Effect of external radiation on the Rayleigh-Bénard system

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Université de Lille 1 SCIENCES ET TECHNOLOGIES

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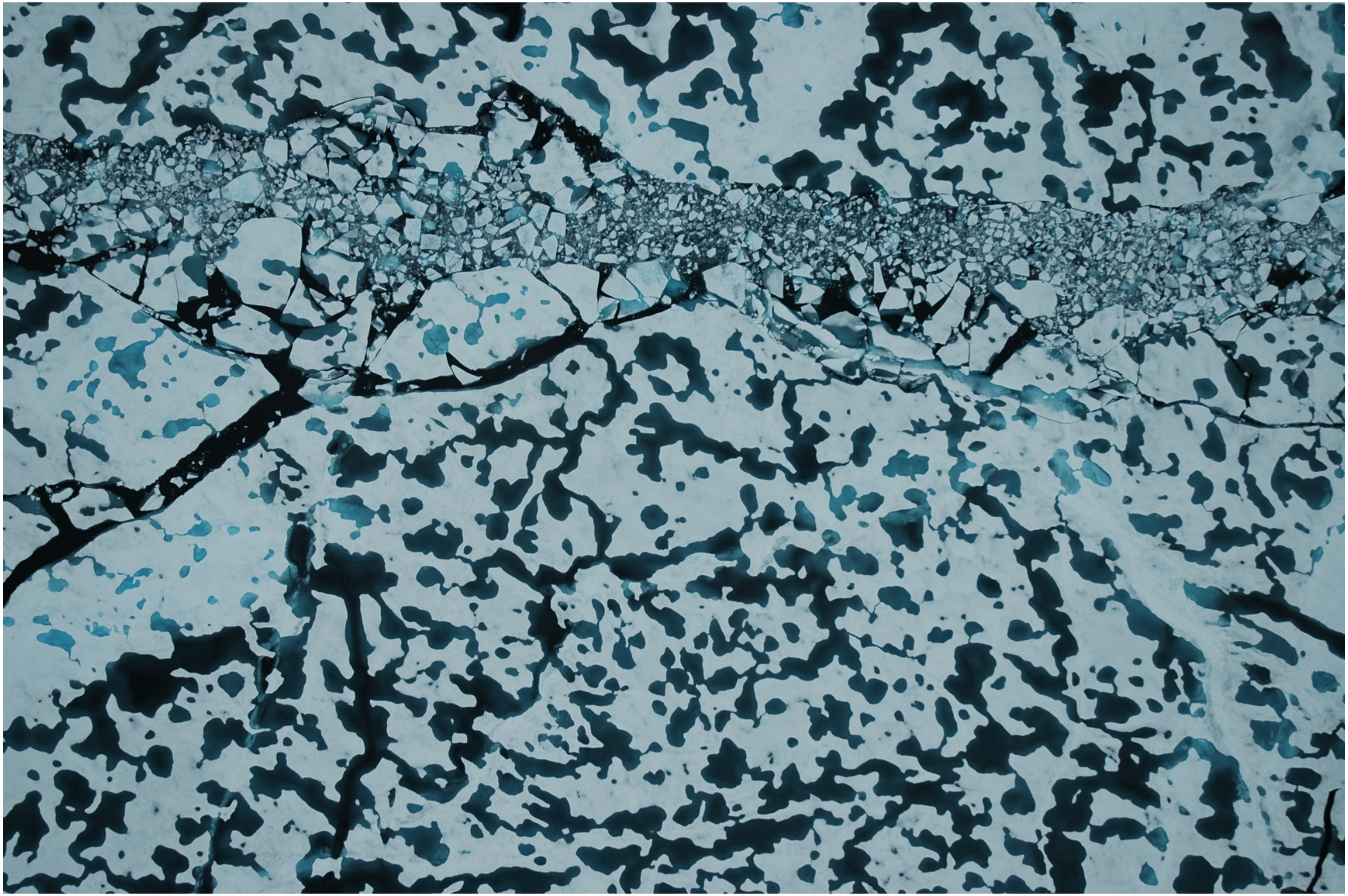
International Conference on Rayleigh-Bénard Turbulence, 1-5 June 2015, Goettingen, Germany

Motivation :

Ice melt ponds during Arctic summer, air view

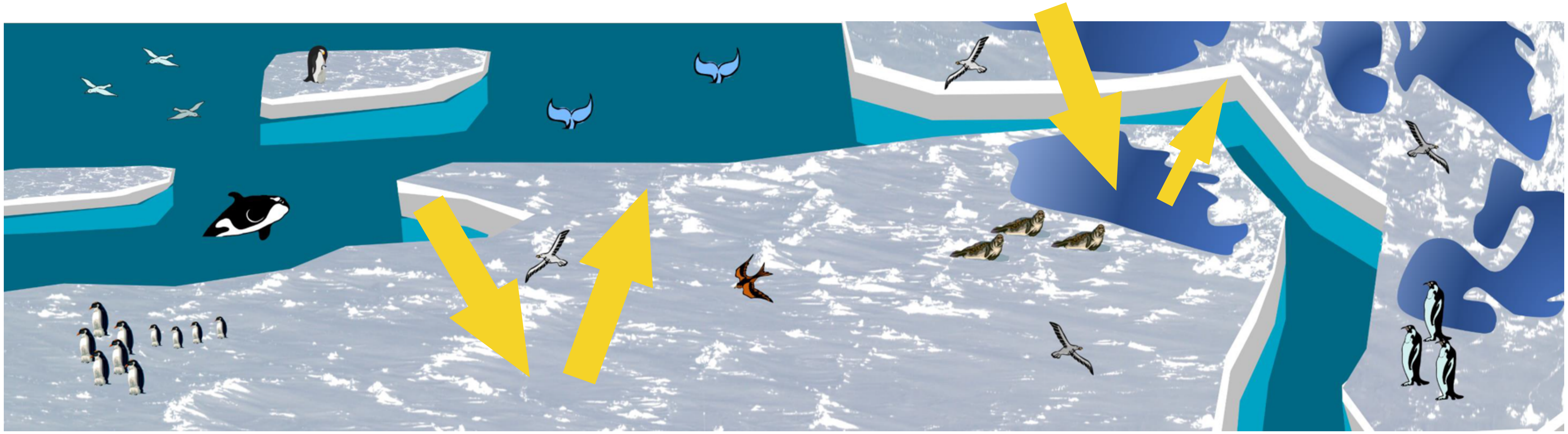


Credits : D. Perovich, ERDC August 2011

See: ***Transition in the fractal geometry of Arctic melt ponds*** C. Hohenegger et al. **The Cryosphere**, 6, 1157–1162, **2012**

Motivation :

Ice-albedo feedback in the Arctic



Credits : O. Lecomte, TECLIM

Albedo: fraction of solar energy (shortwave radiation) reflected from the Earth back into space.

Ice & snow → **high albedo**

water → **low albedo**

Ice-albedo a positive feedback :

Melt ponds formation → reduced albedo → increased Earth temperature → Melt ponds formation

Motivation :

Melt ponds a subgrid scale problem

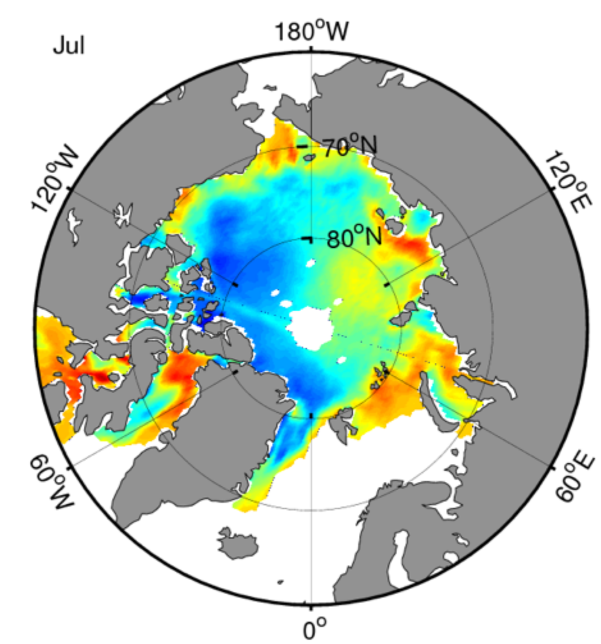
Credits: ICESCAPE mission, NASA, July 12, 2011

