Small buoyant particles in turbulence

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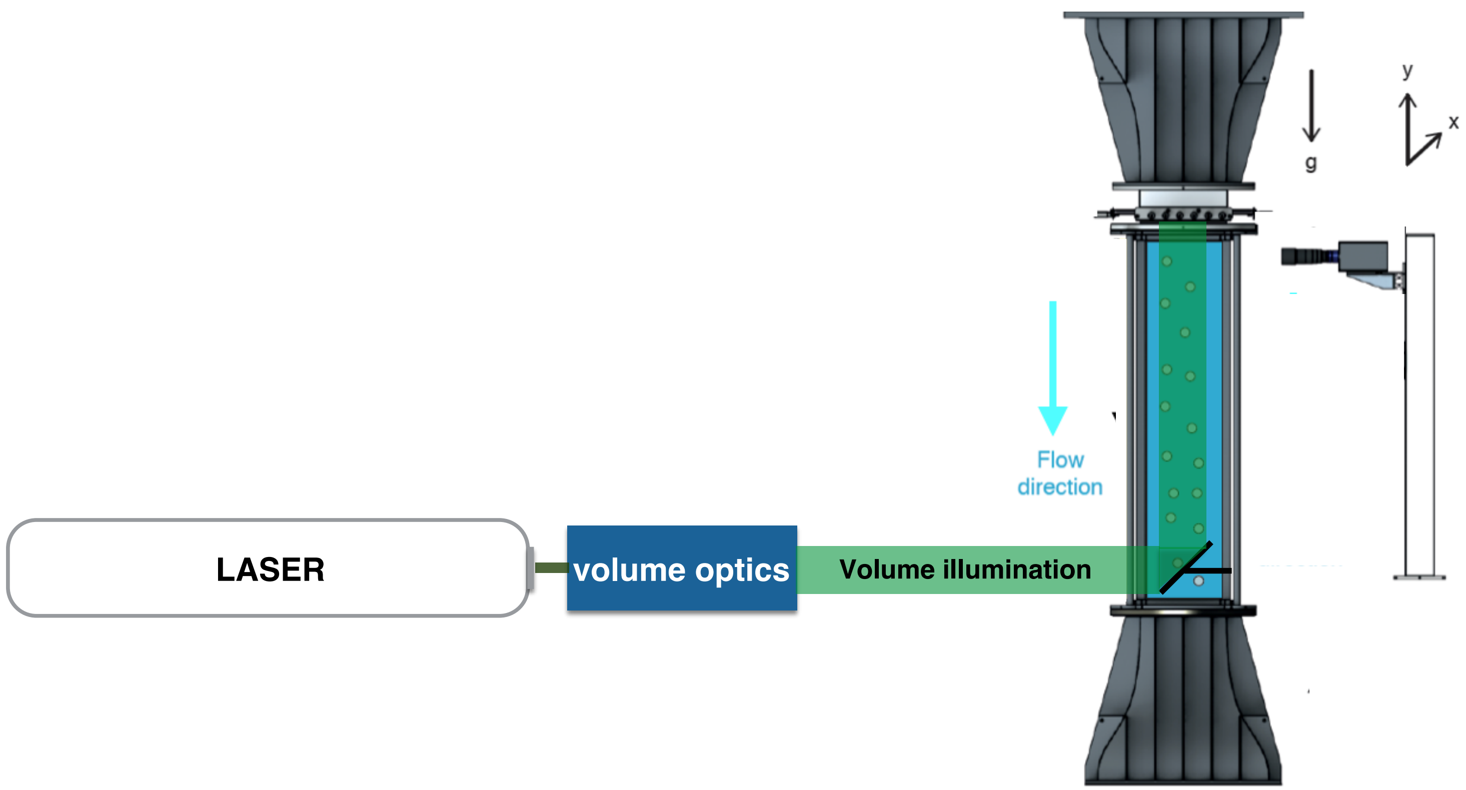
Motivation : experiments (1)

Micro-bubbles in Turbulence @ Twente Water Channel

Experimental Setup



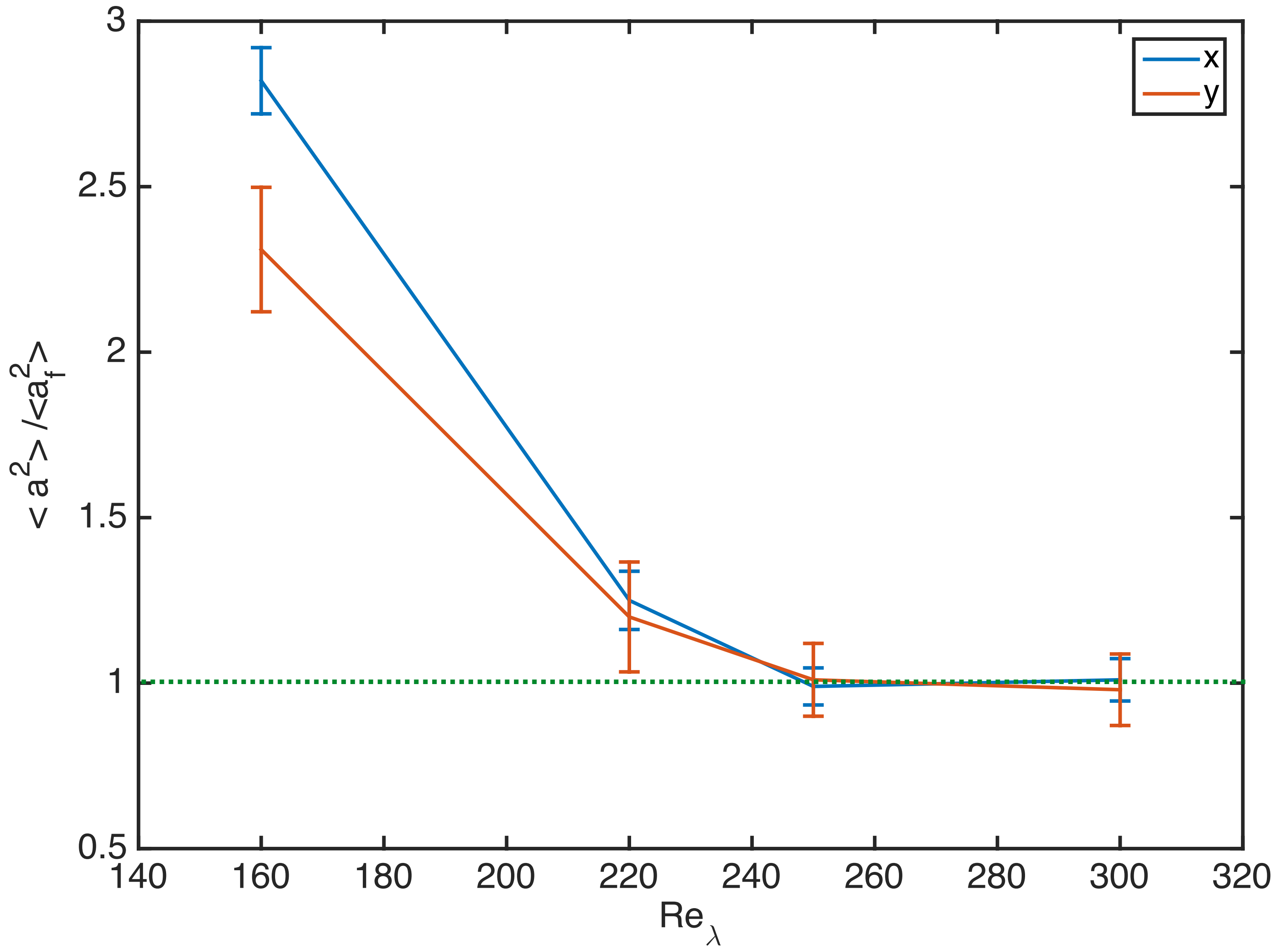
Bubble size $\Phi \sim 150 +/- 25\ \mu m$

Taylor Reynolds number Re_λ up to 300

Credits: Varghese Mathai, Chao Sun , Detlef Lohse

Motivation : experiments (2)

