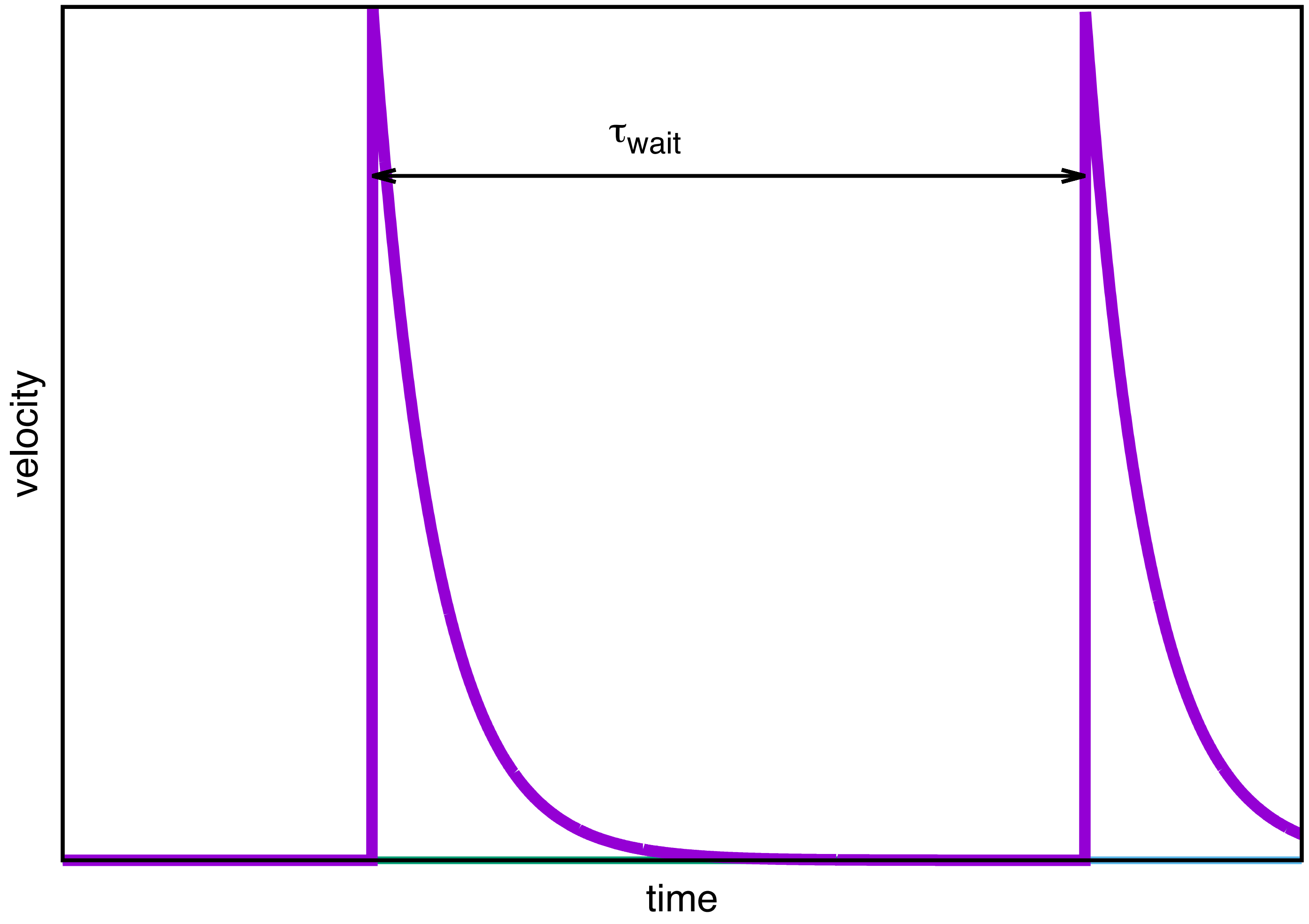
