JDD Lille 2014 edspi

Lattice Boltzmann Method for fluid flows

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Pôle de Recherche et d'Enseignement Supérieur

Mercredi 24 Juin 2014 – Université de Lille 1

INNOCOLD project

INNOCOLD
Institut Technologique du Froid
www.innocold.org

Using LBM method

1 PhD project : Simulation of the dispersion of leaked methane clouds over 1 sq. km

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2 PhD projects : Modeling and Simulation of the source term and phase transition from liquid to gas

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INNOCOLD project : Objective

Subject 3 : ***Numerical Simulation of methane gas dispersion in atmosphere using Lattice Boltzmann Method, at LNG terminal in Dunkerque***

- Risk assessment at Dunkerque site by providing LFL maps
- Reduce the investment cost of installing LNG industry
- Flexible numerical tool, suitable also for other installations and danger studies

Presentation outline

- INNOCOLD project
- Lattice Boltzmann Method (LBM)
  - Stream-collision algorithm
  - Finite Volume LBM (FVLBM) algorithm
- Comparison of LBM algorithms
  - Comparative study of the accuracy of different LBM algorithms
  - Validation of FVLBM: 3D Rayleigh-Benard Convection
- Perspectives

why LBM?

- Could be easily extended to multi-component, multi-phase flows
- Easy implementation for complex geometry : widely used in automobile industry eg. POWERFLOW
- Avoids solving heavy Poisson equation. Pressure and stress tensors are available locally
- Easy parallelization

Although, it has its own limitations

- Lower order of accuracy (2^{nd} order in time and space)
- Limited to weakly compressible
- Based on equi-spaced cartesian grid

LBM

- Developed from the Lattice Gas Cellular Automaton
- LBM equation with BGK model- single relaxation time approximation

$$\frac{\partial f_\alpha}{\partial t} + \boldsymbol{c}_\alpha \cdot \boldsymbol{\nabla} f_\alpha = \frac{1}{\tau}(f_\alpha^{eq} - f_\alpha) + F_\alpha \qquad \text{where, } \alpha = 0, \ldots, N_{pop}$$

$$\rho = \Sigma_\alpha f_\alpha \qquad \rho \boldsymbol{u} = \Sigma_\alpha \boldsymbol{c}_\alpha f_\alpha$$

Velocity Discretization

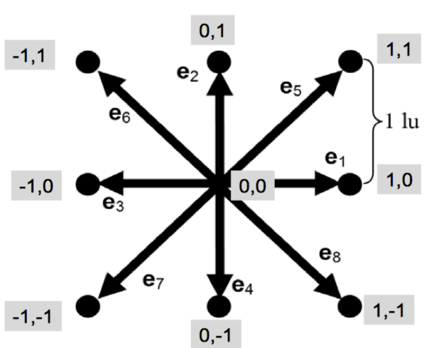


Fig: D2Q9 lattice arrangement

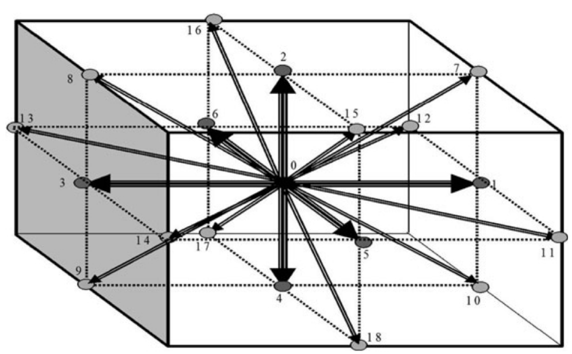


Fig: D3Q19 lattice arrangement

Space and time Discretization

- Stream-collision algorithm
- Finite Volume approach

Stream-Collision algorithm

- Applying method of characteristics to LBM-PDE along the lines $\boldsymbol{x}(t) = \boldsymbol{x}(0) + \boldsymbol{c}_\alpha \; t$

ODE :--> $$\frac{d}{dt} f_\alpha = \frac{1}{\tau}(f_\alpha^{eq} - f_\alpha) + F_\alpha$$

- Redefining the distribution functions for the lattice populations by,

$$\tilde{f}_\alpha = f_\alpha - \frac{\Delta t}{2\tau}(f_\alpha^{eq} - f_\alpha + \tau F_\alpha)$$

$$\tilde{f}_\alpha(\boldsymbol{x} + \boldsymbol{c}_\alpha \Delta t, t + \Delta t) = \tilde{f}_\alpha(\boldsymbol{x}, t) + \frac{\Delta t}{\tilde{\tau}}(\tilde{f}_\alpha^{eq}(\boldsymbol{x}, t) - \tilde{f}_\alpha(\boldsymbol{x}, t)) + \Delta t \left(1 - \frac{\Delta t}{2\tilde{\tau}}\right) F_\alpha$$

where, $\tilde{\tau} = \tau + \Delta t/2 \qquad \tilde{\tau} > \Delta t/2$

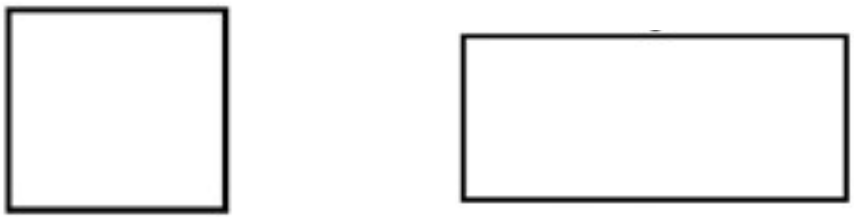
$$\rho = \Sigma_\alpha \tilde{f}_\alpha$$