Bubble acceleration variance



$\Phi \simeq 150\ \mu m$

$\tau_p \simeq 0.625$ ms

$\eta \simeq 200\ \mu m$

$\tau_\eta \simeq 40$ ms

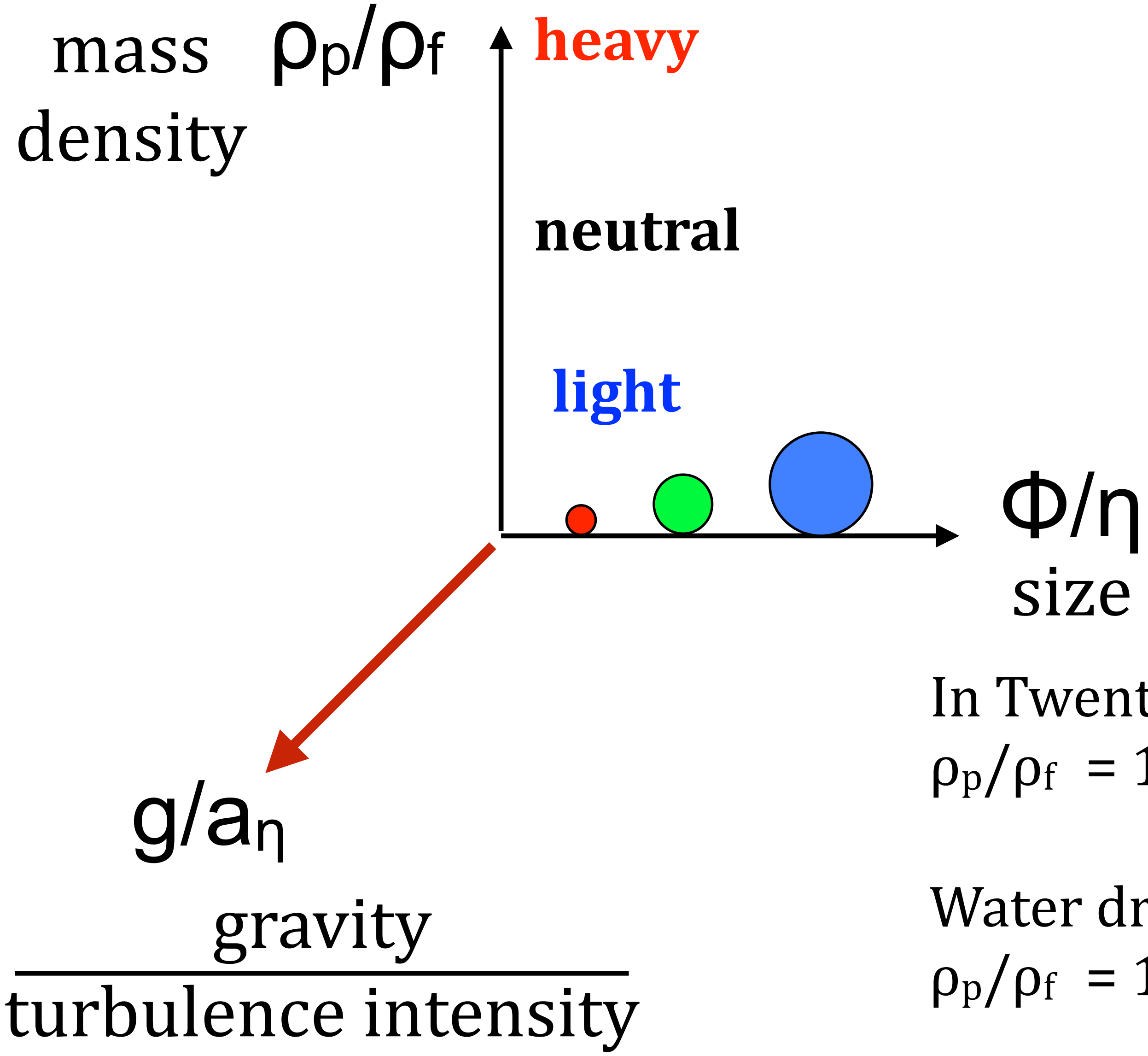
St ~ 0.015

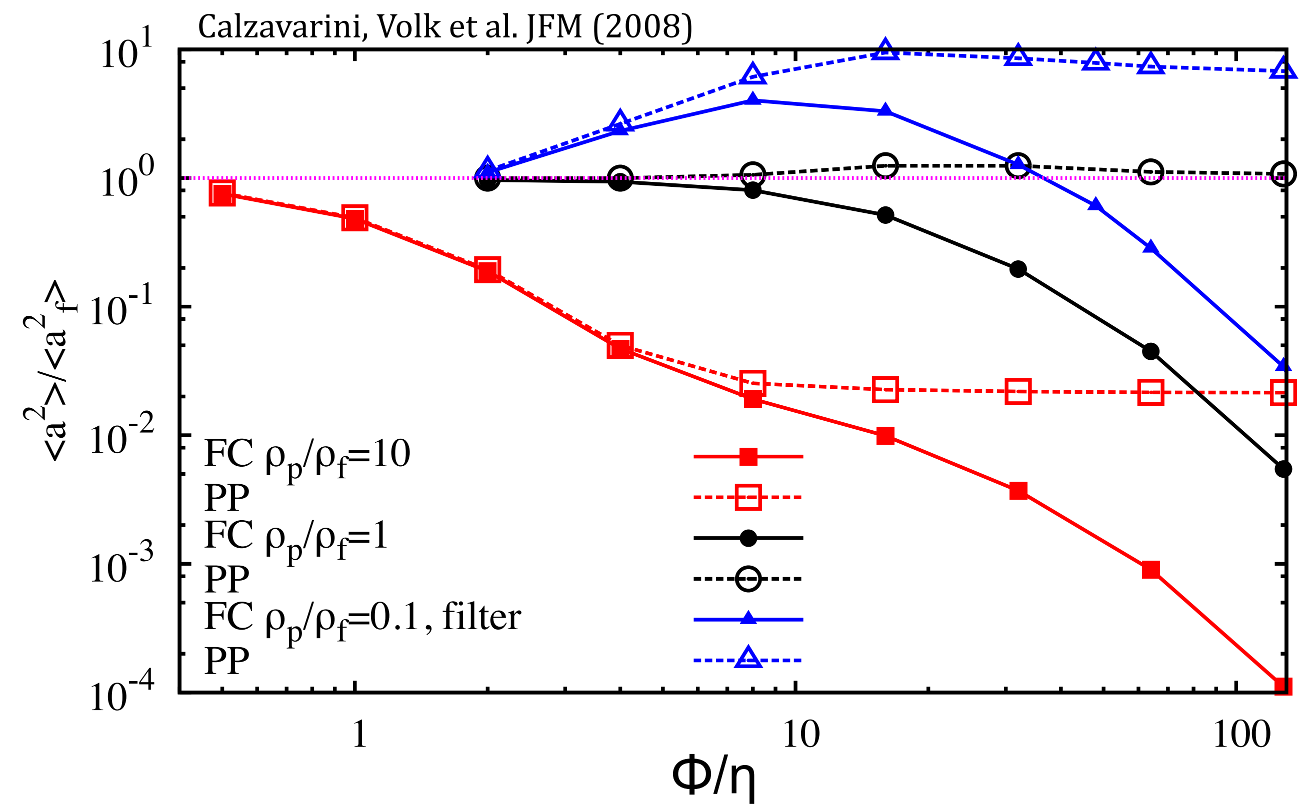
At moderate Re_λ values:

- Increased variance compared to tracers
- Horizontal component larger

Acceleration statistics of drops and bubbles in turbulence

Parameter Space





In Twente bubble experiments :
$\rho_p/\rho_f = 10^{-3}$, $\Phi/\eta < 1$, $g/a_\eta \simeq 78$

Water droplets in cumulus clouds :
$\rho_p/\rho_f = 10^{3}$, $\Phi/\eta < 1$, $g/a_\eta \simeq 20$

Droplets case, see:
Peter J. Ireland, Andrew D. Bragg, Lance R. Collins arXiv:1507.07022 (2015)

Model equation of motion

$\mathbf{U}(\mathbf{X}(\mathcal{T}),\mathcal{T})$ fluid flow field

$$\ddot{\mathbf{X}} = \frac{3\rho_f}{\rho_f + 2\rho_p}\left(\frac{D\mathbf{U}}{D\mathcal{T}} + \frac{3\nu}{a^2}(\mathbf{U} - \dot{\mathbf{X}}) + g\,\hat{\mathbf{e}}_z\right) - g\,\hat{\mathbf{e}}_z$$

added mass

Stokes Drag

buoyancy

We neglect: Lift , History, Faxen (finite-size), two-way coupling (wake)

Dimensionless model

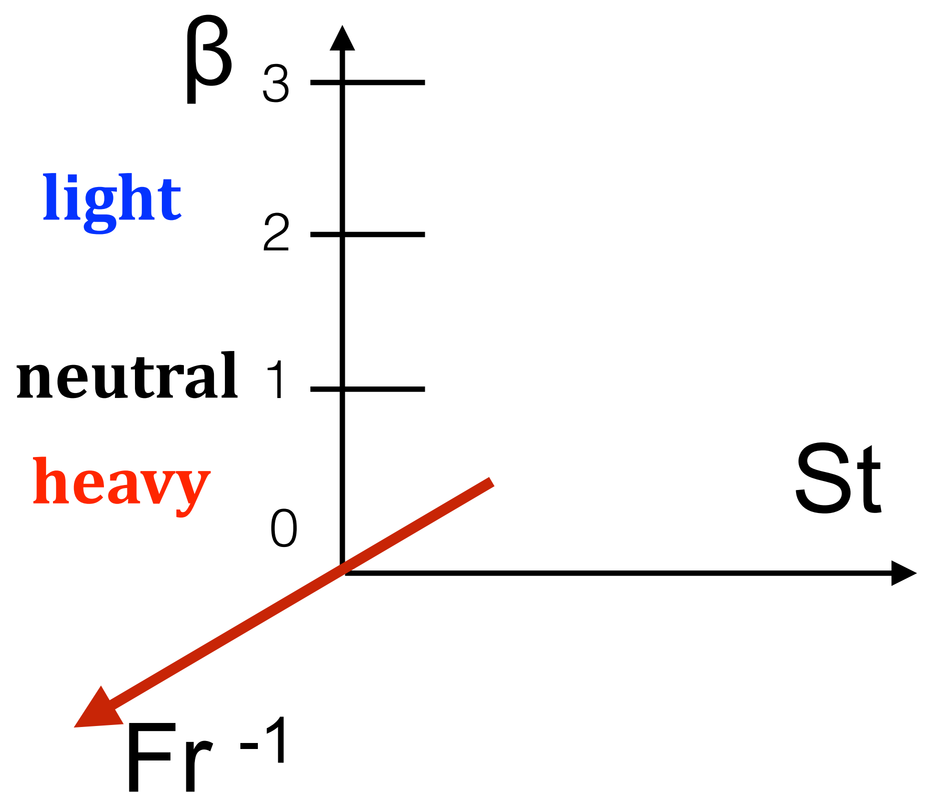
turbulence dissipative scales : η length τ_η time