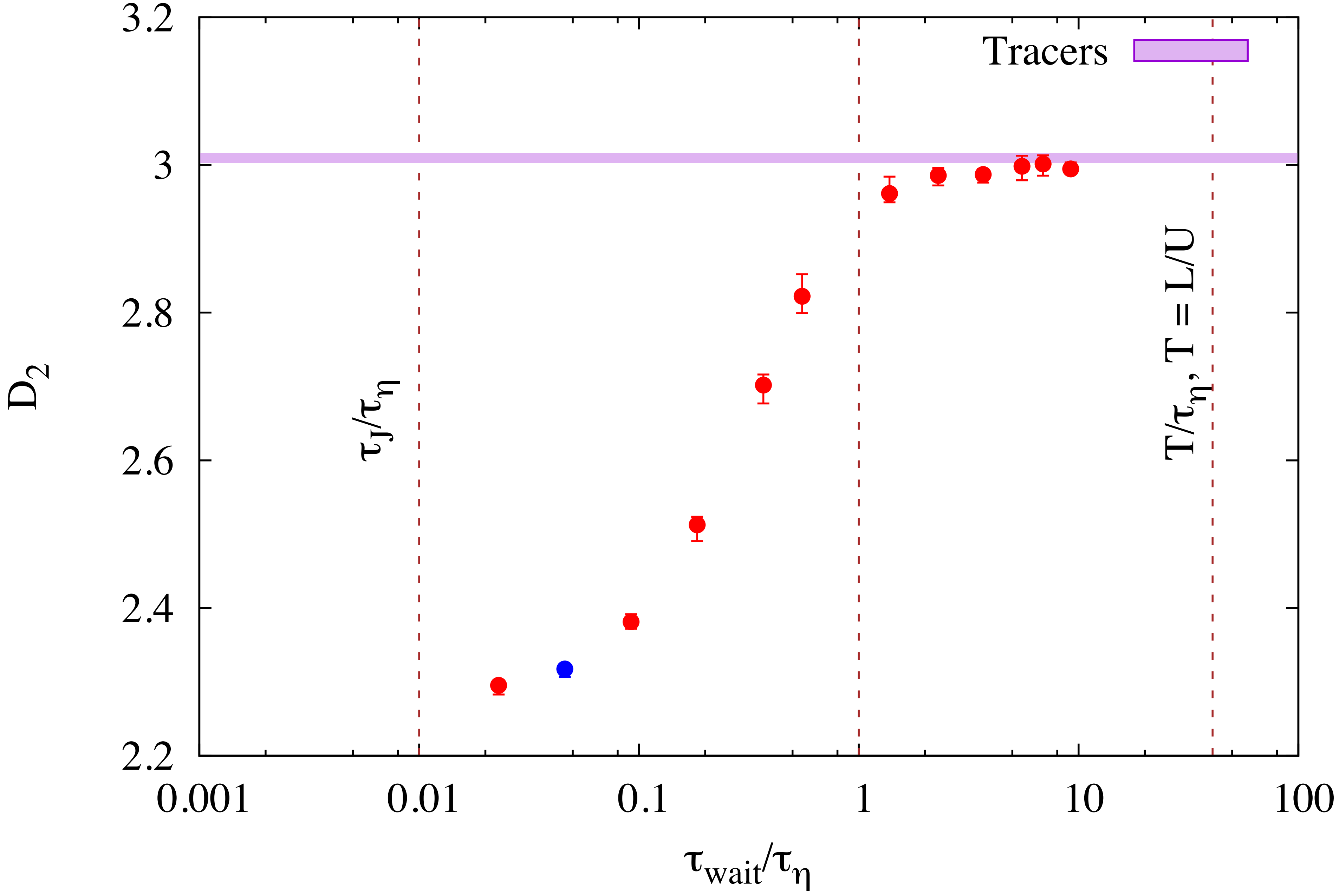