Ponds are too small for Global Sea Ice Models

e.g. CICE Los Alamos, LIM Belgium , MIT model

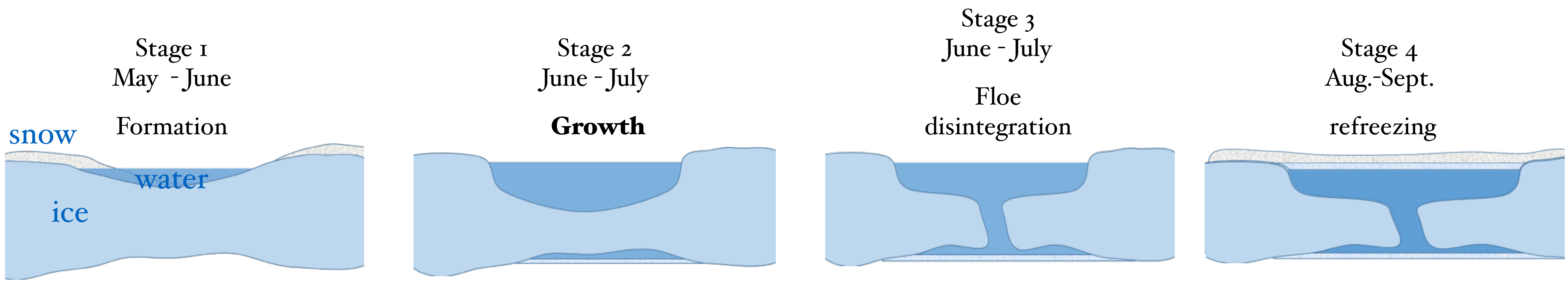
typical grid size ~ 10 x 10 Km

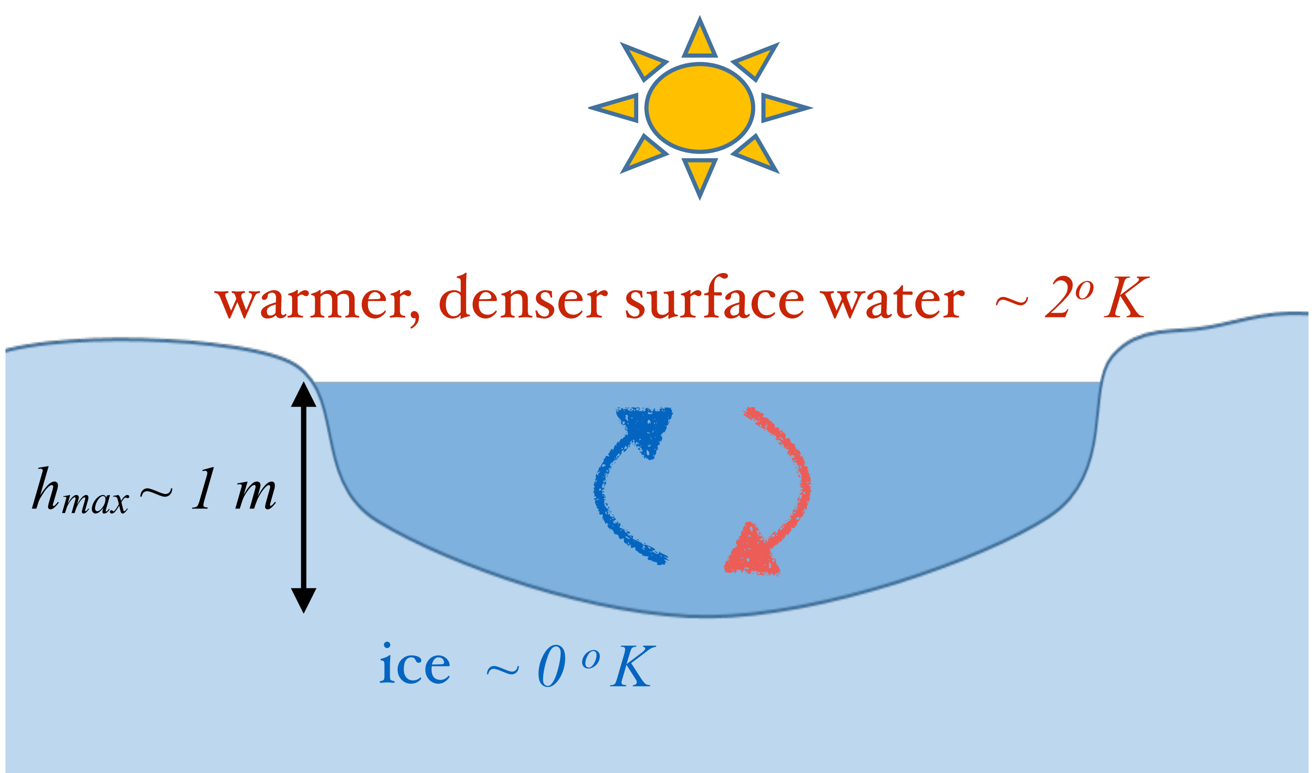
typical time step ~ 6 hours

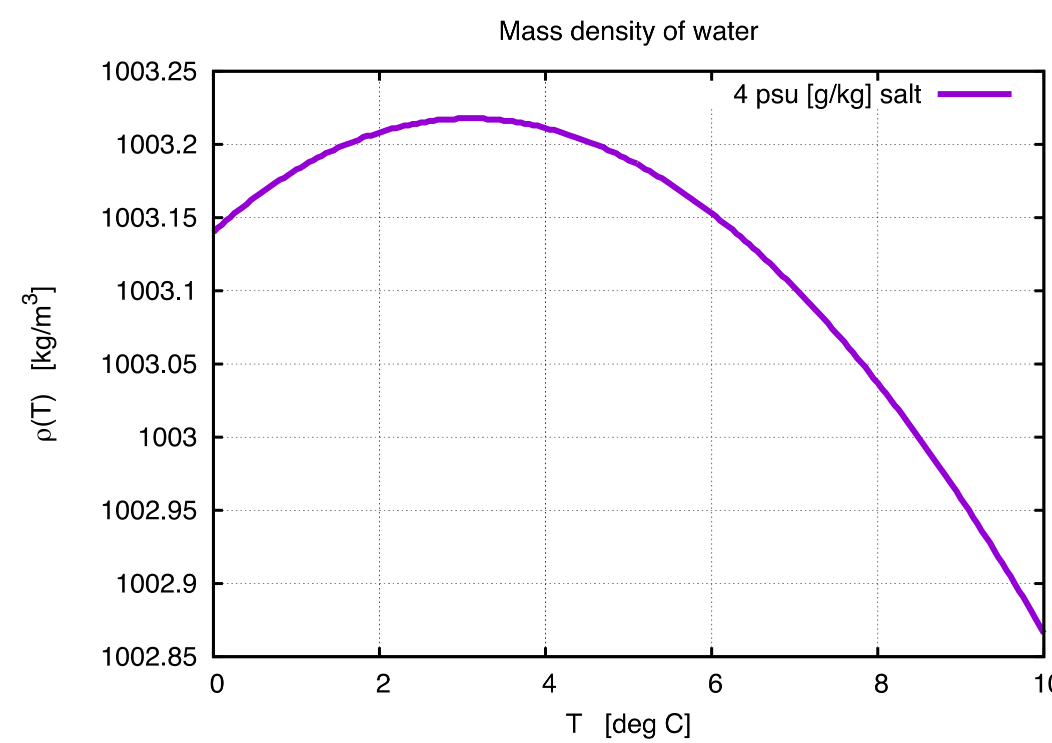


Motivation :

Dynamics of ice melt ponds



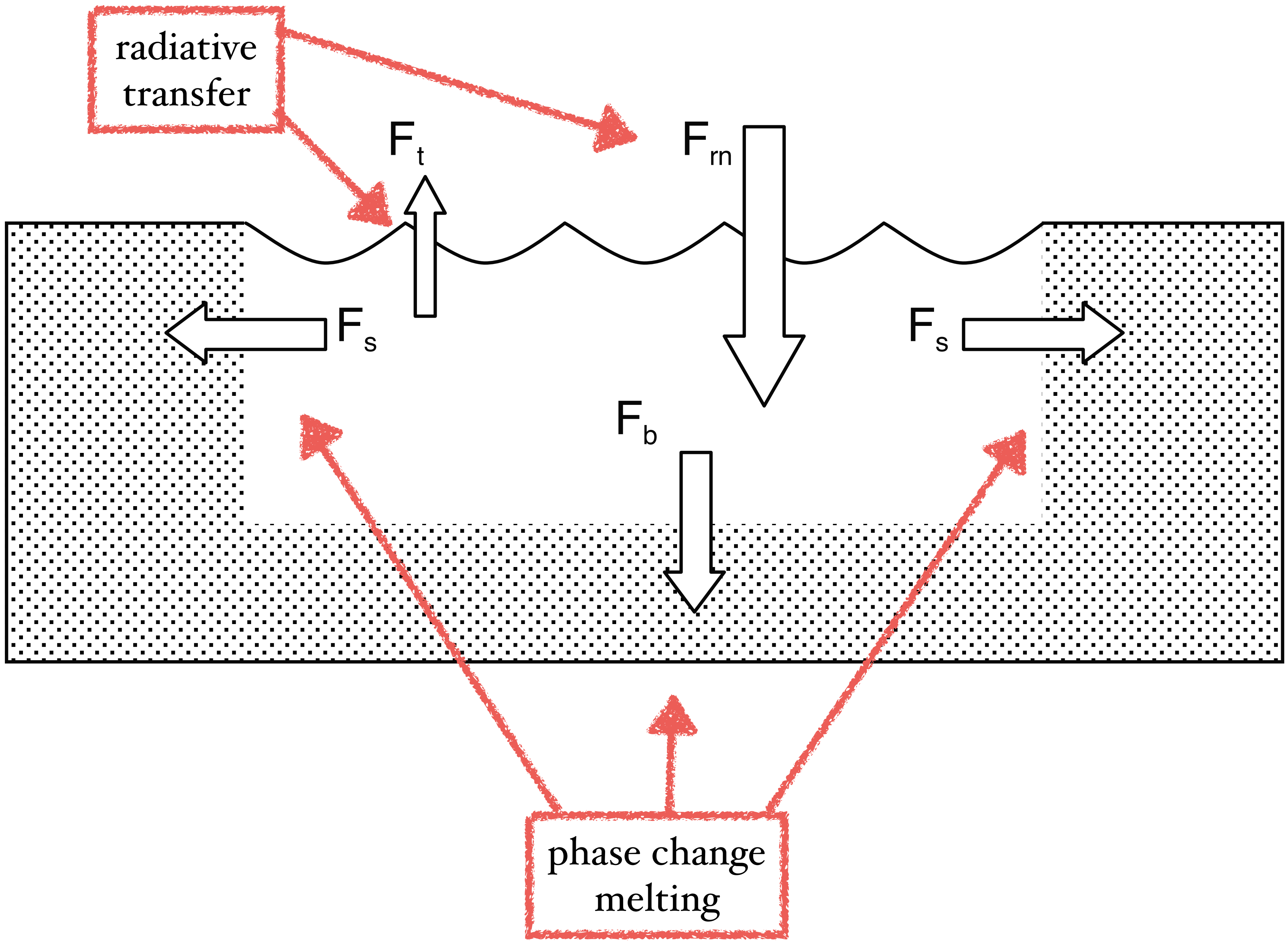




Arctic melt ponds are upside-down RB systems

Eicken, H. et al. **Tracer studies of pathways and rates of meltwater transport through arctic summer sea ice**. *J. Geophys. Res. C: Oceans, 107*(10), SHE 22-1 - 22-20. (2012)

Heat transfer in melt ponds



A numerical study of melt ponds **Eric D. Skyllingstad & Clayton A. Paulson** J. GEOPHYSICAL RESEARCH**, 112, c08015, (2007)**

Radiative energy flux in ponds

The Beer -Lambert law

$$I = I_0 e^{-\alpha z}$$

Irradiance

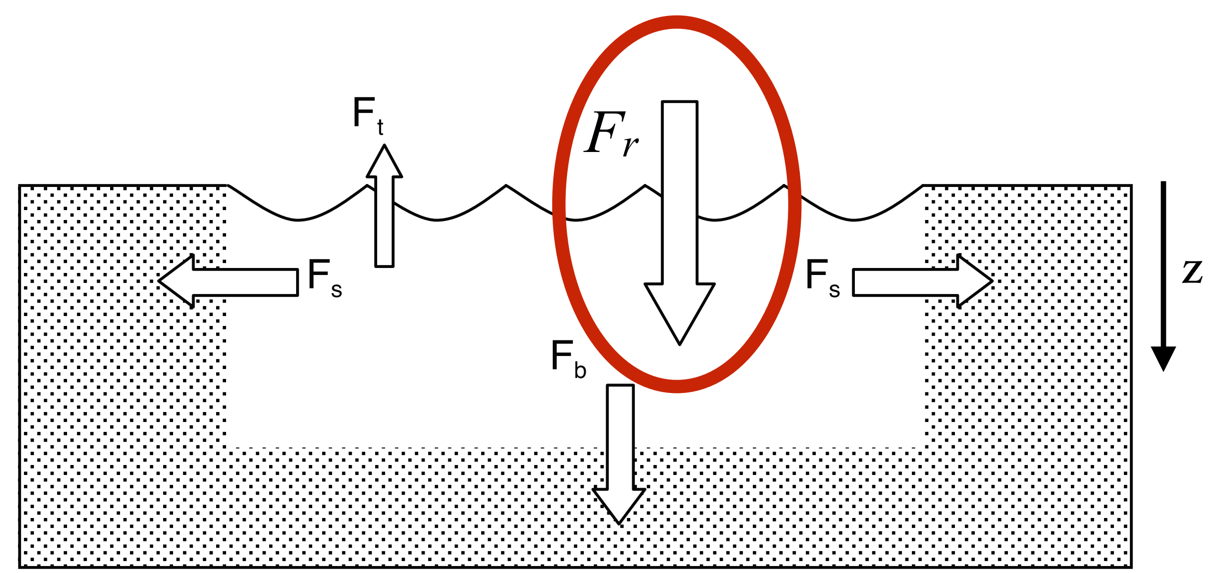
absorption coefficient

$$F_r(z) \;=\; F_{r,in}(z) + \cancel{F_{r,out}(z)}$$

$$F_{r,in}(z) \;=\; I_0 \; (1 - e^{-\alpha z})$$

$$F_{r,out}(z) \;=\; a_b \; F_{r,in}(H)(1 - e^{-(2H - \alpha z)})$$

wall albedo



α	absorption coefficient	m^{-1}
I_0	Irradiance at the top boundary	$W\ m^{-2}$
a_b	Bottom wall albedo	$-$

Irradiance measurement in the Upper Ocean, **C.A. Paulson & J.J. Simpson** J. Phys. Oceanography, **vol 7 , pp. 952 (1977)**

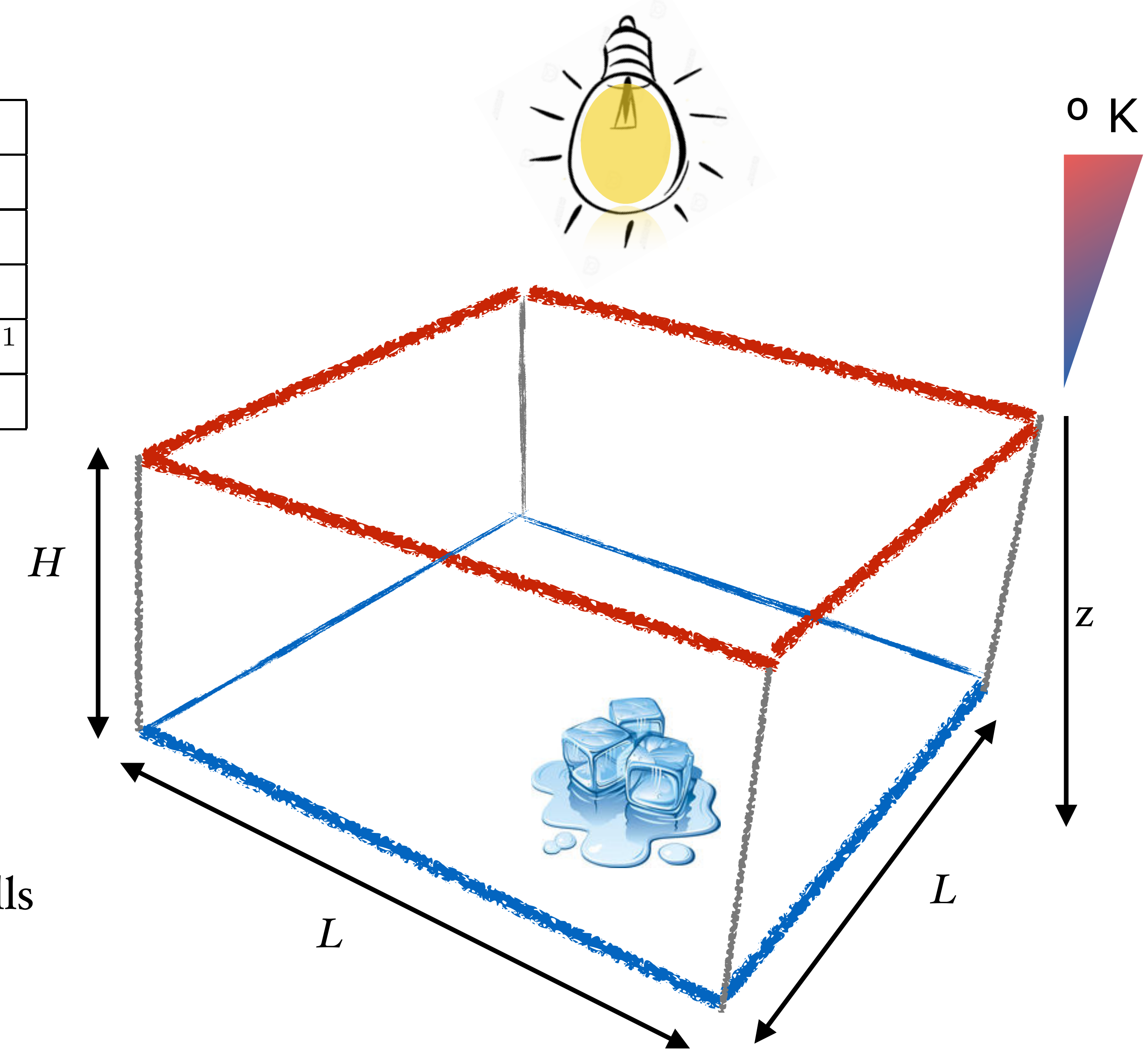
A very idealised model system

Cubic RB cell with monochromatic shortwave-radiation source

$$\partial_t \boldsymbol{u} + (\boldsymbol{u}\cdot\boldsymbol{\partial})\boldsymbol{u} \;=\; -\frac{1}{\rho_0}\boldsymbol{\partial} p + \nu\, \partial^2 \boldsymbol{u} + \beta(T - T_0) g \boldsymbol{e}_z, \quad \boldsymbol{\partial}\cdot\boldsymbol{u} = 0$$

$$\partial_t T + (\boldsymbol{u}\cdot\boldsymbol{\partial})T \;=\; \kappa\, \partial^2 T \;+\; \frac{1}{\rho_0 c_p}\partial_z F_r(z)$$

ν	kinematic viscosity	$m^2 s^{-1}$
κ	thermal diffusivity	$m^2 s^{-1}$
β	thermal expansion coefficient	K^{-1}
g	gravity acceleration	$m\ s^{-2}$
c_p	specific heat capacity at constant pressure	$J\ Kg^{-1} K^{-1}$
ρ_0	fluid density at temperature T_0	$Kg\ m^{-3}$.