$$\ddot{\mathbf{x}} = \beta \frac{D\mathbf{u}}{Dt} + \frac{1}{St}(\mathbf{u} - \dot{\mathbf{x}}) + \frac{1}{Fr}\hat{\mathbf{e}}_z$$

modified density ratio $\beta \equiv \frac{3\rho_f}{\rho_f + 2\rho_p}$

Stokes number $St \equiv \frac{a^2}{3\beta\nu\tau_\eta} = \frac{\tau_p}{\tau_\eta}$

Froude number $Fr^{-1} \equiv \frac{(\beta - 1)\,g}{a_\eta}$

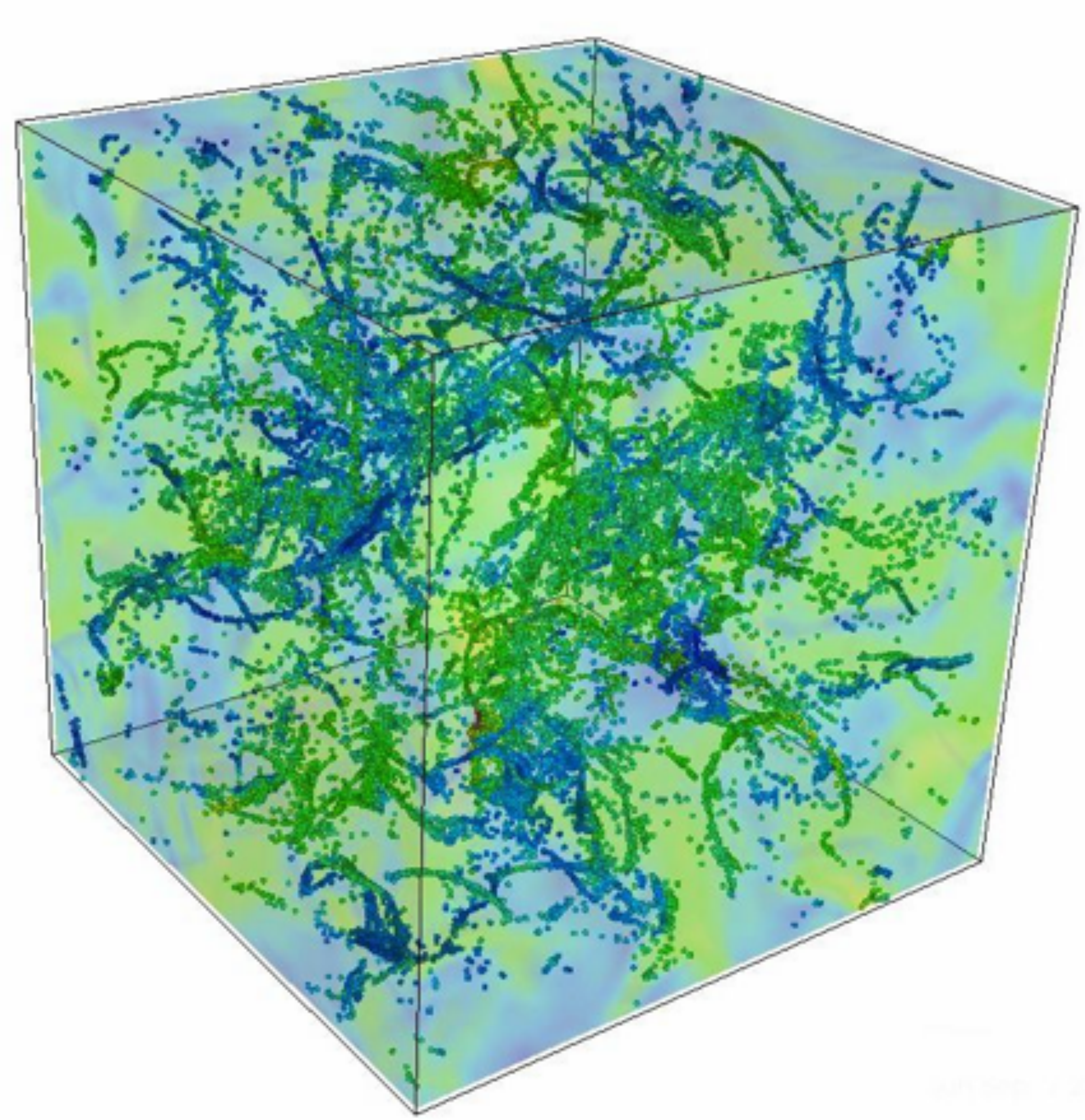


Numerics : Eulerian-Lagrangian DNS

Eulerian

Lattice Boltzmann solver

- periodic cube (256^3)
- constant power, large scale, stochastic forcing
- $Re_\lambda = 80$ ($\eta \sim 1.5\ \Delta x$)



Lagrangian

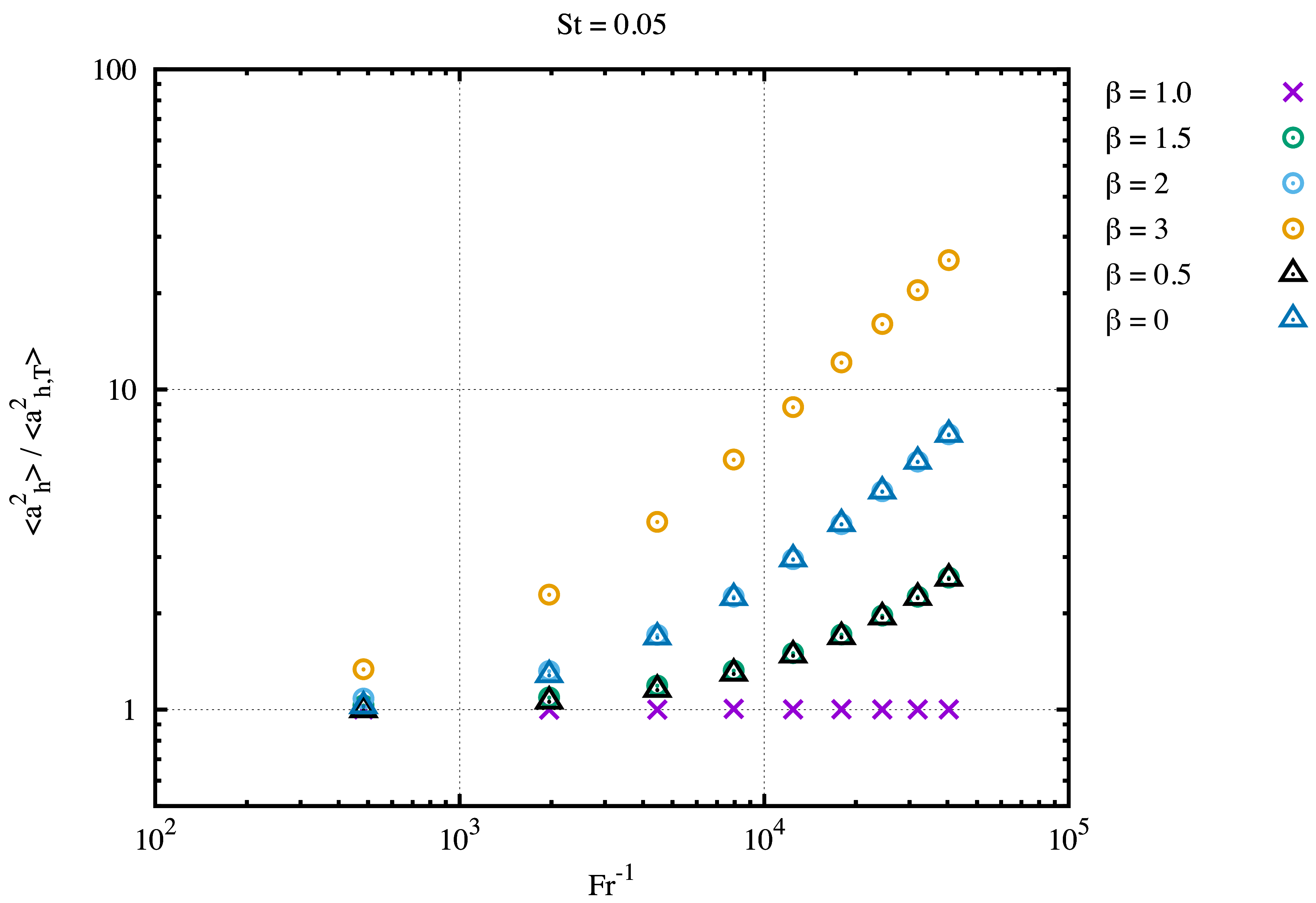
- Trilinear interpolation
- 2^{nd} order in time
- $>5\ 10^6$ particles , ~500 families