Which effect on clustering?

Effect of jump time latency (2)

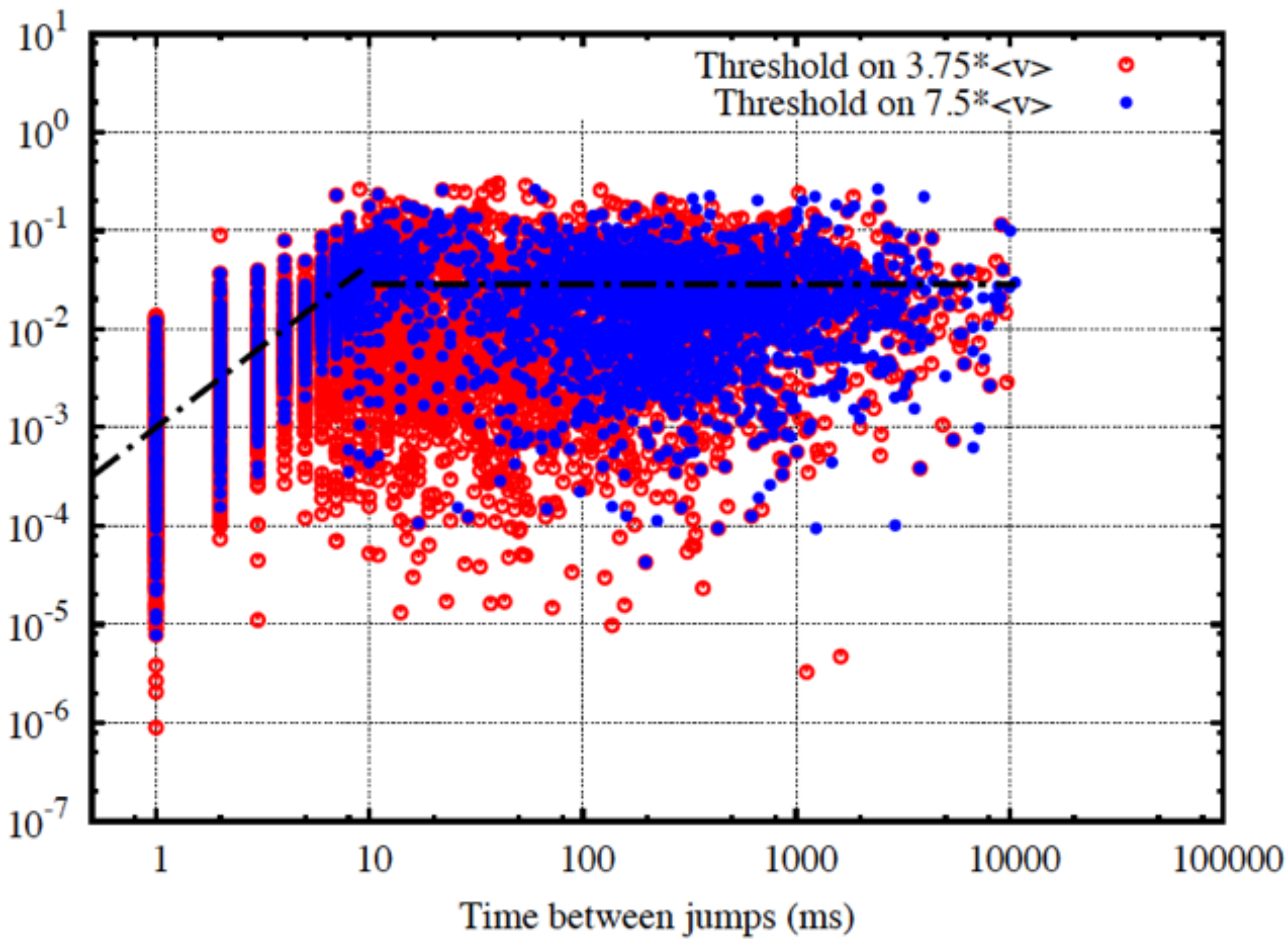


Only a prompt reaction leads to clusters

Jump intensity vs latency in experimental data

Correlation: time gap with peak velocity gap

$\Delta u = |\, u_{peak}(n+1) - u_{peak}(n) \,|$

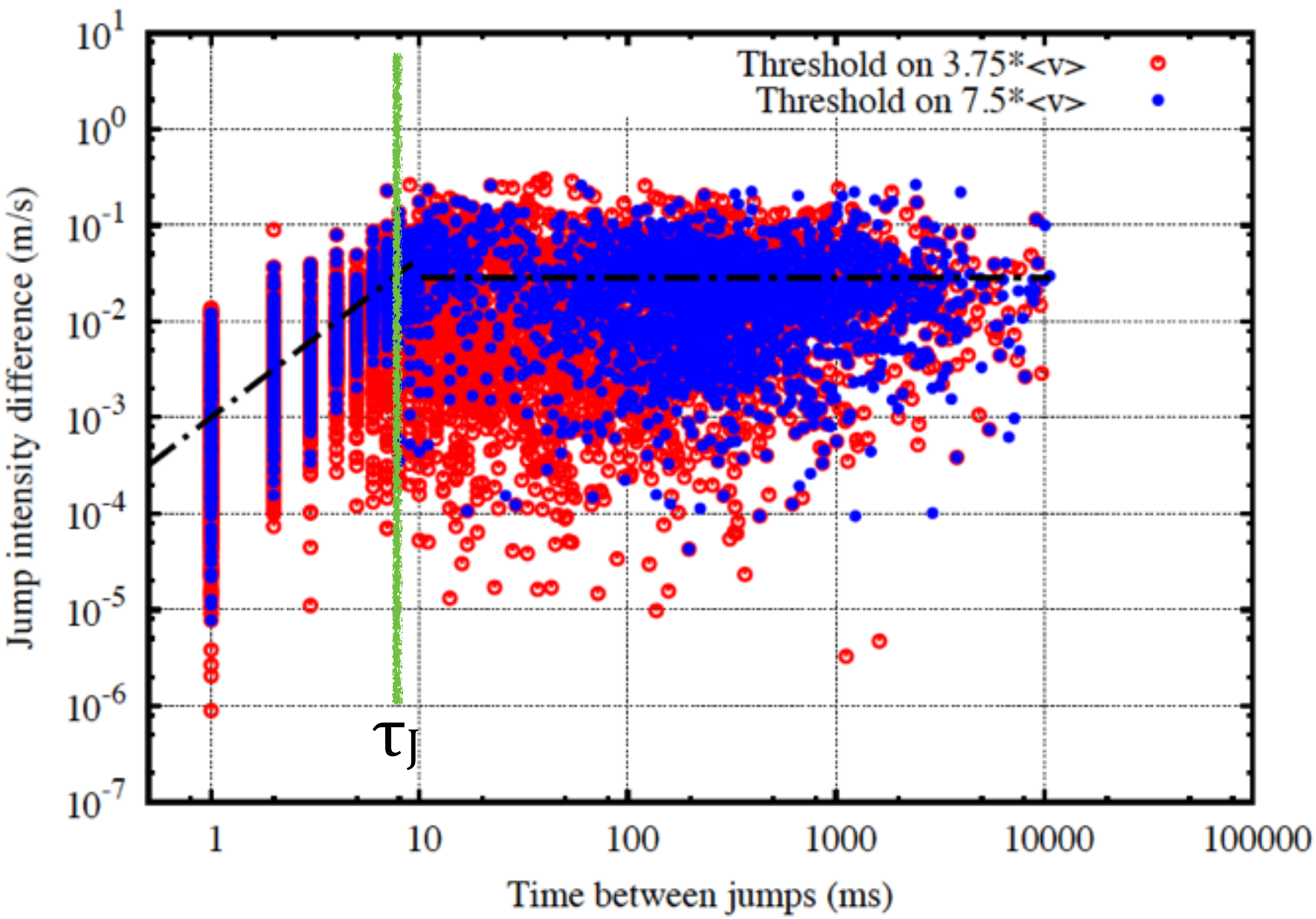


$\Delta t = t_{jump}(n+1) - t_{jump}(n)$

Jump intensity vs latency in experimental data

Correlation: time gap with peak velocity gap

$\Delta u = | u_{peak}(n+1) - u_{peak}(n) |$



$\Delta t = t_{jump}(n+1) - t_{jump}(n)$

Conclusions & Perspectives

- Necessary conditions for Clustering:

  1) high jump speed $u_J > u_{rms}$

  2) sharp sensitivity to shear rate $O(\tau_\eta^{-1})$

  3) high jump frequency $\ll \tau_\eta$

- If reaction is quick, particle orientation is not a crucial aspect
- Eulerian modelling should be possible in these conditions