A bottom-up Rayleigh-Bénard system

- reverted buoyancy
- shortwave incoming radiation flux
- transparent boundaries
- fixed temperature and no-slip on horizontal walls
- periodicity on vertical walls

A very idealised system

dimensionless form

Equations of motion

$$\partial_{\tilde{t}} \tilde{\boldsymbol{u}} + (\tilde{\boldsymbol{u}} \cdot \tilde{\boldsymbol{\partial}}) \tilde{\boldsymbol{u}} \;=\; -\tilde{\boldsymbol{\partial}} \tilde{P} + Pr\ \tilde{\partial}^2 \tilde{\boldsymbol{u}} + Pr\ Ra\ \tilde{T} \boldsymbol{e}_{\tilde{z}}$$

$$\partial_{\tilde{t}} \tilde{T} + (\tilde{\boldsymbol{u}} \cdot \tilde{\boldsymbol{\partial}}) \tilde{T} \;=\; \tilde{\partial}^2 \tilde{T} \;+\; \frac{Ra_r}{Ra}\ Er\ e^{-Er\tilde{z}}$$

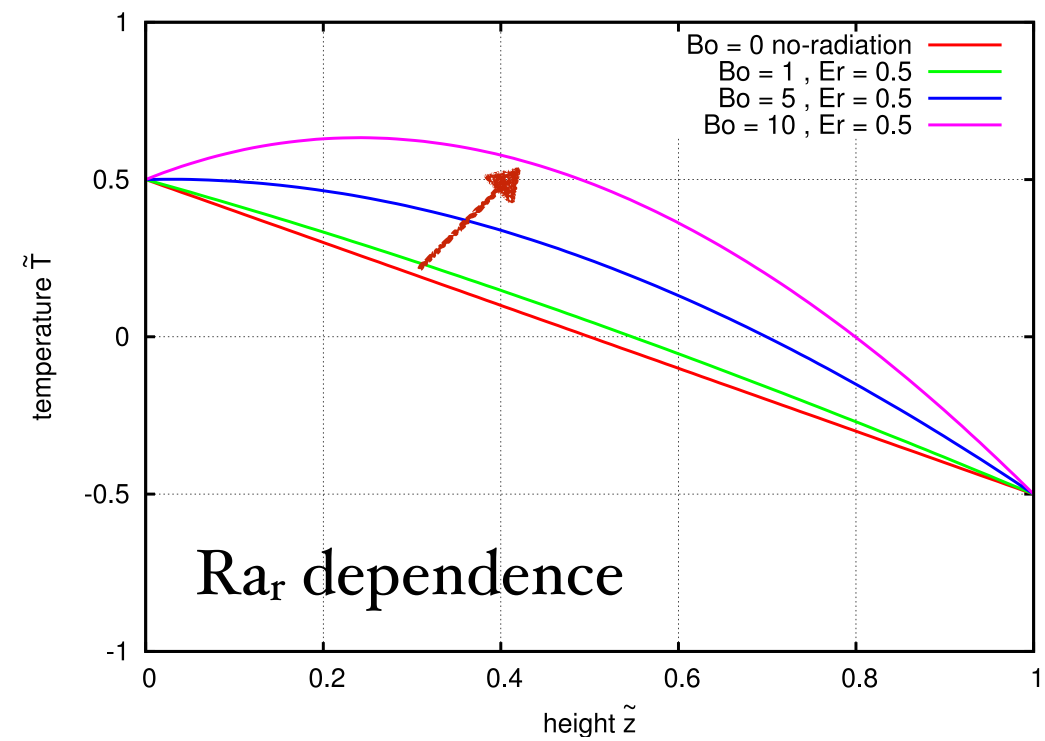
Control parameters

$Ra = \frac{\beta\ g \Delta T H^3}{\nu\kappa}$	Rayleigh number
$Er = \alpha H$	Extinction ratio
$Ra_r = Ra\ Bo = \frac{\beta\ g\ I_0\ H^4}{\rho_0 c_p \nu\ \kappa^2}$	Radiant Rayleigh number
$Bo = \frac{I_0 H}{\rho_0 c_p \kappa \Delta T}$	Radiative over conductive flux ratio
$Pr = \frac{\nu}{\kappa}$	Prandtl number
$Ar = \frac{L}{H}$	Aspect ratio

Vertical Temperature profile

for the conductive state

$$\tilde{T}(\tilde{z}) = -\tilde{z} + \frac{1}{2} + \frac{Ra_r}{Ra\ Er}\left((1-e^{-Er\tilde{z}}) - (1-e^{-Er})\tilde{z}\right)$$

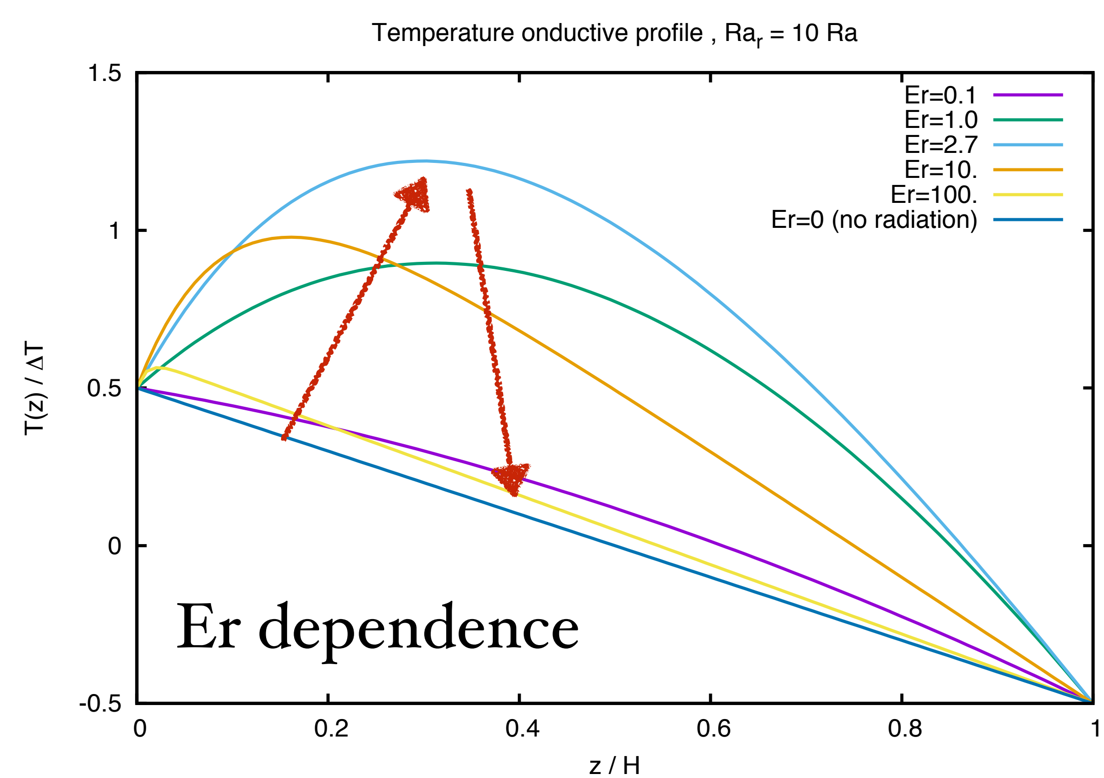


Average bulk temperature

$$\langle\tilde{T}\rangle_V = \frac{Ra_r}{Ra\ Er}\left(\frac{1}{2} + \frac{1}{Er}e^{-Er} - \frac{1}{Er} + \frac{1}{2}e^{-Er}\right) \quad \neq \frac{\tilde{T}_{top} + \tilde{T}_{bot}}{2}$$

Max:

$\langle\tilde{T}\rangle_V \simeq 0.07\ \frac{Ra_r}{Ra}$ for $Er = 2.688$



Heat flux

From time and surface average of temperature equation

$$\partial_{\tilde z}\left(\underbrace{\overline{\tilde{\boldsymbol{u}}_z \tilde T} - \tilde\partial_z \overline{\tilde T}}_{\text{not constant}} - \; Bo(1 - e^{-Er\tilde z}) \right) = 0$$

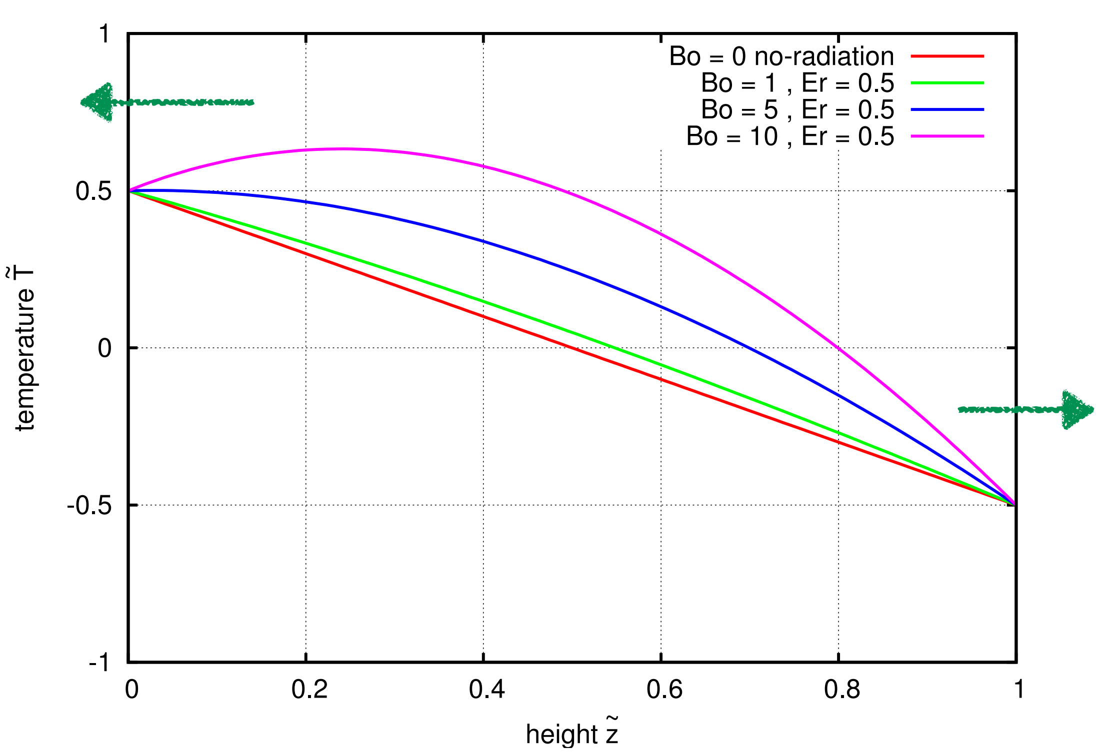
mean heat flux
across a horizontal surface
at height z

$$\overline{\tilde J}(\tilde z) \equiv \overline{\tilde u_{\tilde z} \tilde T} - \partial_{\tilde z} \overline{\tilde T} = 1 + \underbrace{\frac{Ra_r}{Ra}\left(\frac{1 - e^{-Er}}{Er} - e^{-Er\,\tilde z} \right)}_{\text{radiative}} + \langle \tilde u_{\tilde z} \tilde T \rangle_V$$