$\beta = 0,\ldots,3$

$St = 0.05,\ldots,1$

$|Fr| = 0.02,\ldots,40$

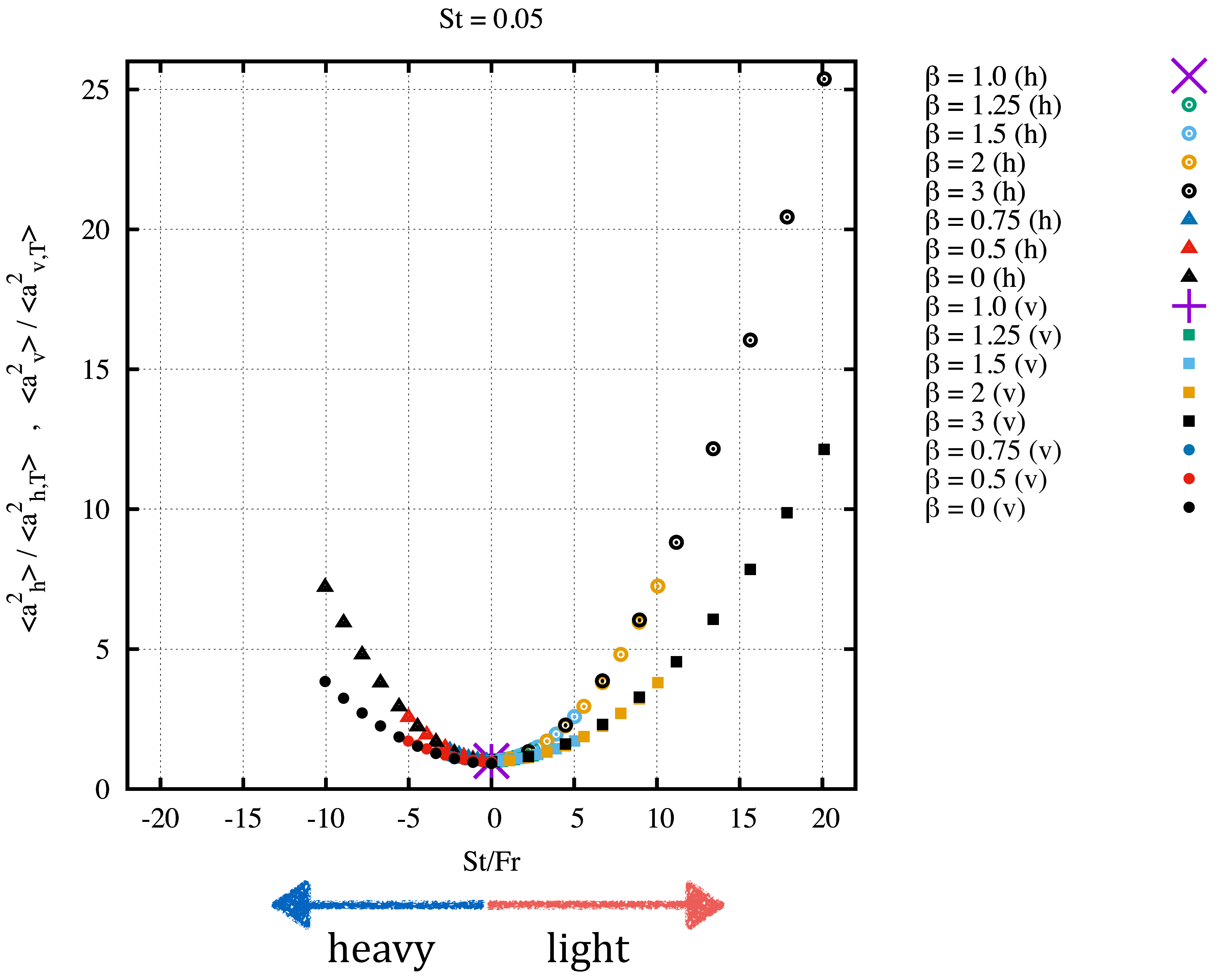
Acceleration variance from DNS



β = 0 already studied in:

- Bec, Homann, Ray , PRL 184501 (2014)
- Parishani, Ayala, Rosa, Wang, Grabowski Phys Fluids 27, 033304 (2015)
- Ireland, Bragg, Collins arXiv:1507.07022 (2015)

Acceleration variance from DNS



Instantaneous Particle Acceleration

For very small particles (small Stokes)

$$Fr^{-1} \to 0 \qquad \dot{\mathbf{x}} \simeq \mathbf{u} \longrightarrow \ddot{\mathbf{x}} \simeq D_t\mathbf{u}$$

$$Fr^{-1} \text{ finite} \qquad \dot{\mathbf{x}} \simeq \mathbf{u} + \frac{St}{Fr}\hat{\mathbf{e}}_z$$

$$\frac{St}{Fr} = \frac{(\beta-1)}{\beta}\frac{a^2 g}{3\nu u_\eta} = \frac{u_T}{u_\eta}$$

Acceleration

$$\ddot{\mathbf{x}} \simeq \frac{d}{dt}(\mathbf{u} + \frac{St}{Fr}\hat{\mathbf{e}}_z) = \frac{\partial \mathbf{u}}{\partial t} + \dot{\mathbf{x}} \cdot \nabla(\mathbf{u} + \frac{St}{Fr}\hat{\mathbf{e}}_z) = D_t\mathbf{u} + \frac{St}{Fr}\partial_z\mathbf{u}$$

Evaluation of Acceleration variance

horizontal

$$\langle \ddot{x}^2 \rangle \simeq \langle (D_t u_x)^2 \rangle + \left(\frac{St}{Fr} \right)^2 \langle (\partial_z u_x)^2 \rangle$$

vertical

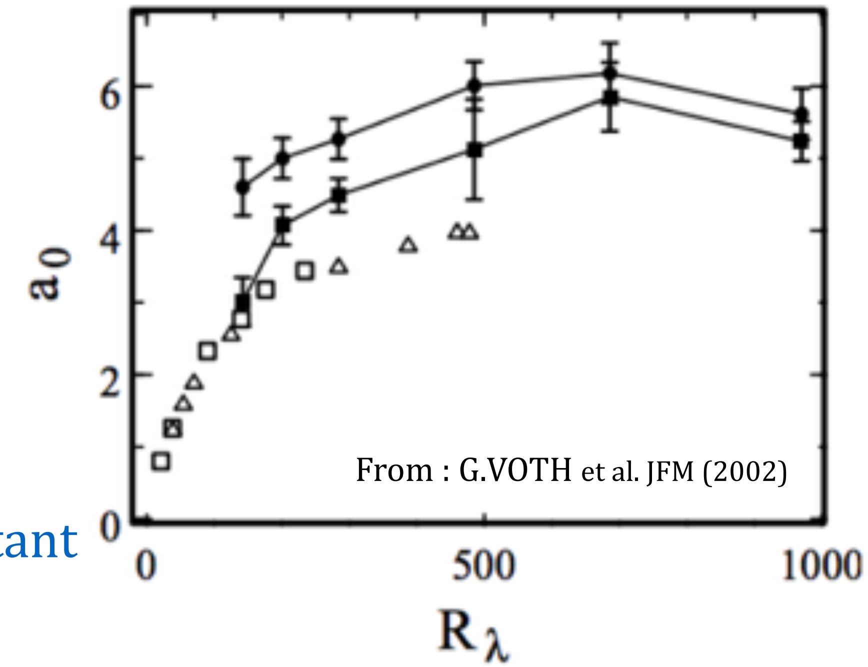
$$\langle \ddot{z}^2 \rangle \simeq \langle (D_t u_z)^2 \rangle + \left(\frac{St}{Fr} \right)^2 \langle (\partial_z u_z)^2 \rangle$$

In homogeneous & isotropic turbulence

$$\langle (\partial_Z U_X)^2 \rangle \simeq \frac{2}{15} \frac{\epsilon}{\nu} \qquad \langle (\partial_Z U_Z)^2 \rangle \simeq \frac{1}{15} \frac{\epsilon}{\nu}$$

$$\langle (D_T U_X)^2 \rangle = \langle (D_T U_Y)^2 \rangle = \langle (D_T U_Z)^2 \rangle = a_0 \left(\frac{\eta}{\tau_\eta^2} \right)^2$$