  Effective Diffusivity(local share-rate)

arXiv:1601.01438

A Lagrangian model of Copepod dynamics: clustering by escape jumps in turbulence

Hamidreza Ardeshiri, Ibtissem Benkeddad, François G. Schmitt, Sami Souissi, Federico Toschi, Enrico Calzavarini

Conclusions & Perspectives

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  Effective Diffusivity(local share-rate)

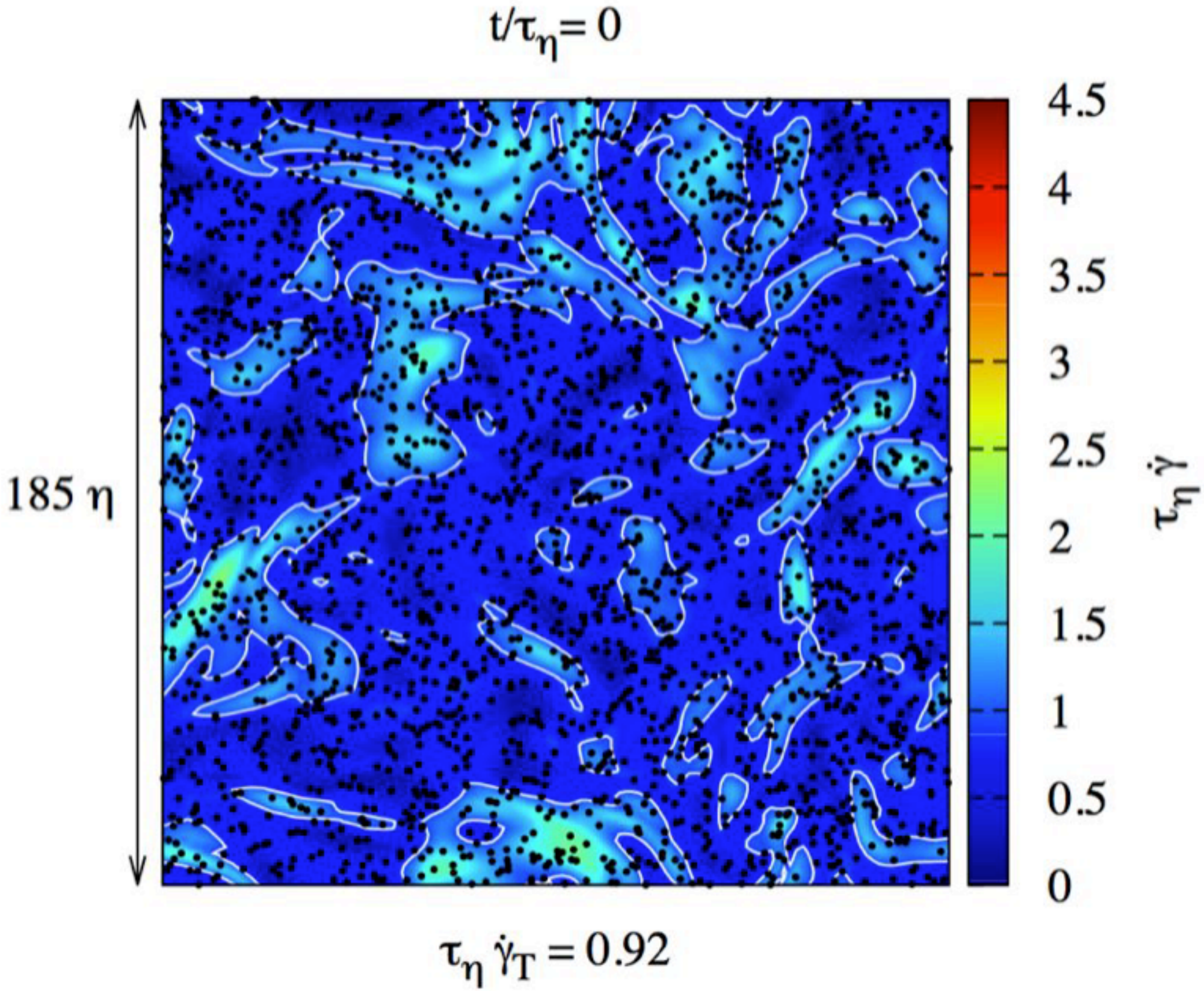
Model refinements

- Memory effect (explore delay - amplitude relation)
- Considering finite-size of domain of perception

Test the model against real measurements in turbulence

(Needs new experimental data)

Thank you!