conductive

radiative

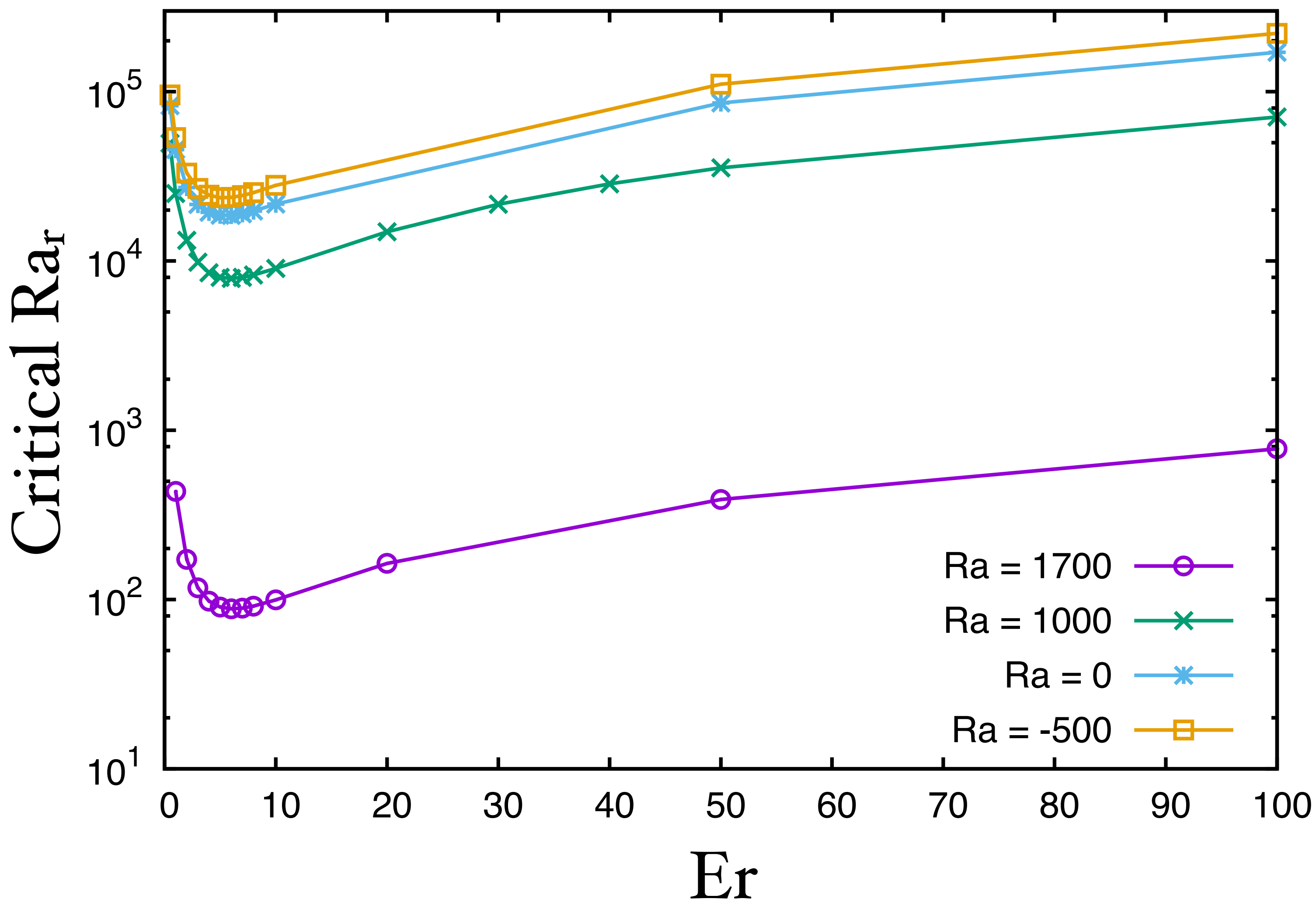
convective



Heat can flow out from both plates

Convective instability

From linear stability analysis



- Radiation always destabilises the system, even when stably stratified ($\Delta T < 0$)
- Optimal Er value ~ 6

Convective and absolute instabilities in Rayleigh–Bénard–Poiseuille mixed convection for viscoelastic fluids
S. C. Hirata, *et al.* J. Fluid Mech. (2015), vol. 765, pp. 167–210

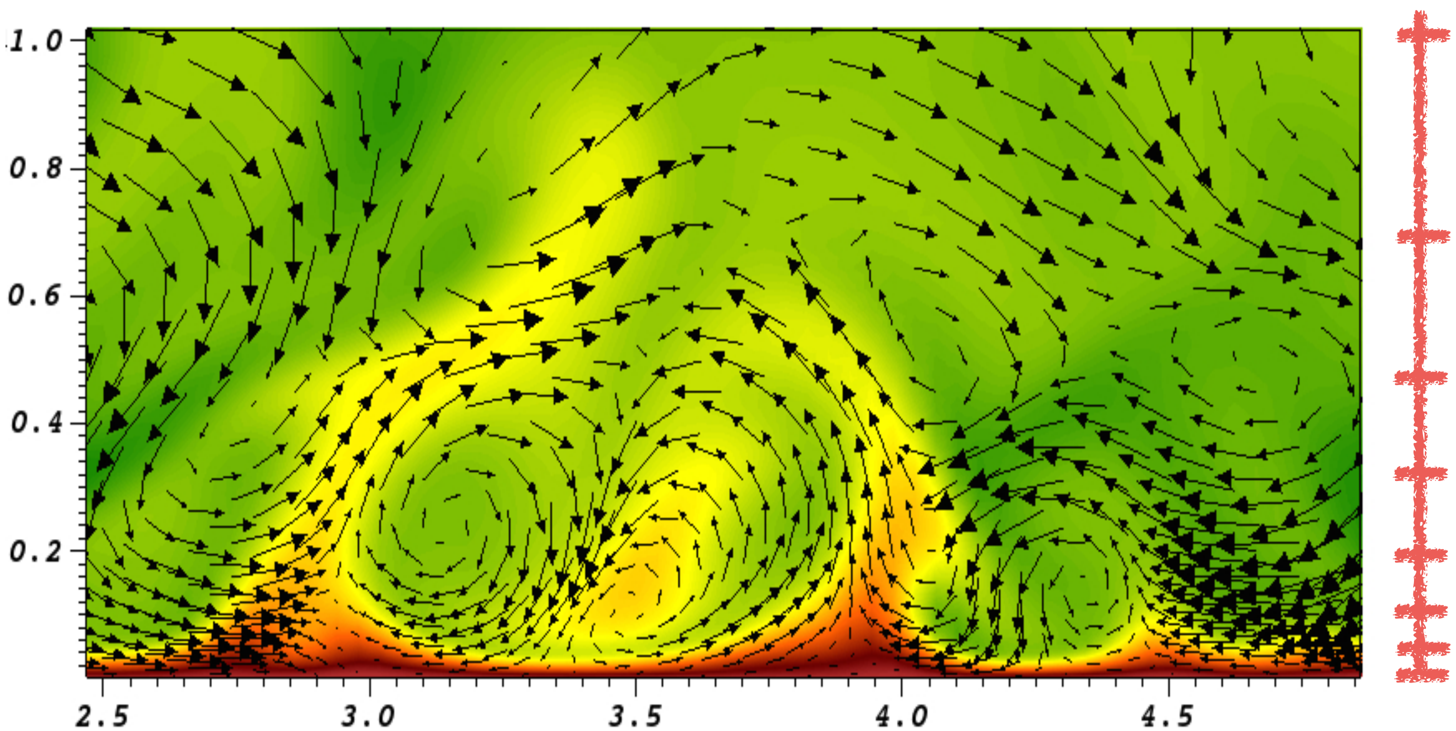
Convective state

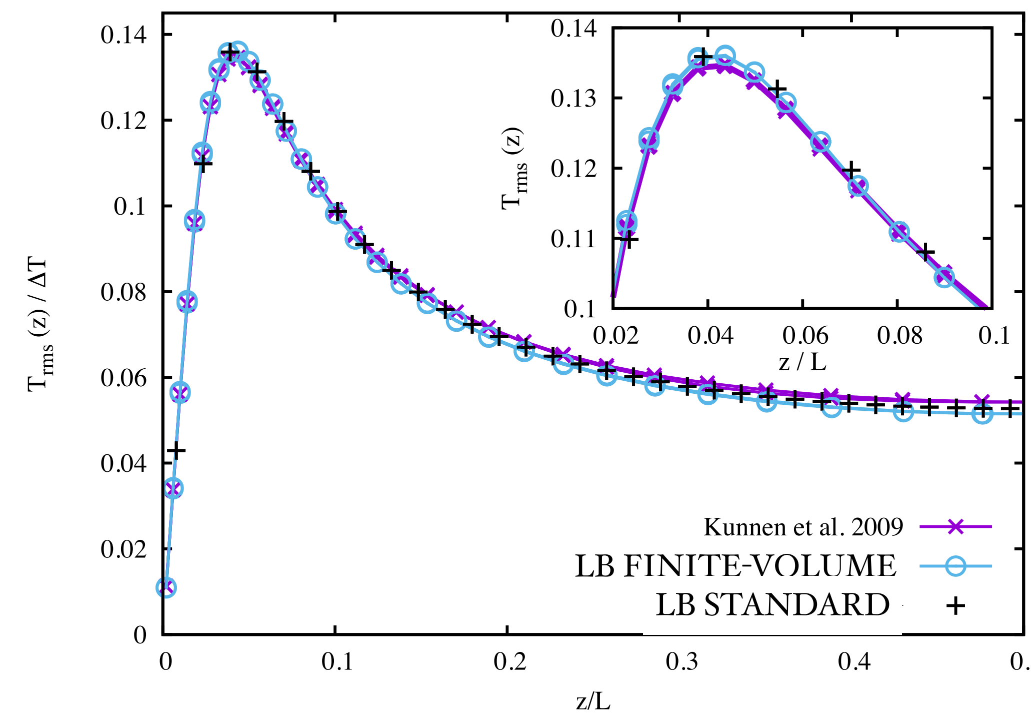
Numerical simulations

Lattice Boltzmann
method
2 implementations

standard, efficient, regular grid

finite-volume , expensive, wall-stretched grid



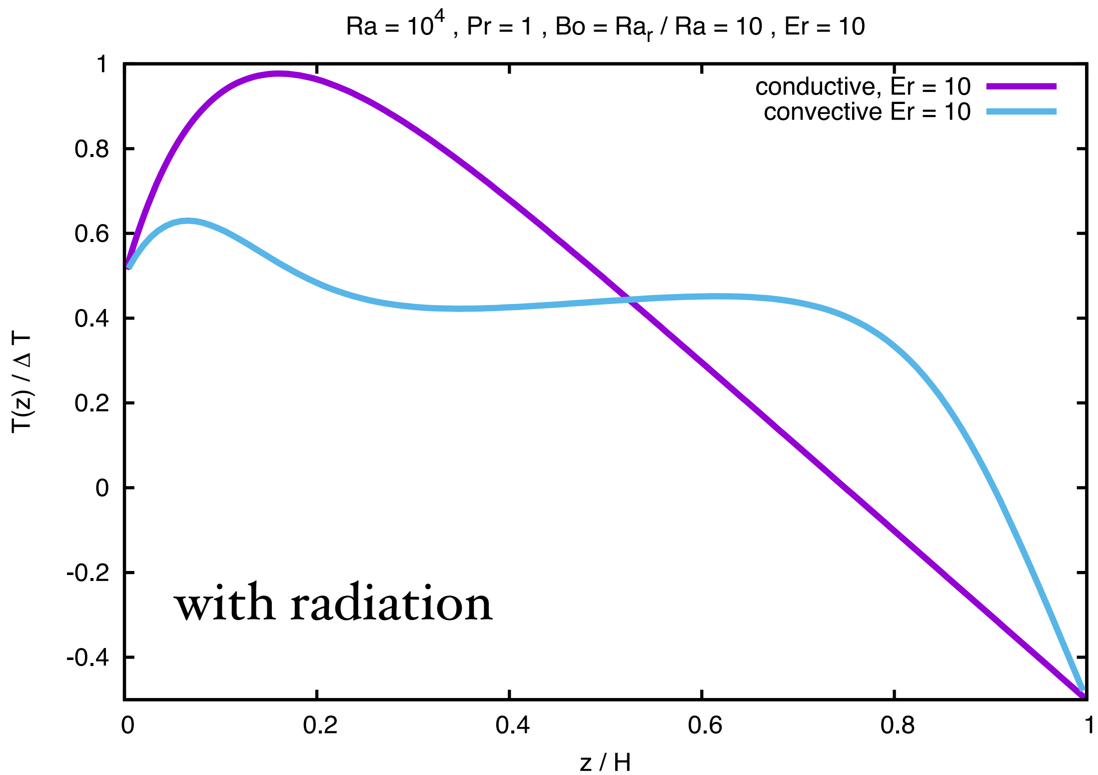


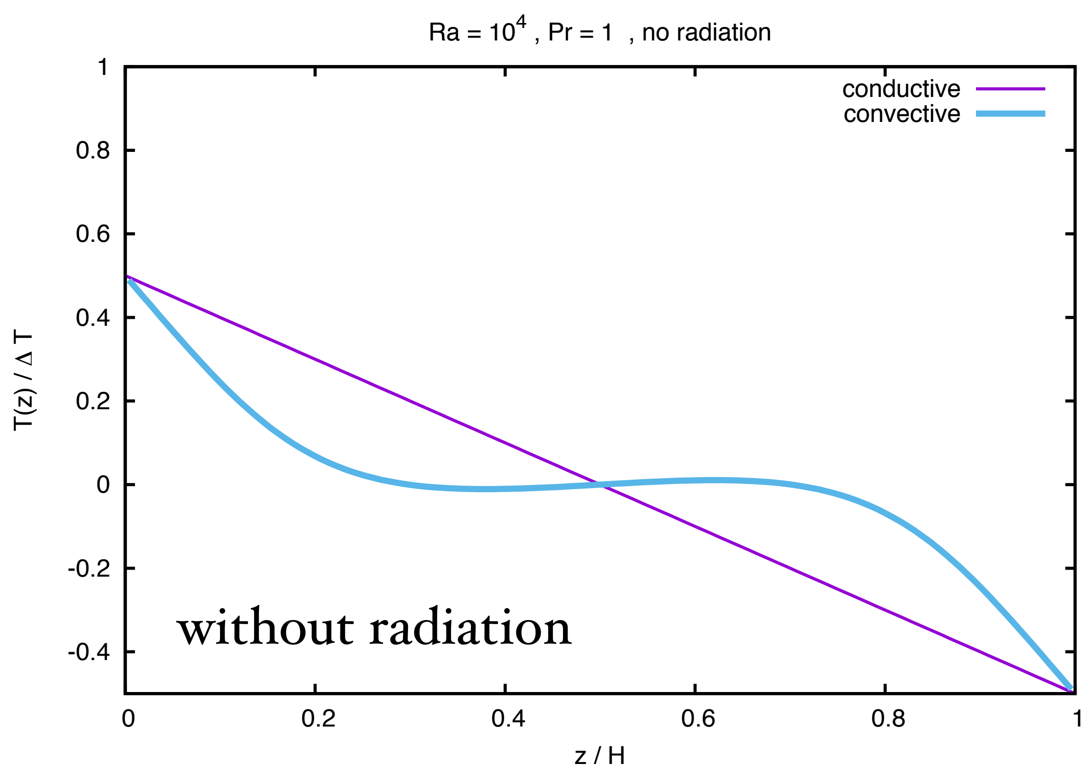
More details in:

Finite Volume vs. Streaming-based Lattice Boltzmann algorithm for fluid-dynamics simulations: a one-to-one accuracy and performance study
Kalyan Shrestha, Gilmar Mompean, Enrico Calzavarini   ArXiv.org/abs/1505.03271

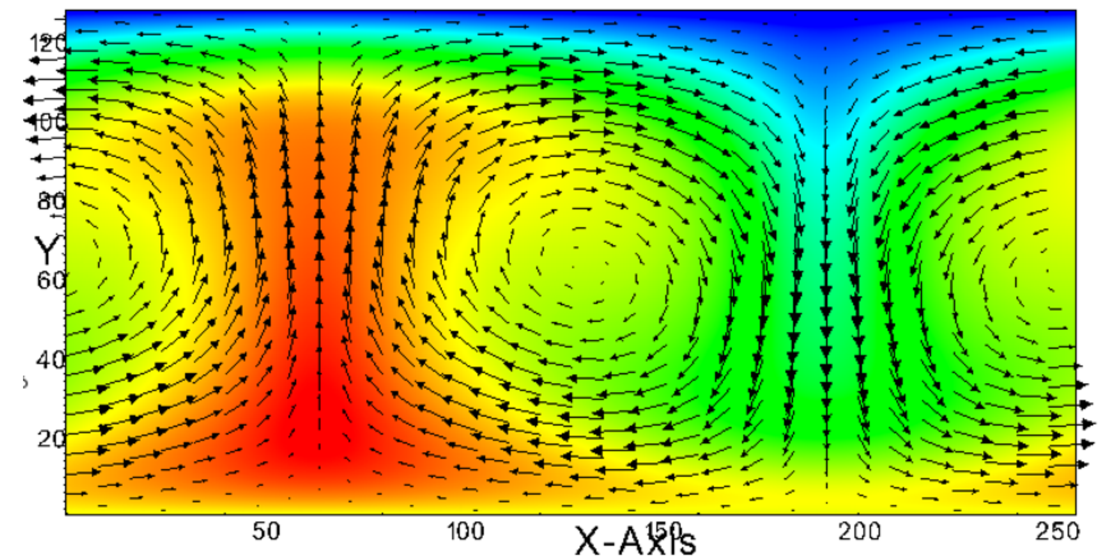
Steady Convection

Close to onset

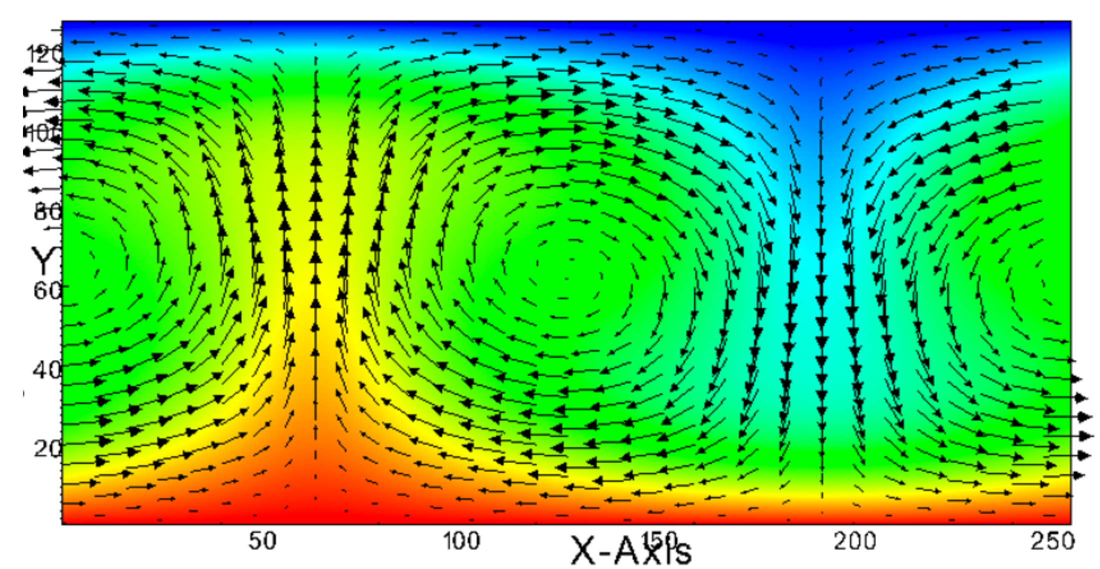




$Ra = 10^4 \ , Ra_r/Ra = 10 \ , Er = 10$

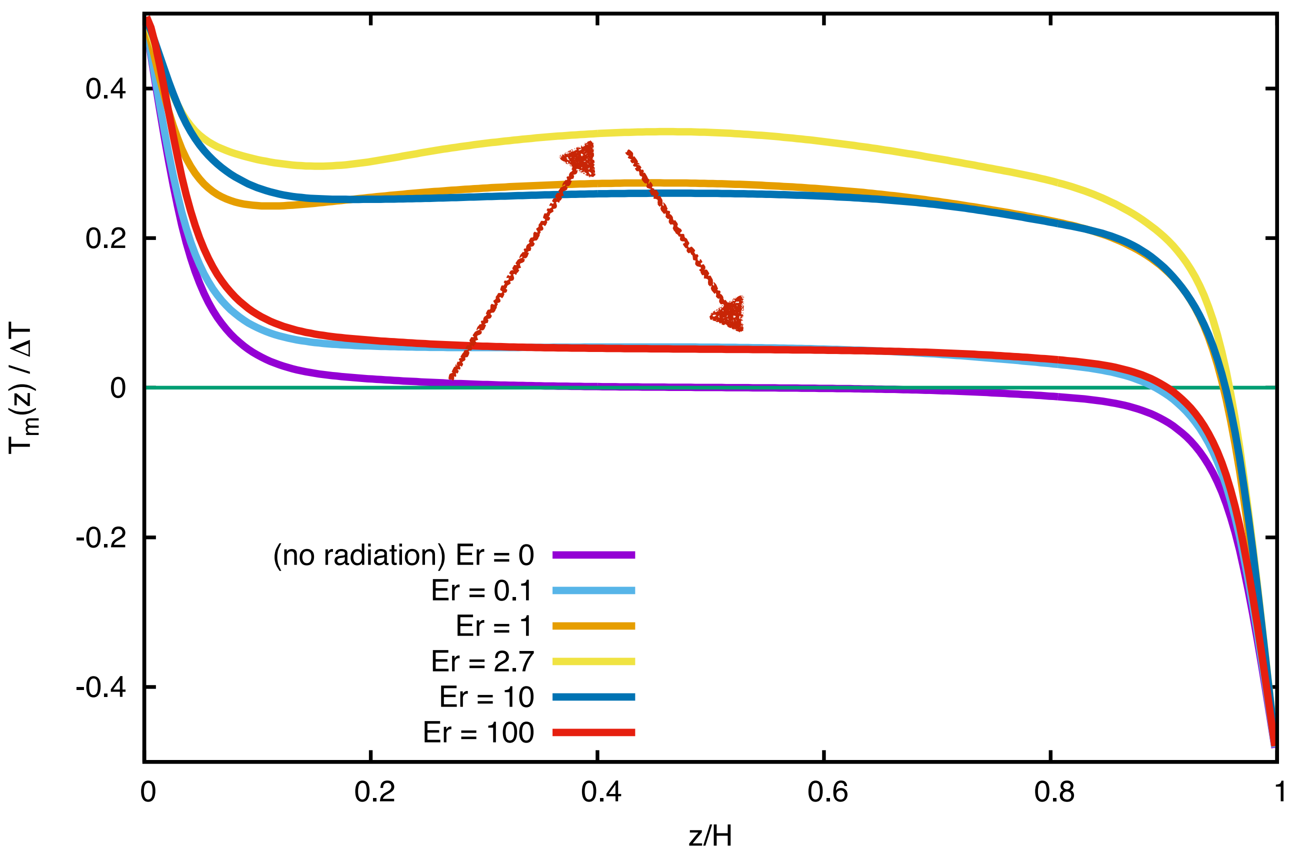


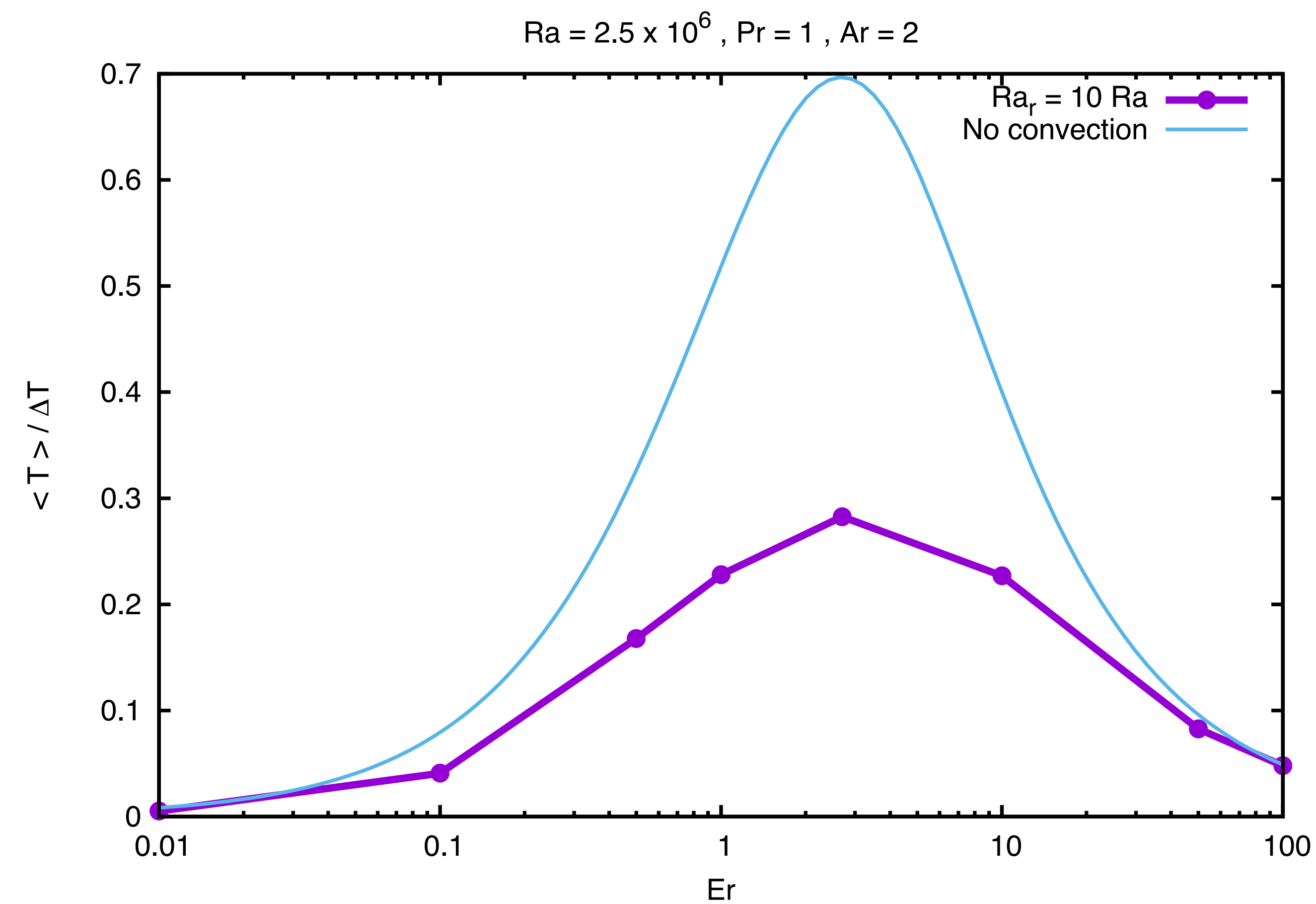
$Ra = 10^4 \ , Ra_r = 0$



Effect of absorption depth (Er)