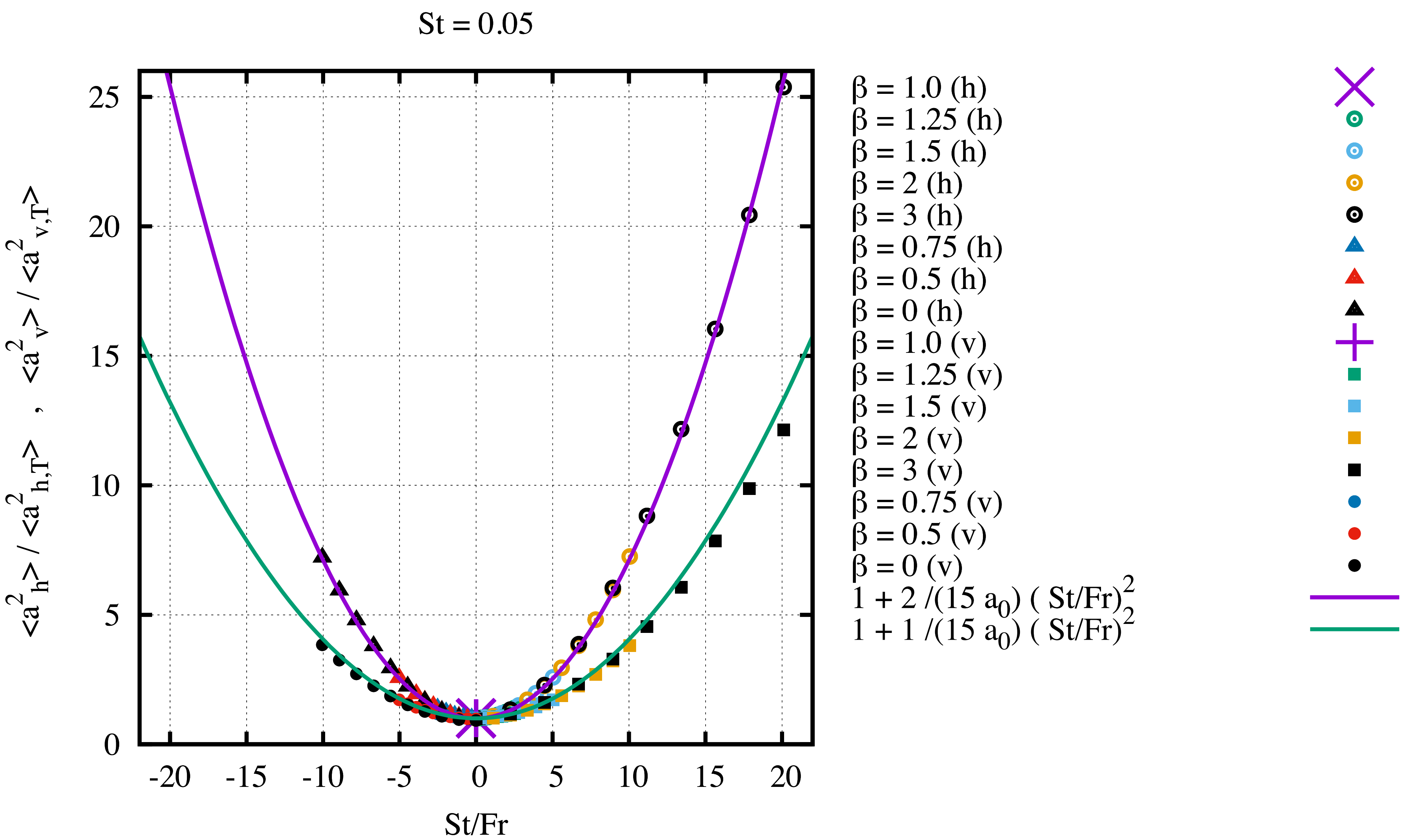
Heisenberg-Yaglom constant



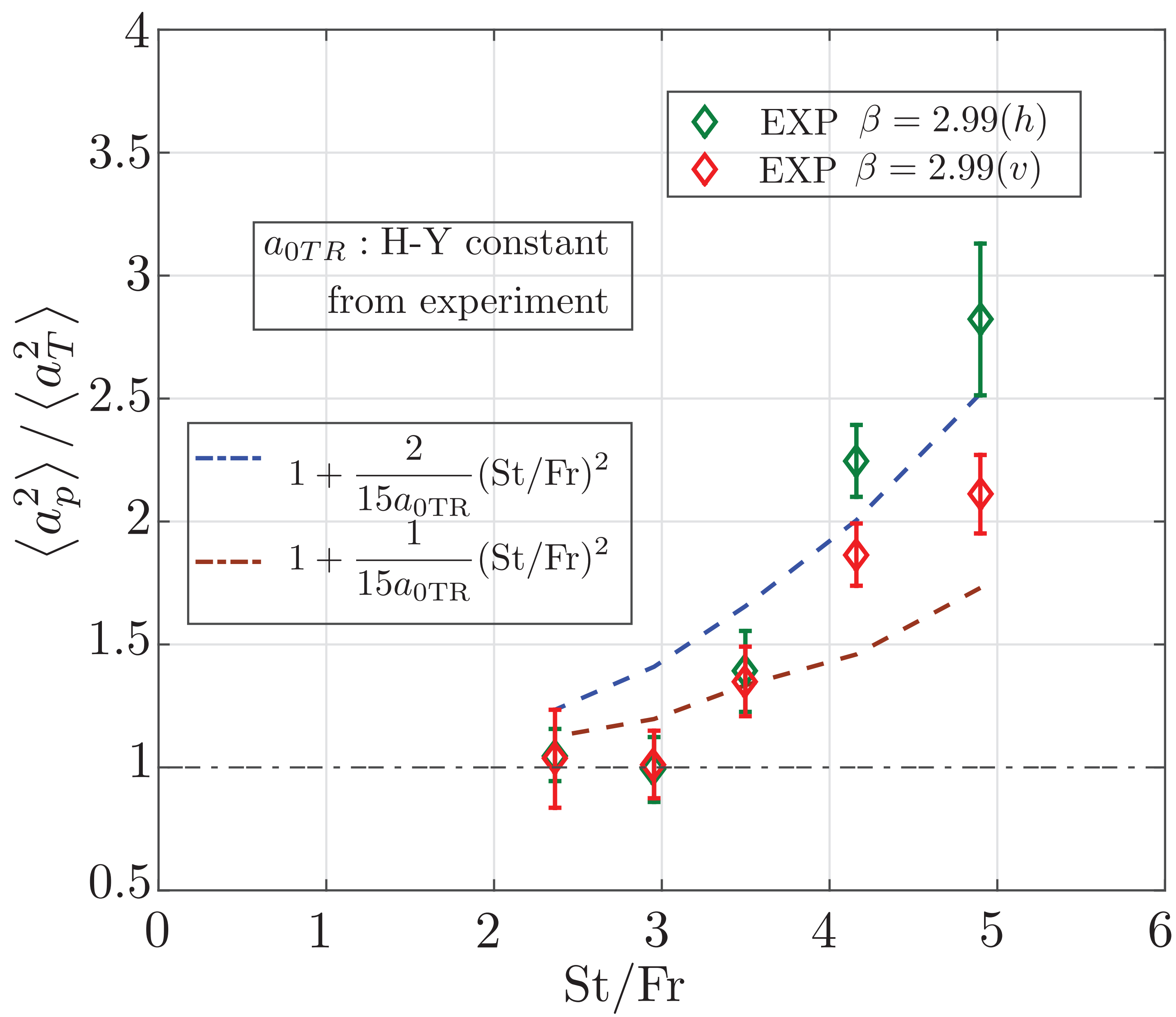
$$\frac{\langle \ddot{x}^2 \rangle}{\langle (D_t u_x)^2 \rangle} \simeq 1 + \frac{2}{15 a_0} \left(\frac{St}{Fr} \right)^2$$

$$\frac{\langle \ddot{z}^2 \rangle}{\langle (D_t u_x)^2 \rangle} \simeq 1 + \frac{1}{15 a_0} \left(\frac{St}{Fr} \right)^2$$