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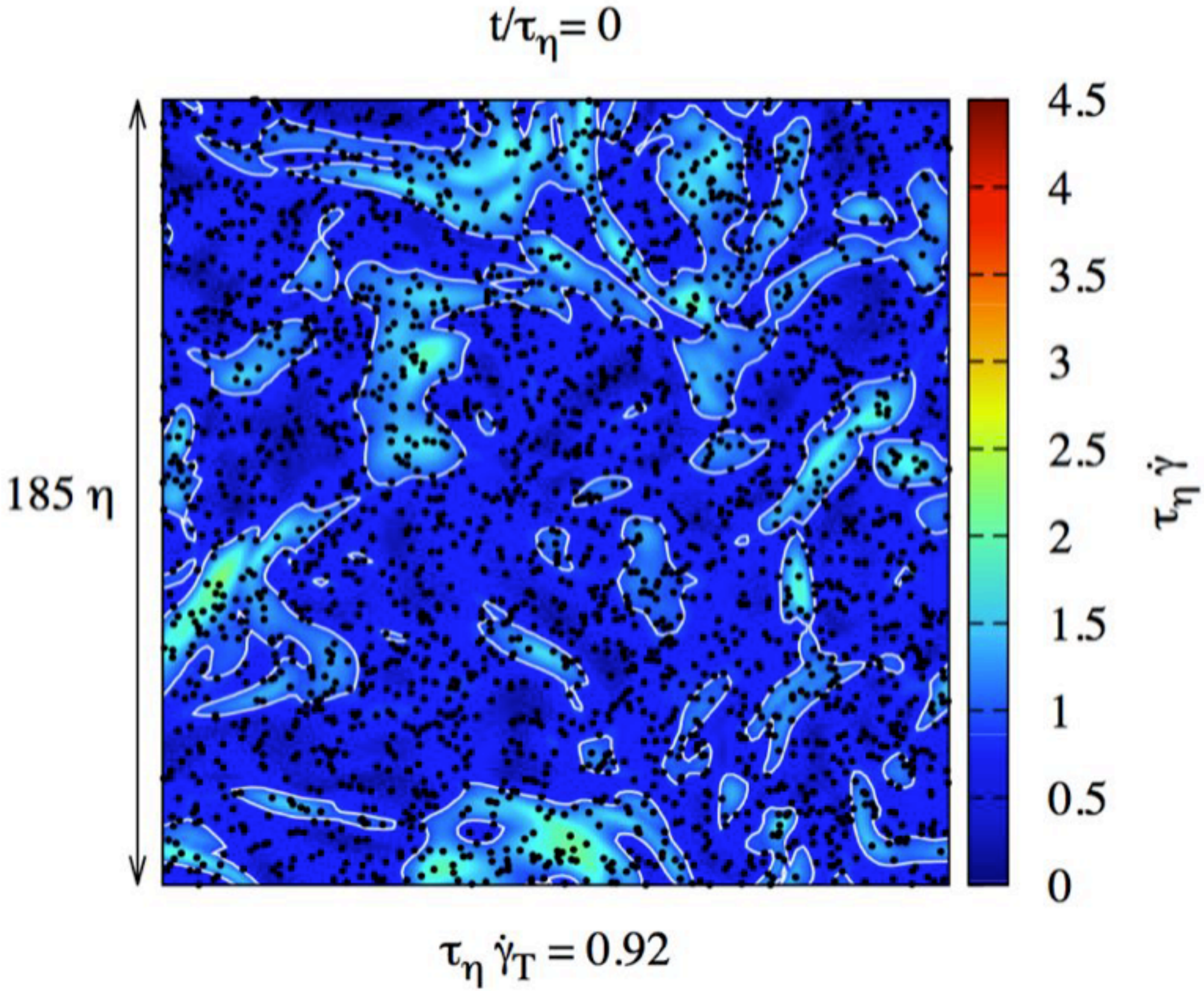