$Ra = 2.5\ 10^6\ ,\ Ra_r/Ra = 10$

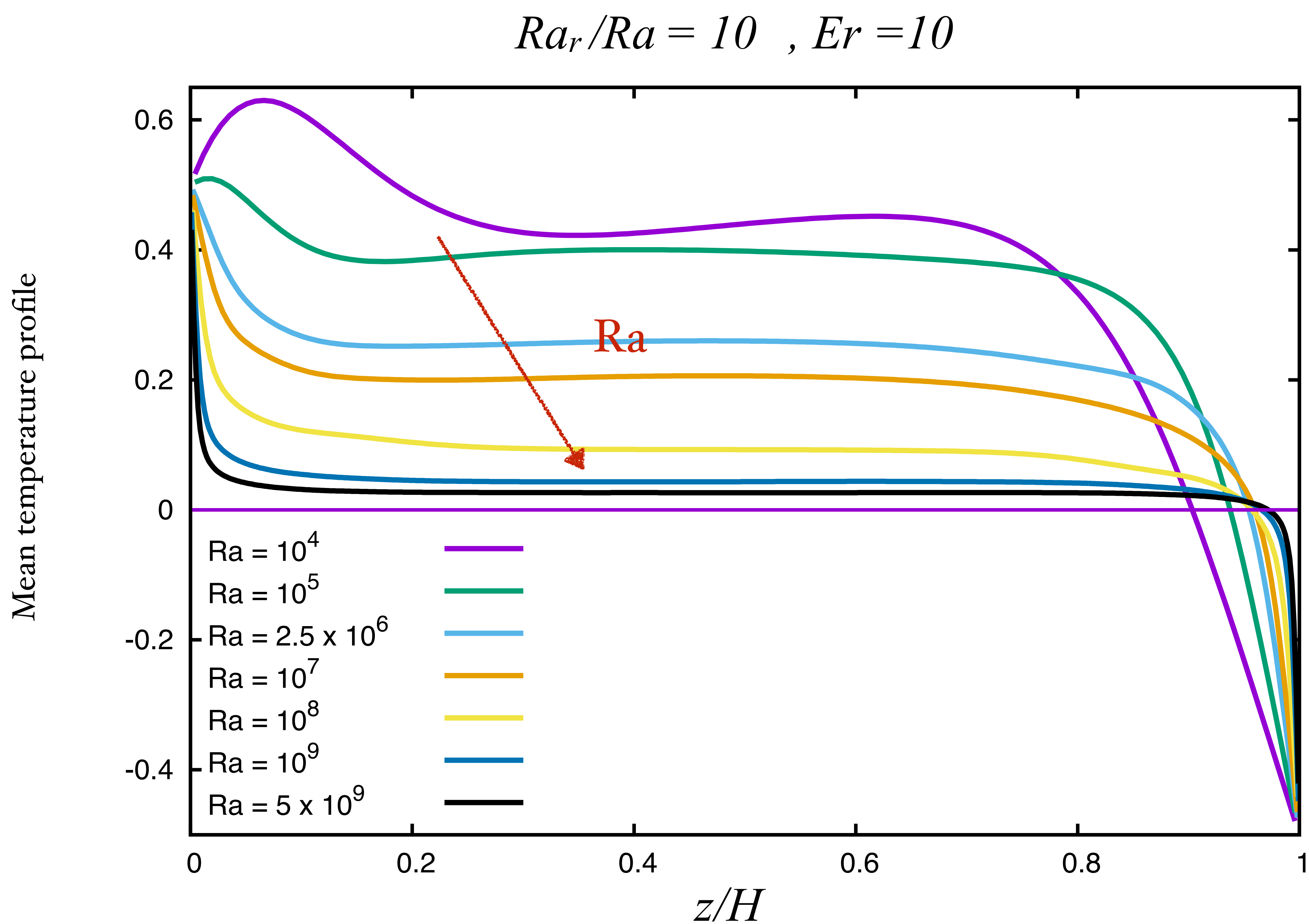




Non-monotonous behaviour with Er

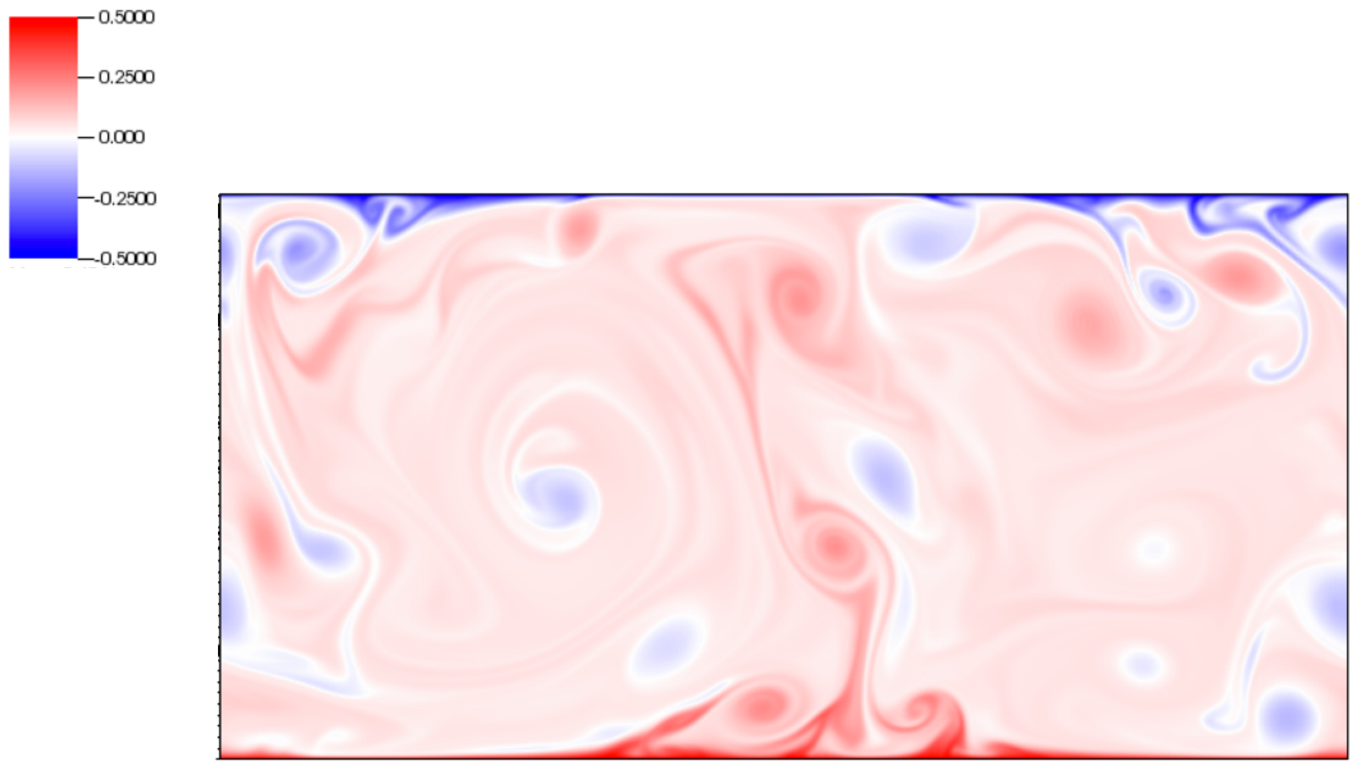
The mean global temperature maximum at Er ~ 3
reflects the conductive state trend

Effect of Rayleigh number



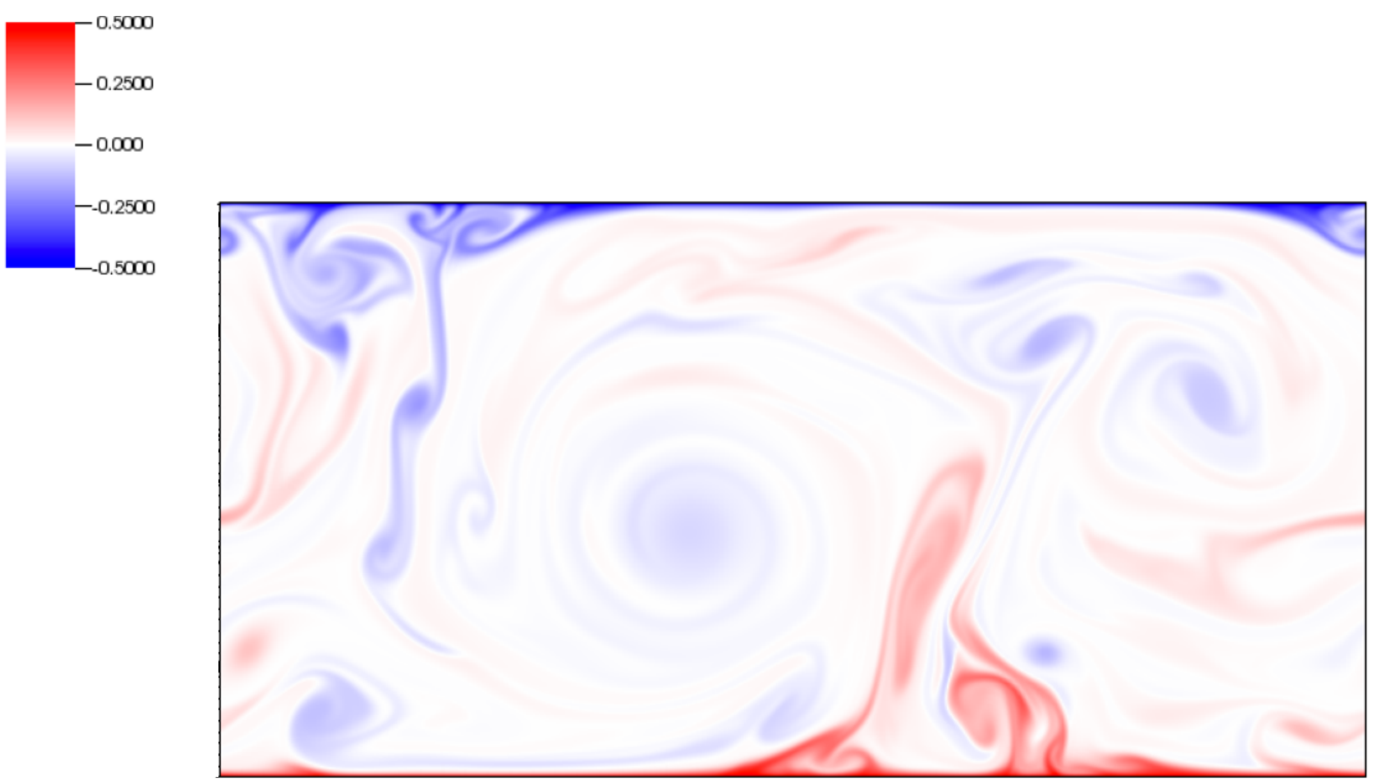
Increased system temperature

with radiation



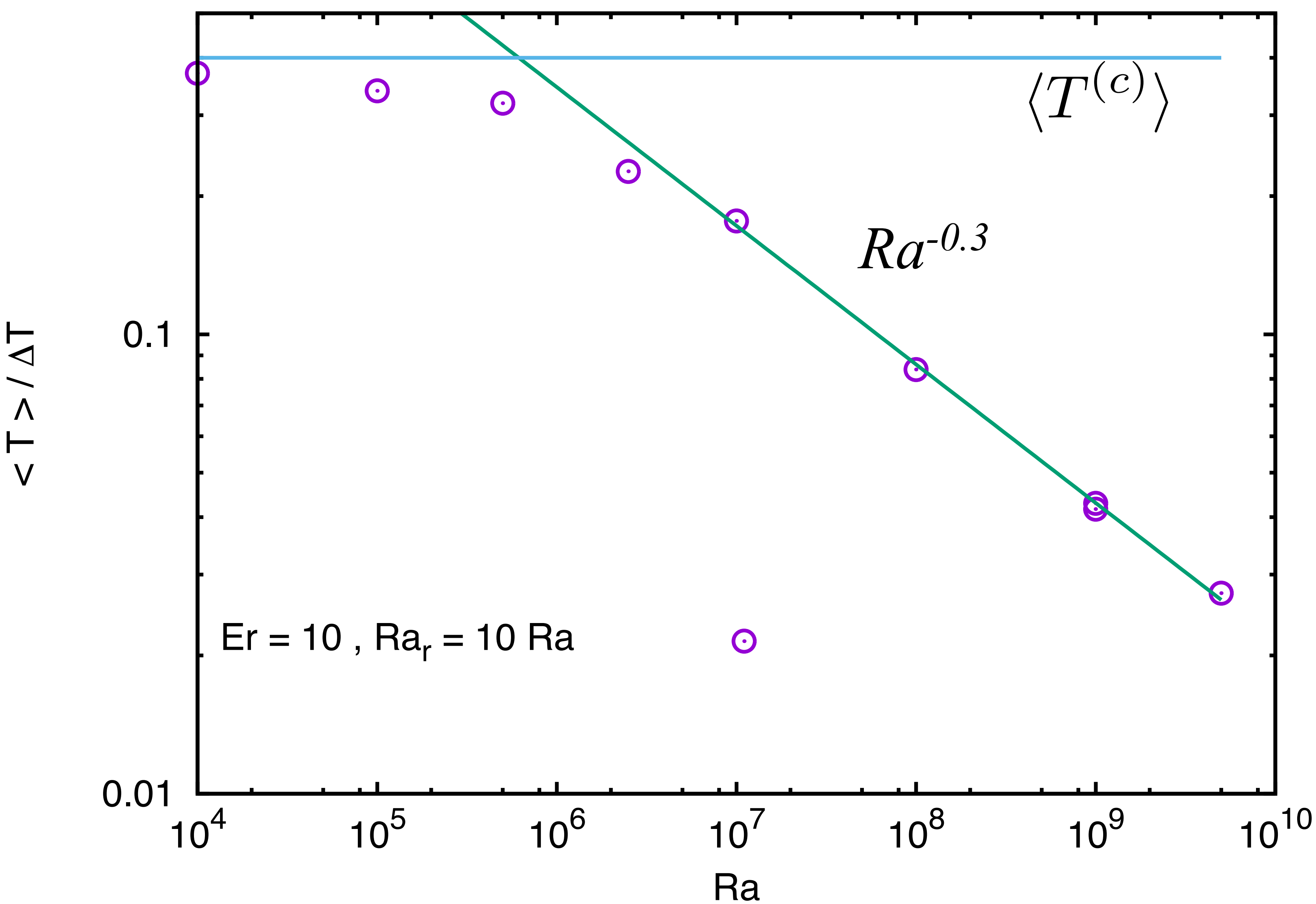
$Ra = 10^9$
$Pr = 1$
$Ra_r/Ra = 10$
$Er = 10$