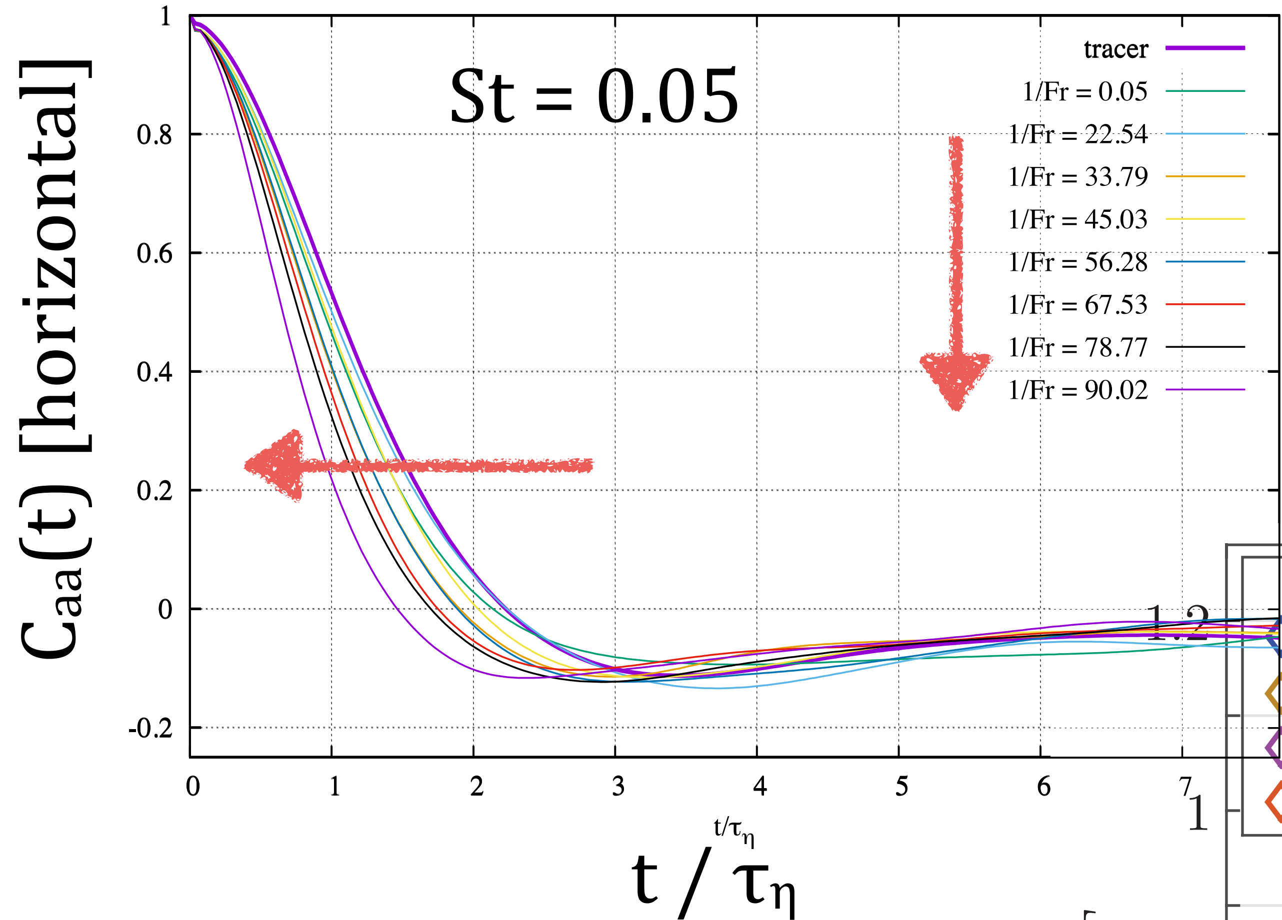
Acceleration variance comparison



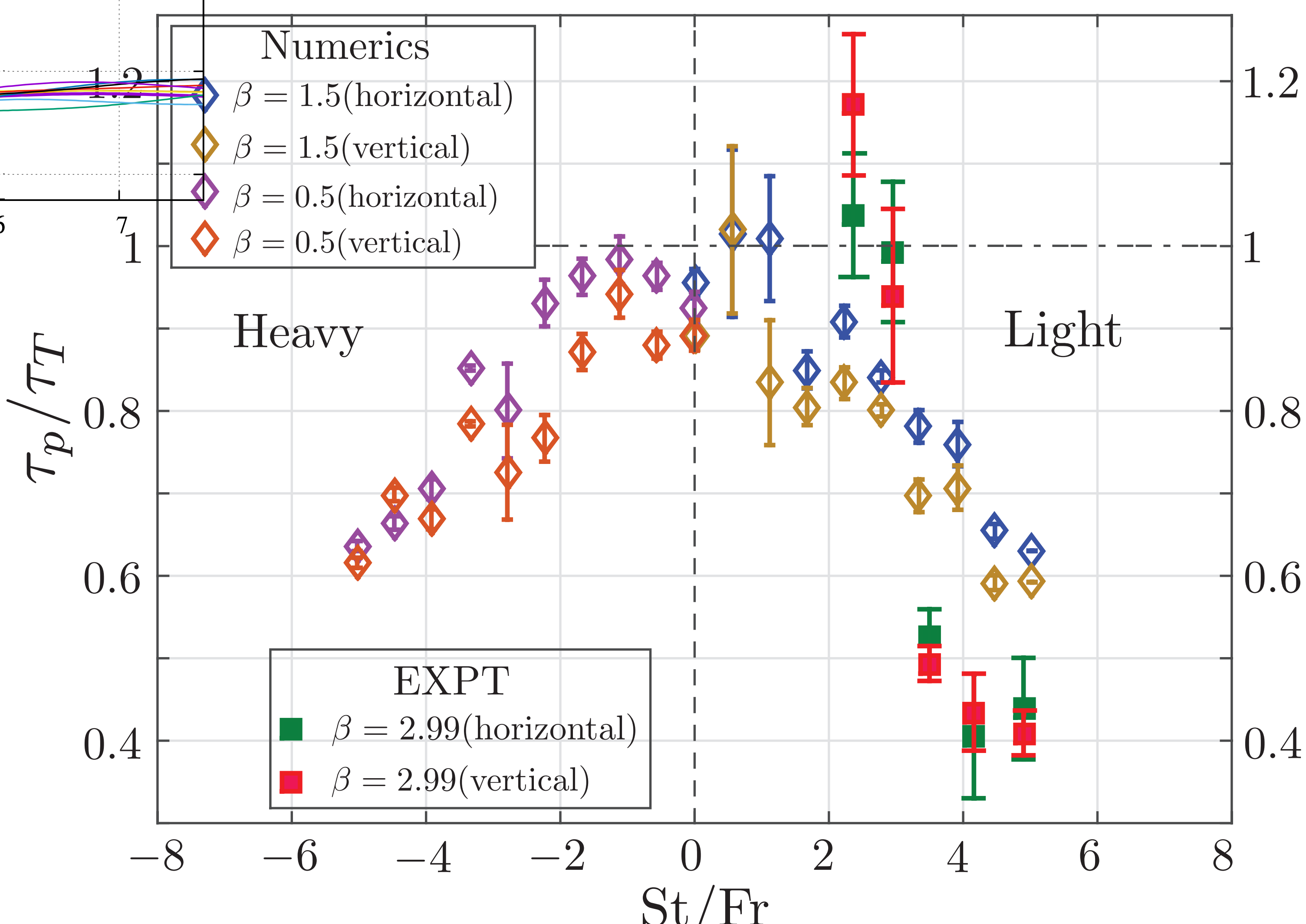
Acceleration variance: comparison with experiments