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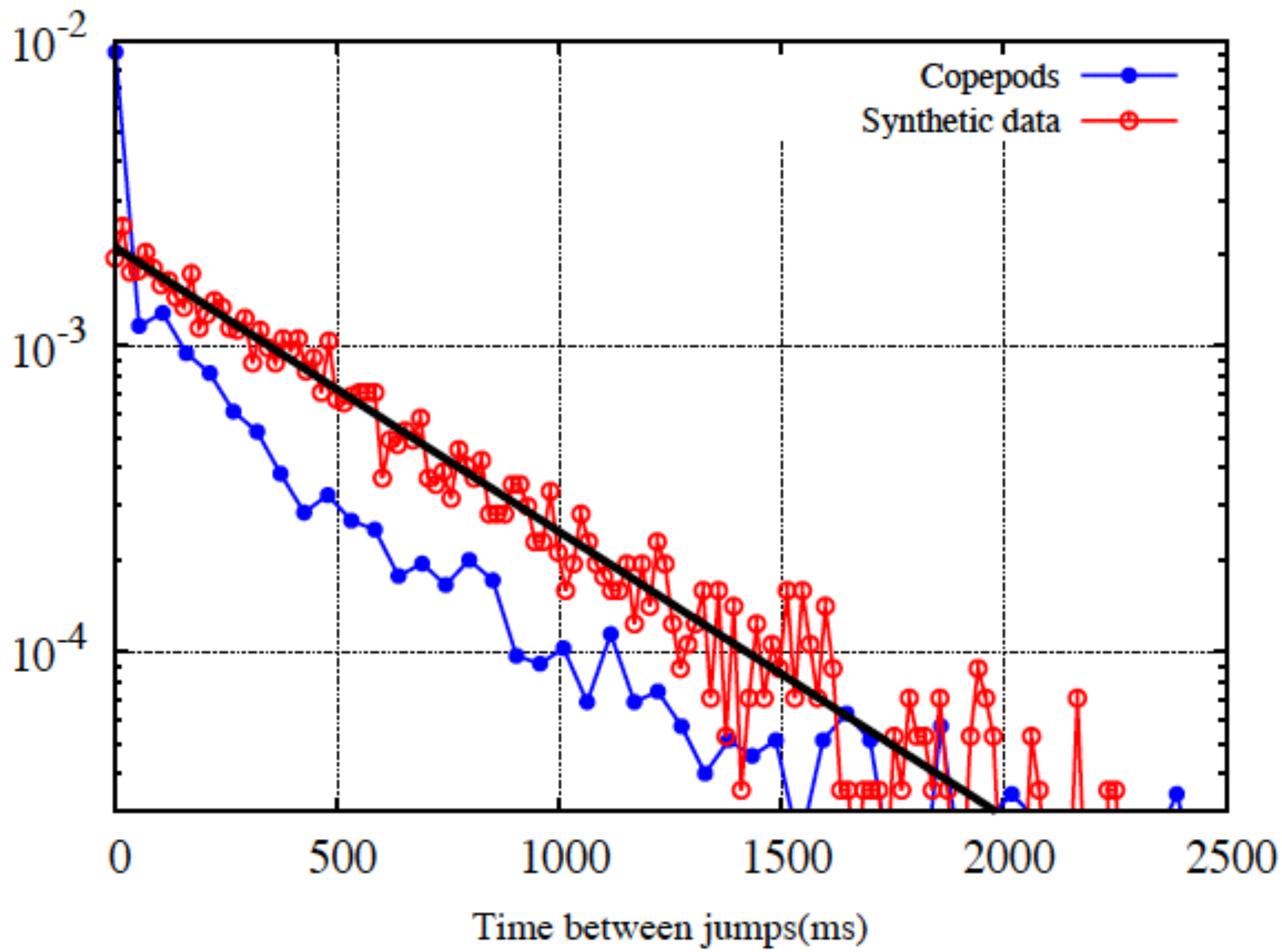
Back to data analysis