without radiation



$Ra = 10^9$
$Pr = 1$

Mean global temperature *vs.* Ra

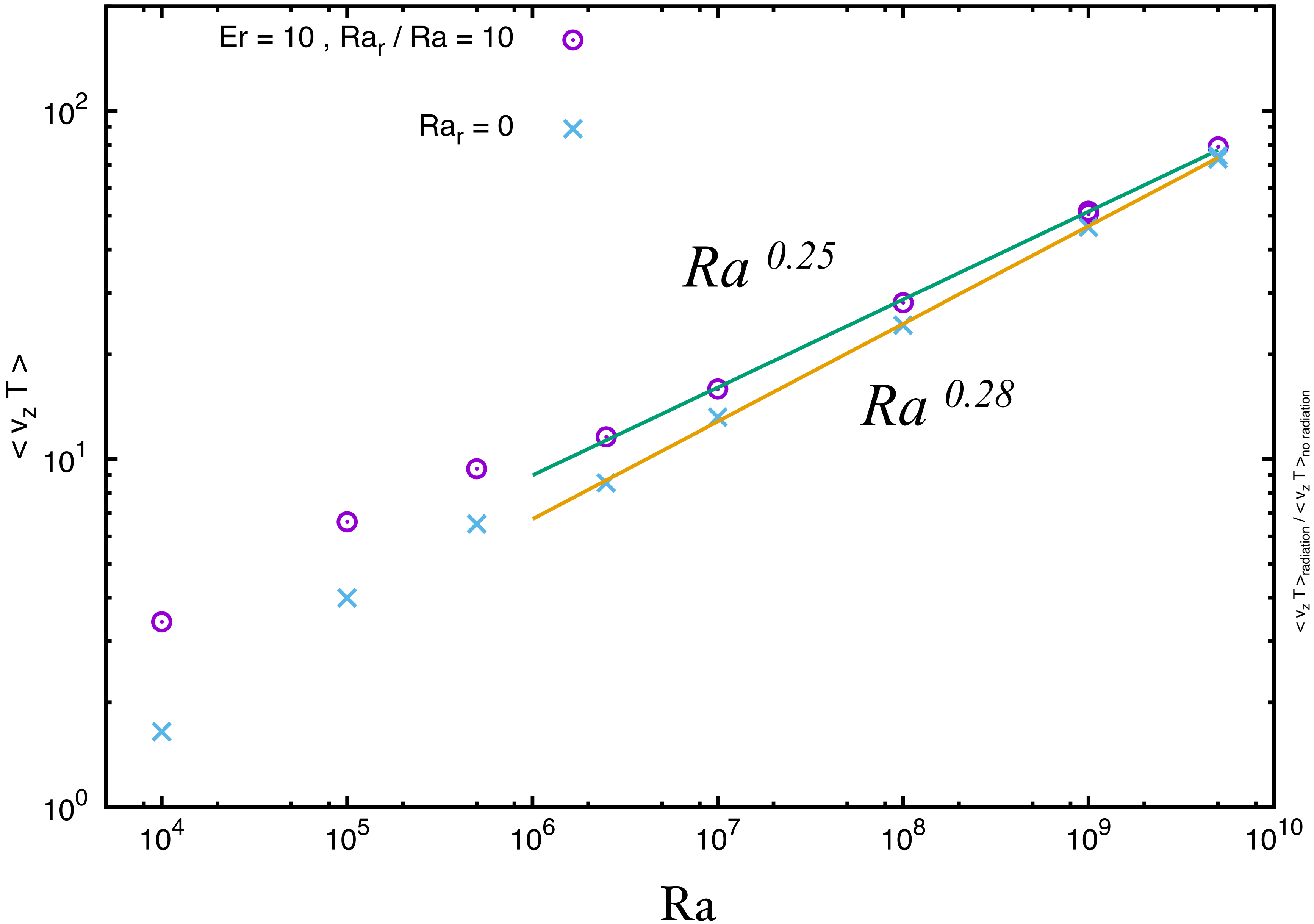


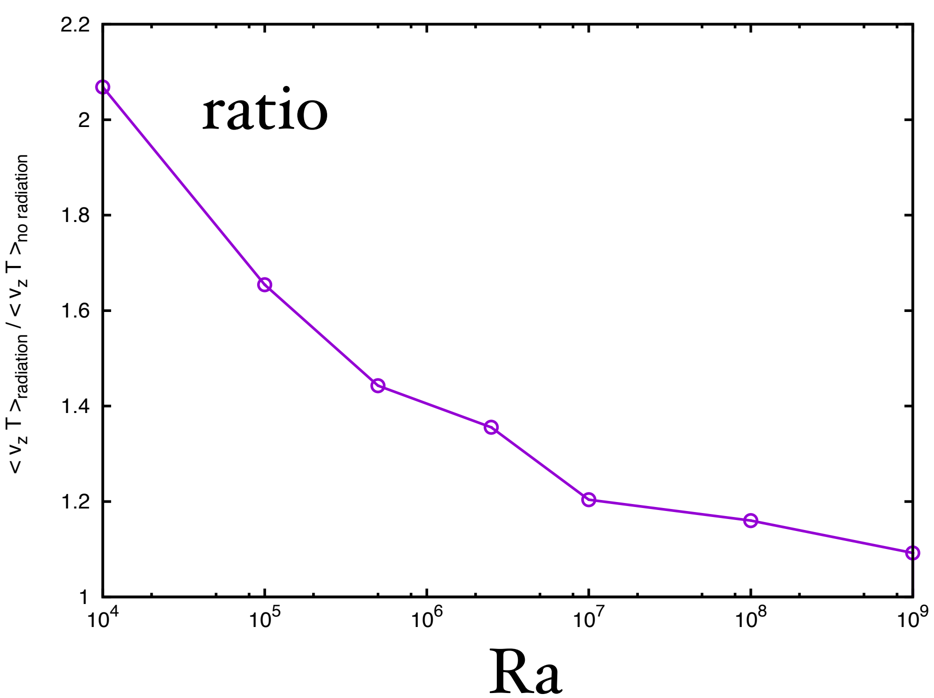
exact relation

$$\langle T \rangle = \langle T^{(c)} \rangle - \left\langle \left(z - \frac{1}{2} \right) u_z T \right\rangle$$

D. Goluskin, E.A. Spiegel, **Convection driven by internal heating**, Phys. Lett. A 377, 83–92 (2012).

Global heat flux *vs.* Ra

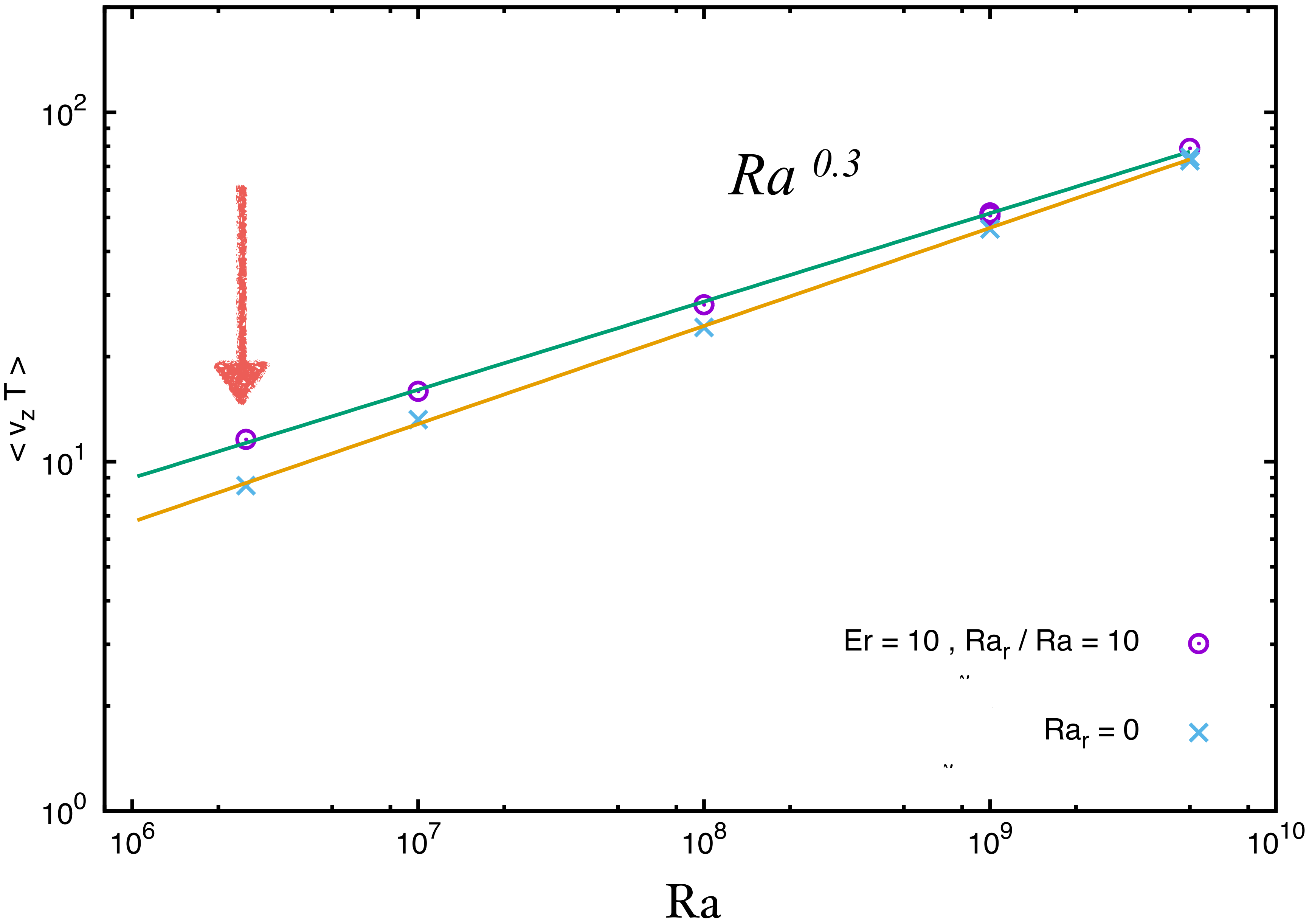




- Radiation heating increases the heat flux
- becomes negligible as turbulence (Ra) increases at $Ra_r/Ra = const$ and $Er = const$.

Consequences for heat flux in ponds

Increasing pond depth $h \longrightarrow Ra \sim h^3$, $Ra_r \sim h^4$, $Er \sim h$