Acceleration Time Correlation

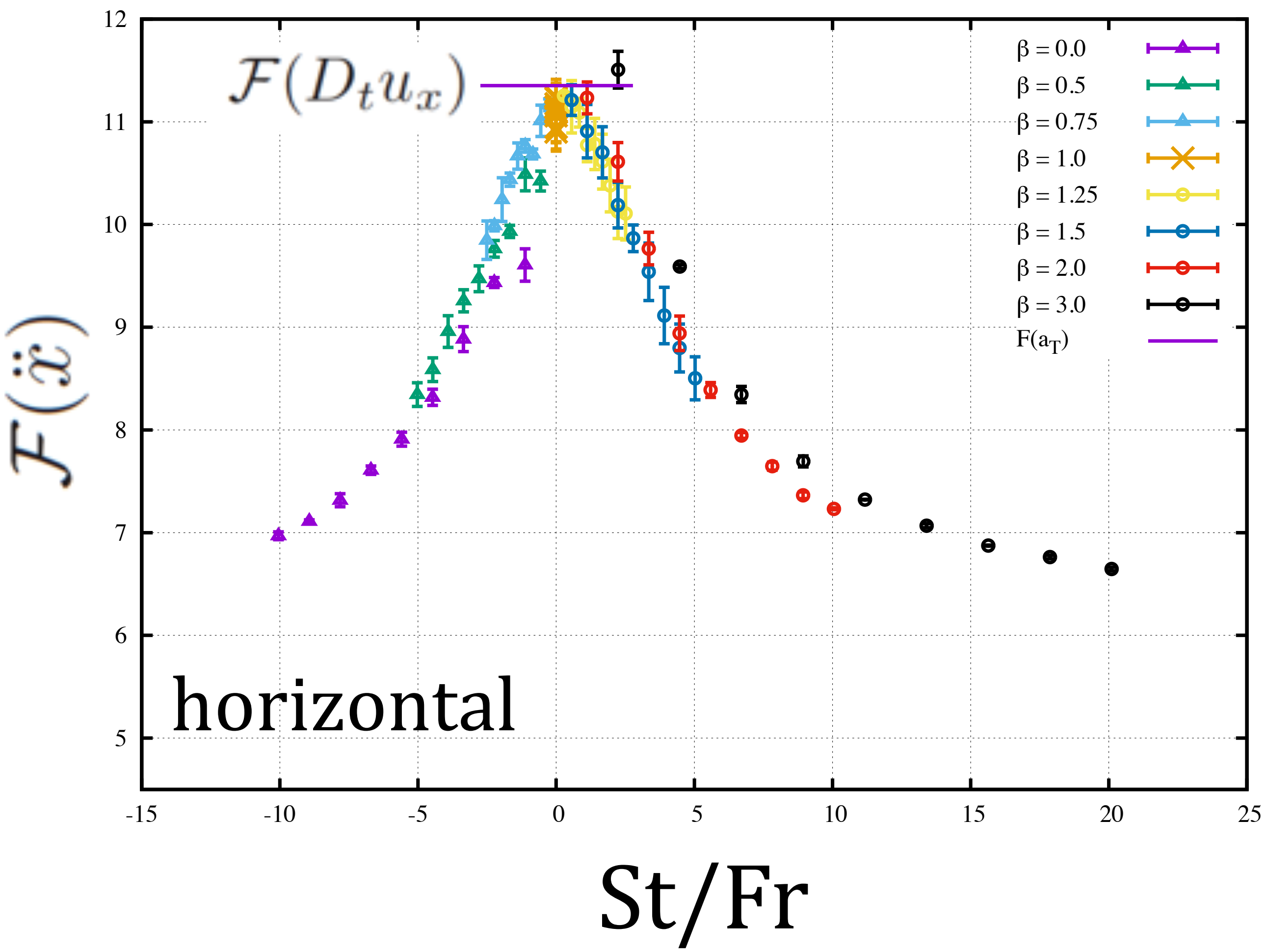


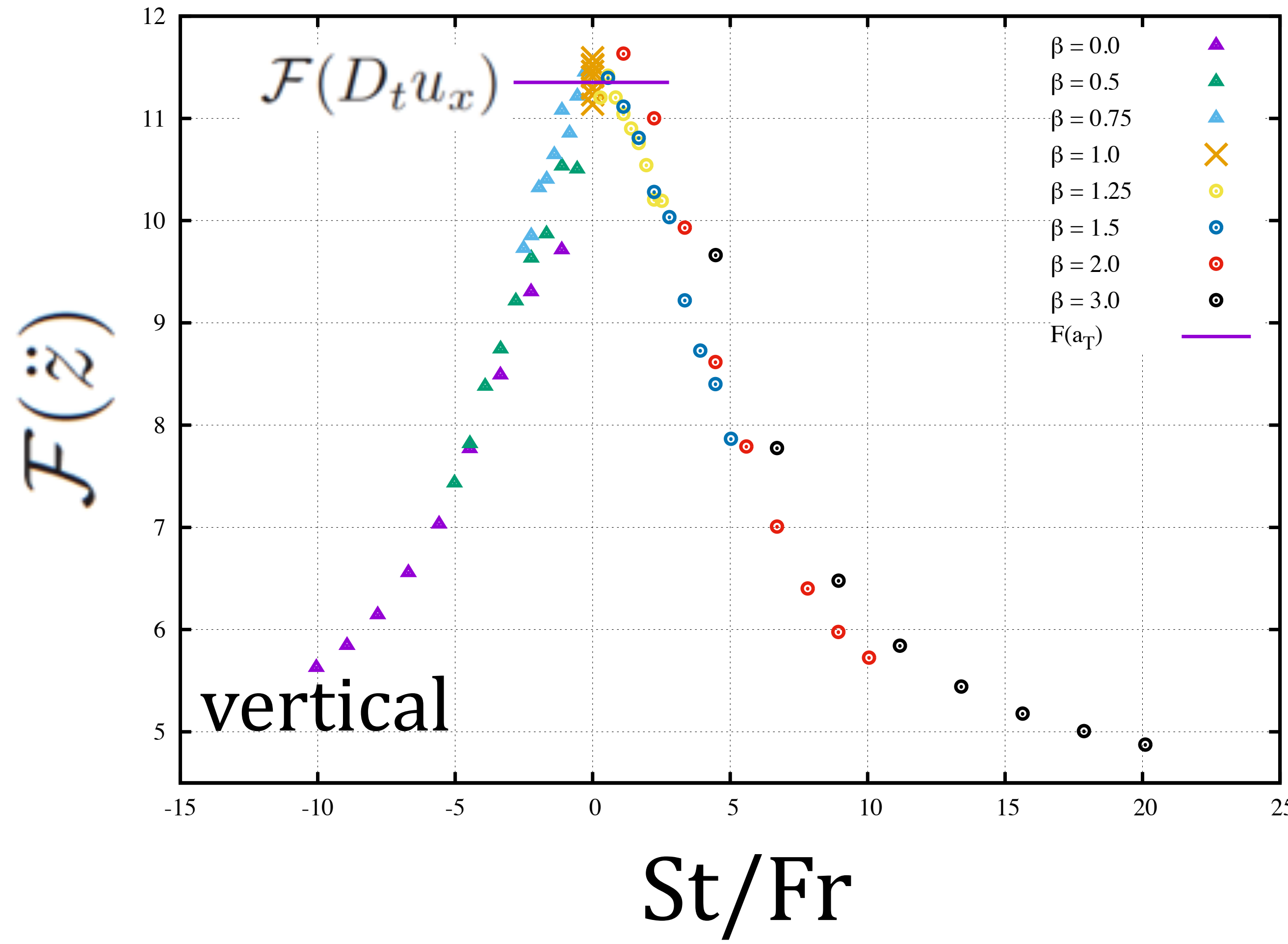
Gravity decreases the correlation time



Time correlation (0.5 crossing time)

Higher moments: Acceleration Flatness

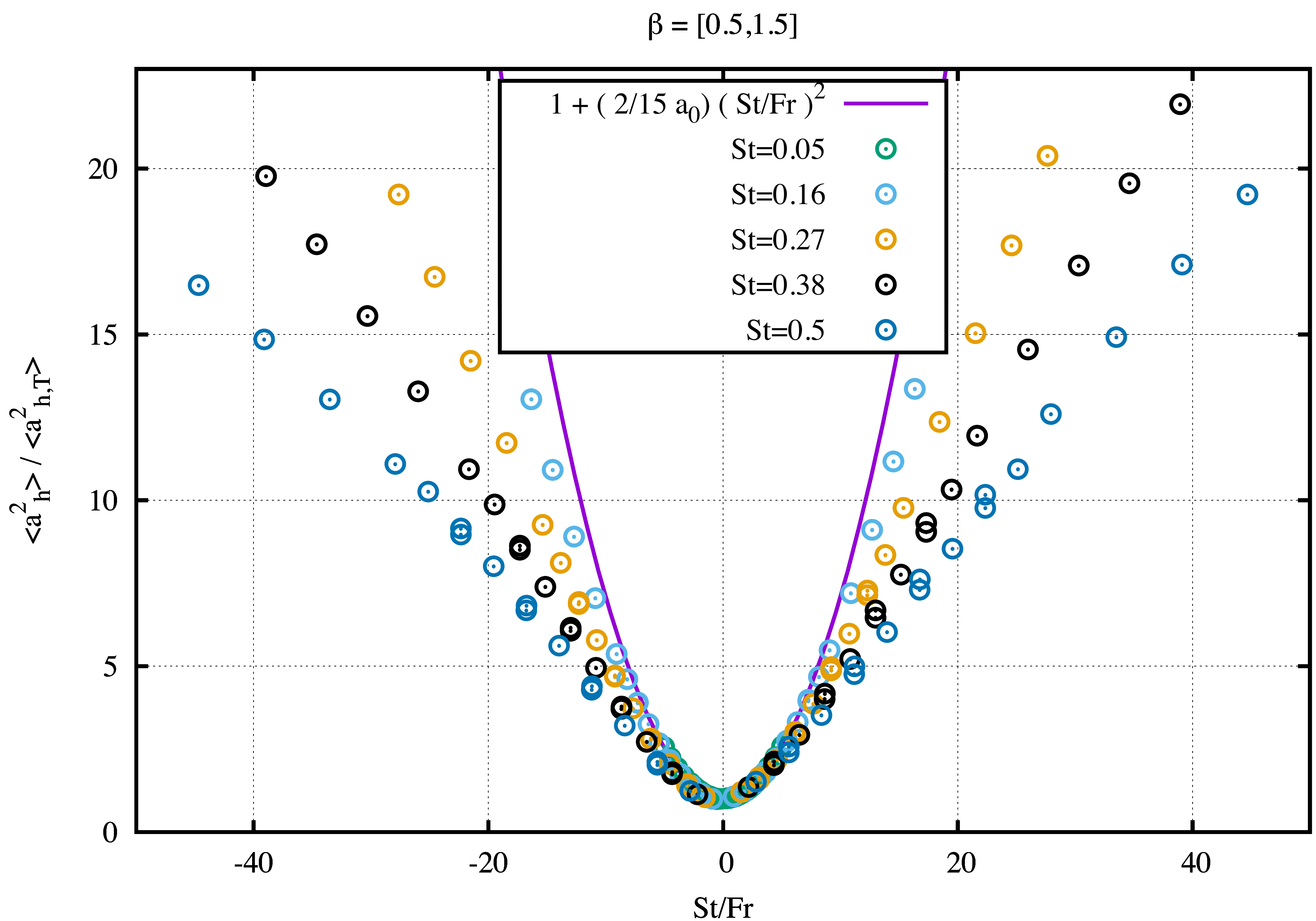




For small |St/Fr| ⟶ $$\frac{\mathcal{F}(\ddot{x})}{\mathcal{F}(D_t u_x)} \simeq 1 + 2\left(\frac{3}{\mathcal{F}(D_t u_x)} - 1\right)\frac{2}{15\, a_0}\left(\frac{St}{Fr}\right)^2$$