jump time-gap statistics

Pdf(Δt)



$$\Delta t = t_{jump}(n+1) - t_{jump}(n)$$

Back to data analysis

jump time-gap statistics

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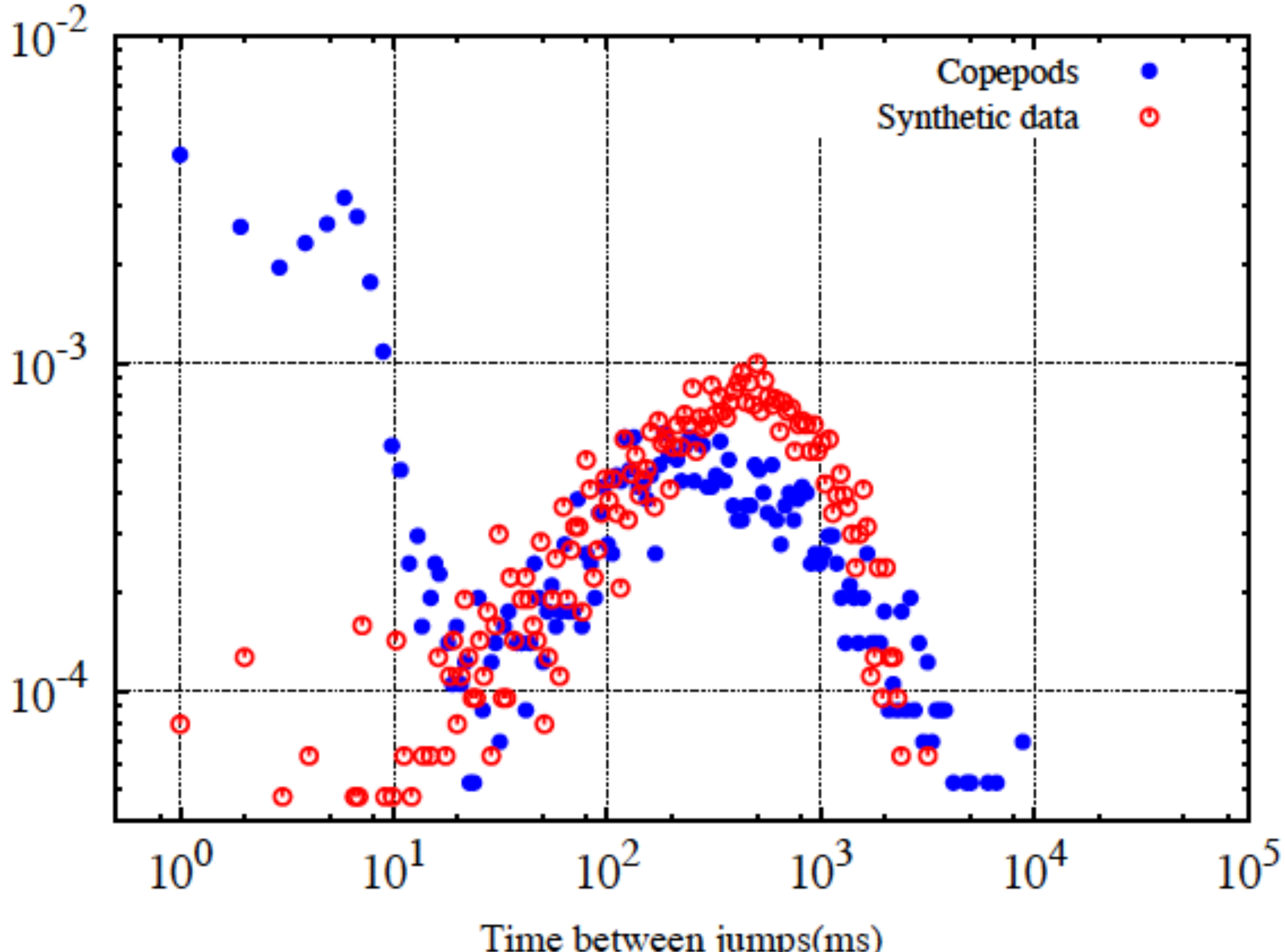

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