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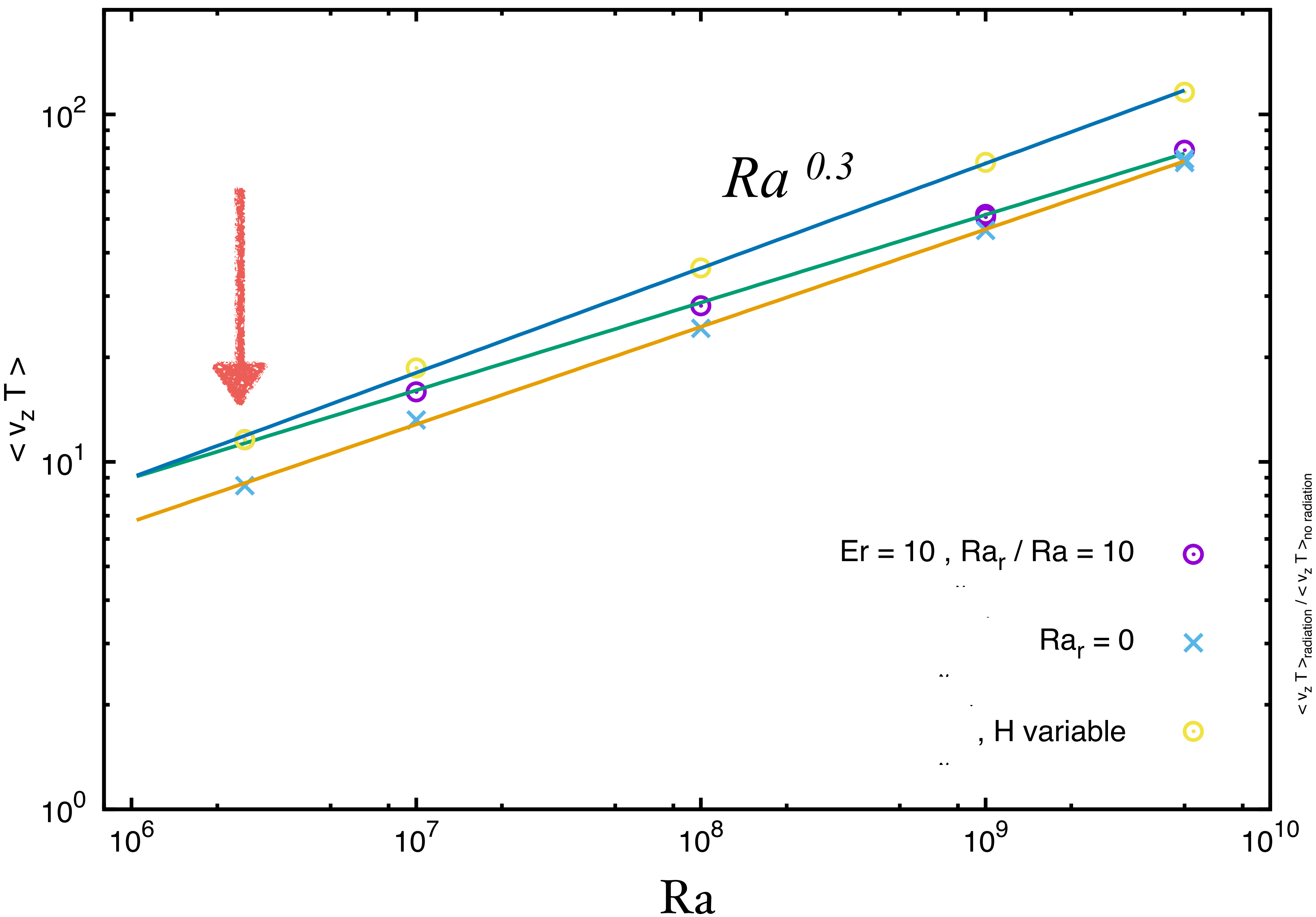
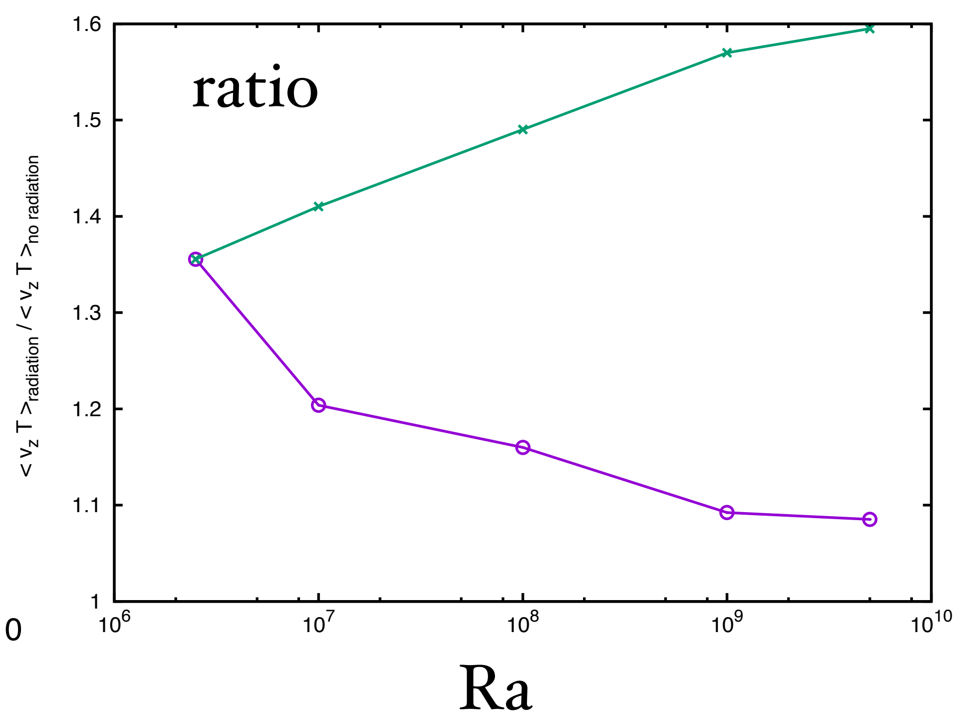




Summary & Perspectives

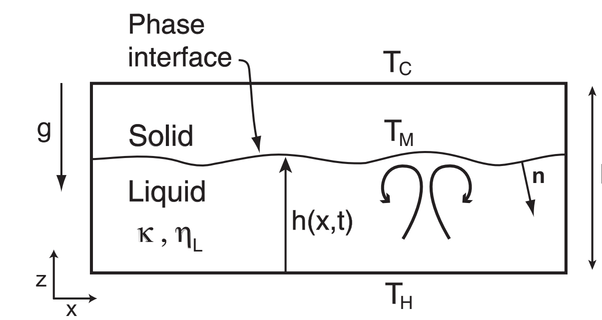
- An optimal extinction ratio (Er) exists.
- Radiation heating increases the heat flux but becomes negligible as turbulence (Ra) increases and $Ra_r/Ra = const$ and $Er = const$.
- However the pond grows (h larger), relative importance of radiation can not be overlooked

- Better explore the $Nu(Ra, Ra_r, Er)$ relation
- Implement more refined boundary conditions (imposed temperature flux, radiative bc, upper wind shear, etc.)
- Introducing bottom ice melting effect

Melting

$$\partial_{\tilde t}\tilde{\boldsymbol{u}} + (\tilde{\boldsymbol{u}}\cdot\tilde{\boldsymbol{\partial}})\tilde{\boldsymbol{u}} \;=\; -\tilde{\boldsymbol{\partial}}\tilde P + Pr\;\tilde\partial^2\tilde{\boldsymbol{u}} + Pr\;Ra\;\tilde T\boldsymbol{e}_{\tilde z}$$

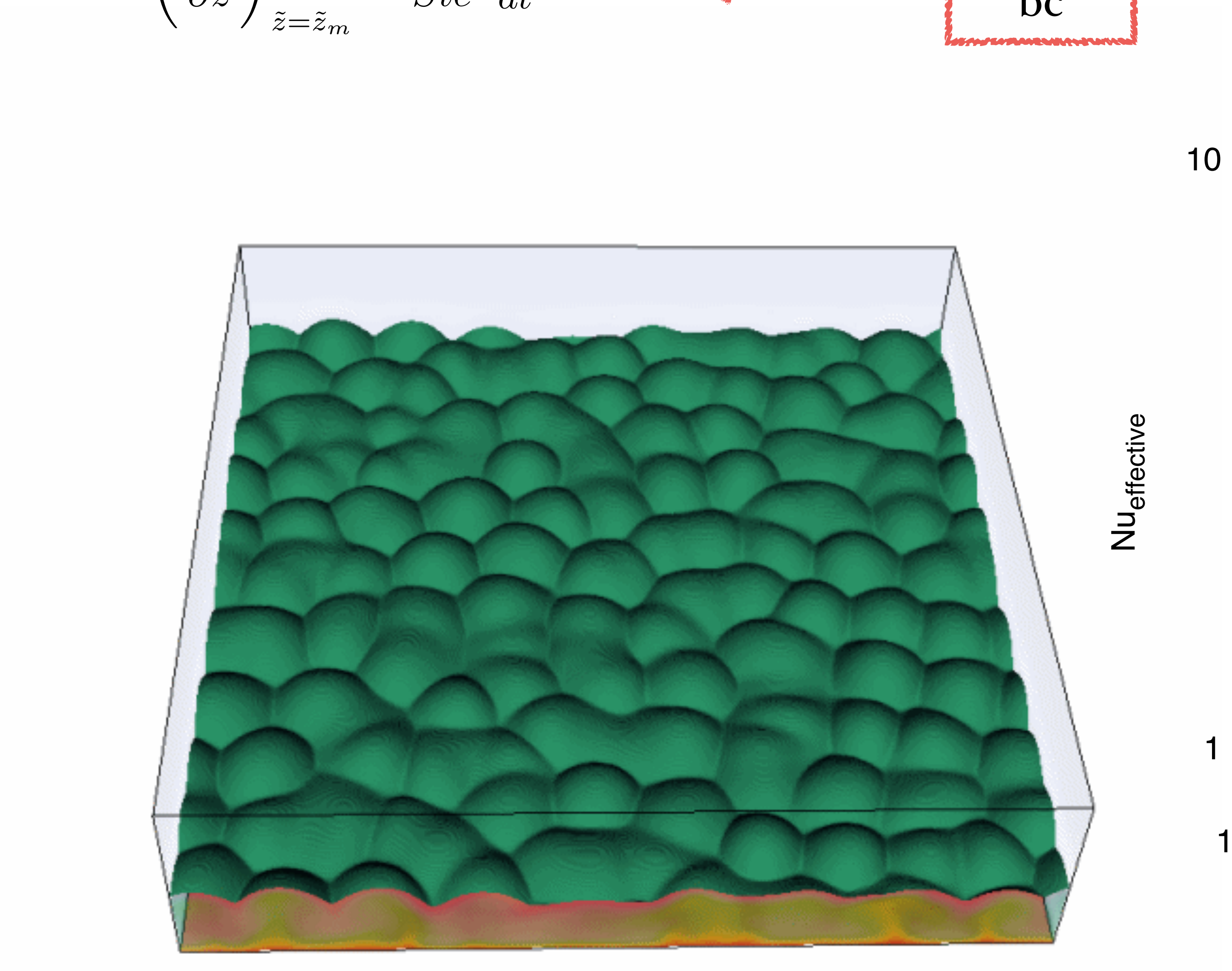
$$\partial_{\tilde t}\tilde T + (\tilde{\boldsymbol{u}}\cdot\tilde{\boldsymbol{\partial}})\tilde T \;=\; \tilde\partial^2\tilde T \;+\; \frac{Ra_r}{Ra}\;Er\;e^{-Er\tilde z}$$

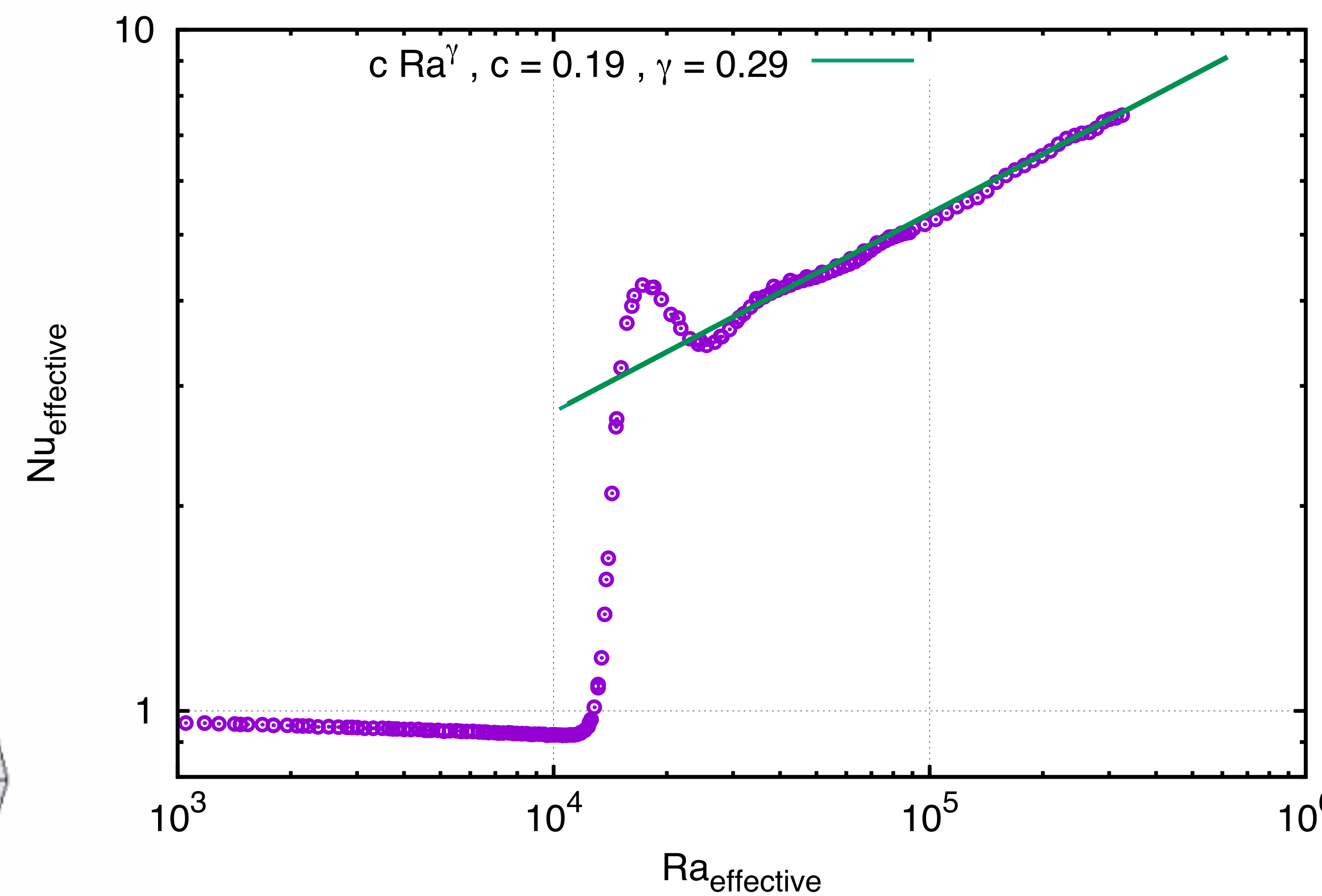


$$\left(\frac{\partial\tilde T}{\partial\tilde z}\right)_{\tilde z=\tilde z_m} = \frac{1}{Ste}\,\frac{d\tilde z_m}{d\tilde t}$$

melting bc

Stefan number $\quad Ste = \dfrac{c_p\;\Delta T}{L}$





Thanks!

Acknowledgments:

Martin Vancopenolle Paris

Olivier Lecomte Belgium

More infos at:

www.ecalzavarini.info/research/projects/melt-ponds