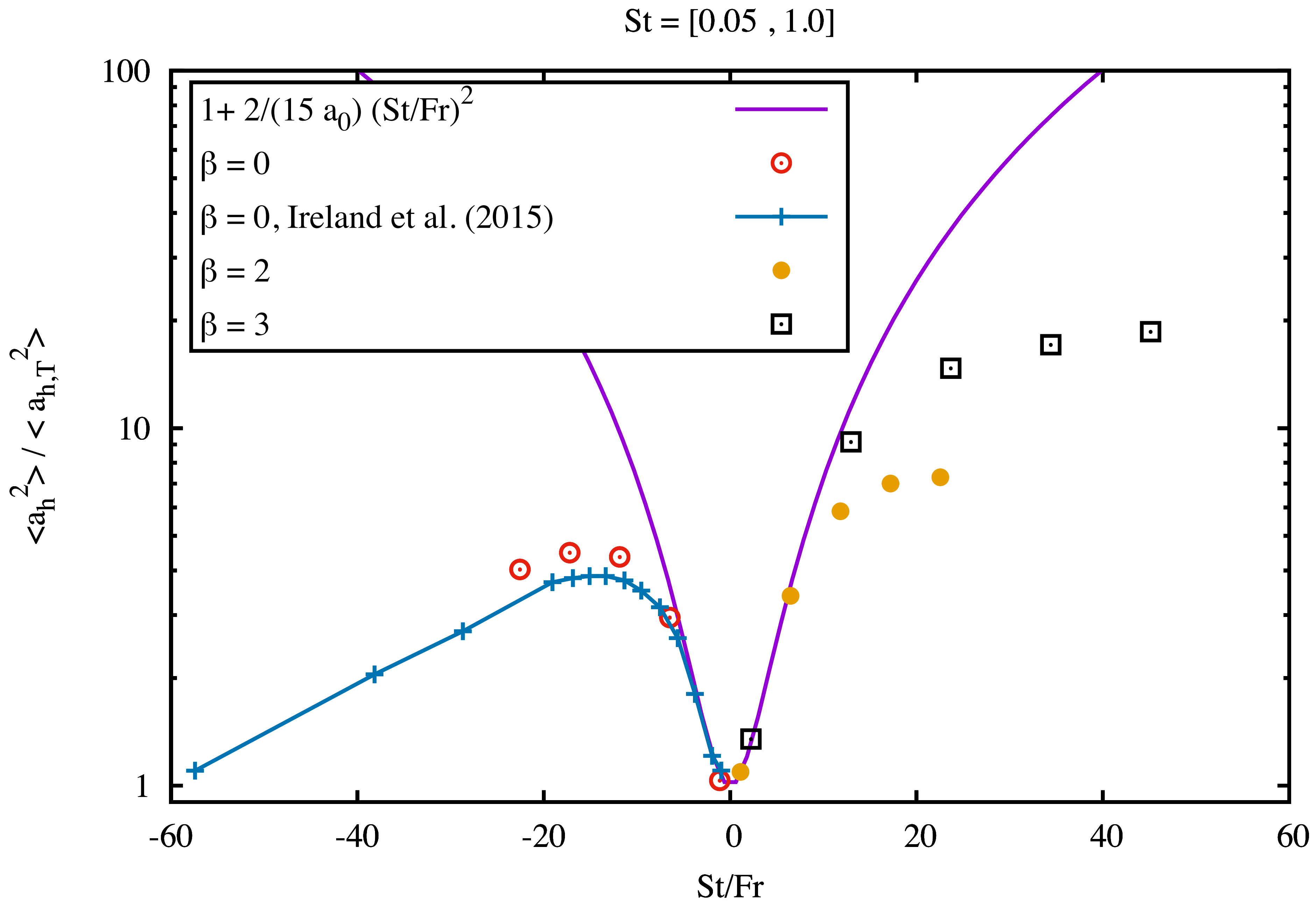
Asymptotically goes to ⟶ $$\mathcal{F}(\partial_z u_x) < \mathcal{F}(D_t u_x)$$
$$\mathcal{F}(\partial_z u_z) < \mathcal{F}(D_t u_x)$$

Larger Stokes (fixed gravity and Re_λ)



Inertial Filtering reduces
the acceleration enhancement effect

Larger Stokes (fixed gravity and Re_λ)



Inertial Filtering reduces
the acceleration enhancement effect

Conclusions & Perspectives

A tiny bubble (or a droplet) is not a good fluid acceleration proxy

Finite Froude numbers,
Small Stokes limit:

- Increase of acceleration variance
- Decrease of the correlation time
- Decrease of acceleration flatness

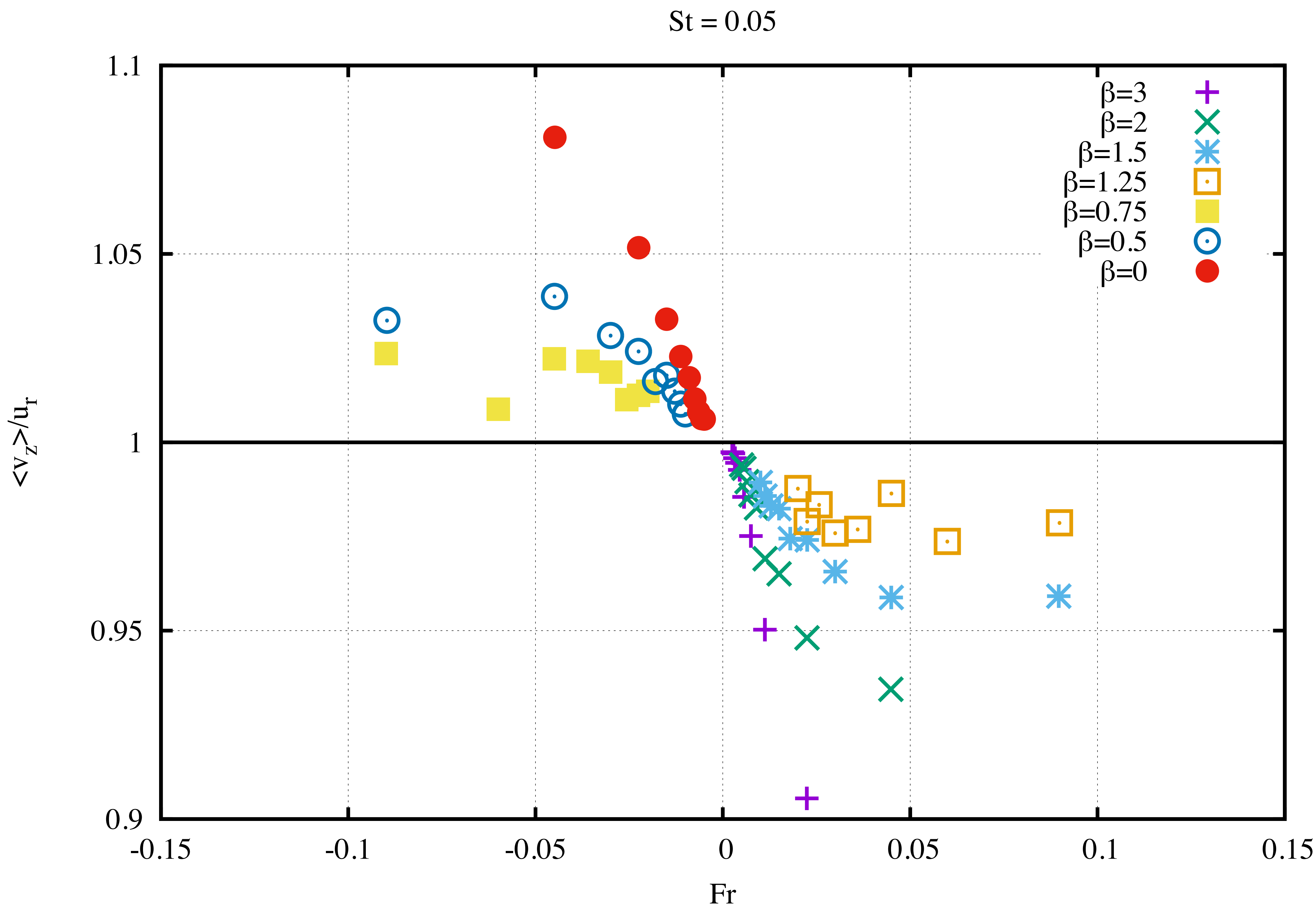
→ confirmed by bubble experiments

To be explored:

- Intermediate and large *St* limit (clustering)
- Interplay with finite-size, lift and wake effects

Thanks!

Mean vertical velocity *vs.* Fr



What about Lift force?

$$\ddot{\mathbf{x}} = \beta \frac{D\mathbf{u}}{Dt} + \frac{1}{St}(\mathbf{u} - \dot{\mathbf{x}}) + \frac{1}{Fr}\hat{\mathbf{e}}_z + \frac{\beta}{3}(\mathbf{u} - \dot{\mathbf{x}}) \times \boldsymbol{\omega}$$

important for large bubbles