$$\rho \boldsymbol{u} = \Sigma_\alpha \boldsymbol{c}_\alpha (\tilde{f}_\alpha + \tfrac{\Delta t}{2} F_\alpha)$$

$$\nu = (\tilde{\tau} - \Delta t/2) c_s^2$$

FVLBM algorithm

- ***Target : Applicable to stretched 3D cartesian grids***



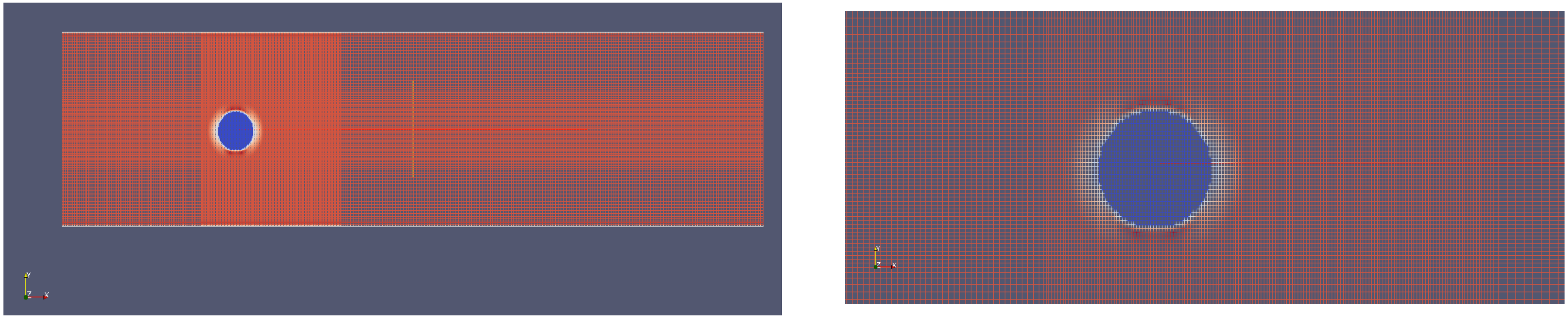


Fig: *Grid Refinement (ParaView)*

FVLBM algorithm (cell-centered)

- Integrating LBE over a volume V (with surface S)

$$\int_V \frac{\partial f_\alpha}{\partial t}\, dV + \int_S \boldsymbol{c}_\alpha \cdot \boldsymbol{n}\ f_\alpha\, dS = \int_V \frac{1}{\tau}(f_\alpha^{eq} - f_\alpha)\, dV + \int_V F_\alpha\, dV$$

$$\frac{\partial f_\alpha}{\partial t} + \frac{S_j}{V}\boldsymbol{c}_\alpha \cdot \boldsymbol{n}_j\, [f_\alpha]_j = \frac{1}{\tau}(f_\alpha^{eq} - f_\alpha) + F_\alpha$$

Representation of 2-D structured FV grid

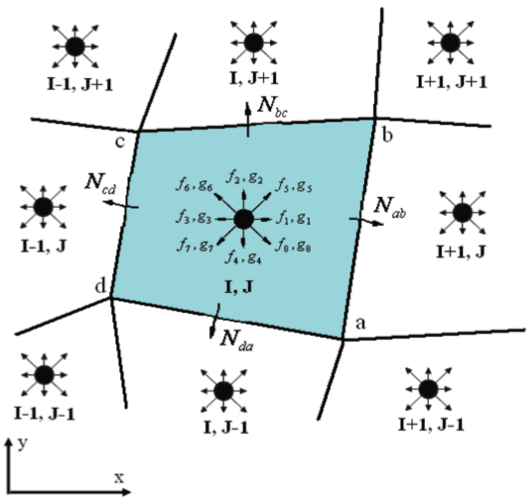


Fig: *Schematic of the FV discretization with cell-centered lattice (2-D case)*

FVLBM algorithm

Time discretization :

Applying explicit Euler scheme, $\partial f_\alpha / \partial t = (f_\alpha^{(t+\Delta t)} - f_\alpha^{(t)})/\Delta t$

$$f_\alpha^{(t+\Delta t)} = f_\alpha^{(t)} - \Delta t \frac{S_j}{V} \boldsymbol{c}_\alpha \cdot \boldsymbol{n}_j \left[f_\alpha^{(t)} \right]_j + \frac{\Delta t}{\tau} (f_\alpha^{eq\,(t)} - f_\alpha^{(t)}) + \Delta t \; F_\alpha$$

Space discretization : At-least quadratic interpolation scheme required. QUICK scheme is chosen.

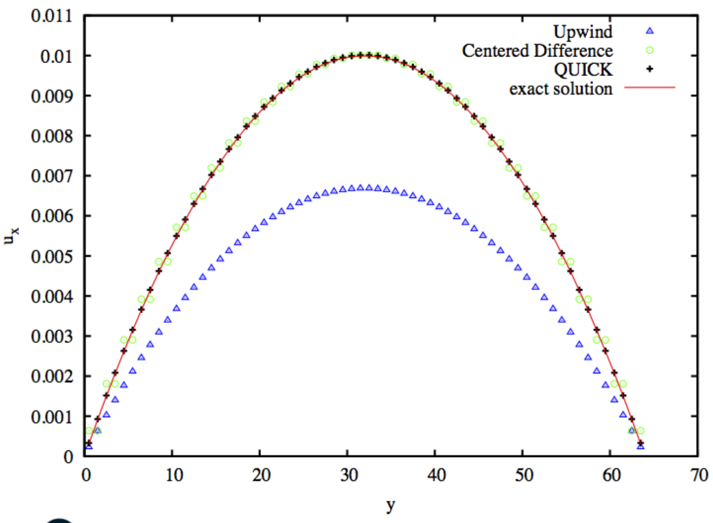
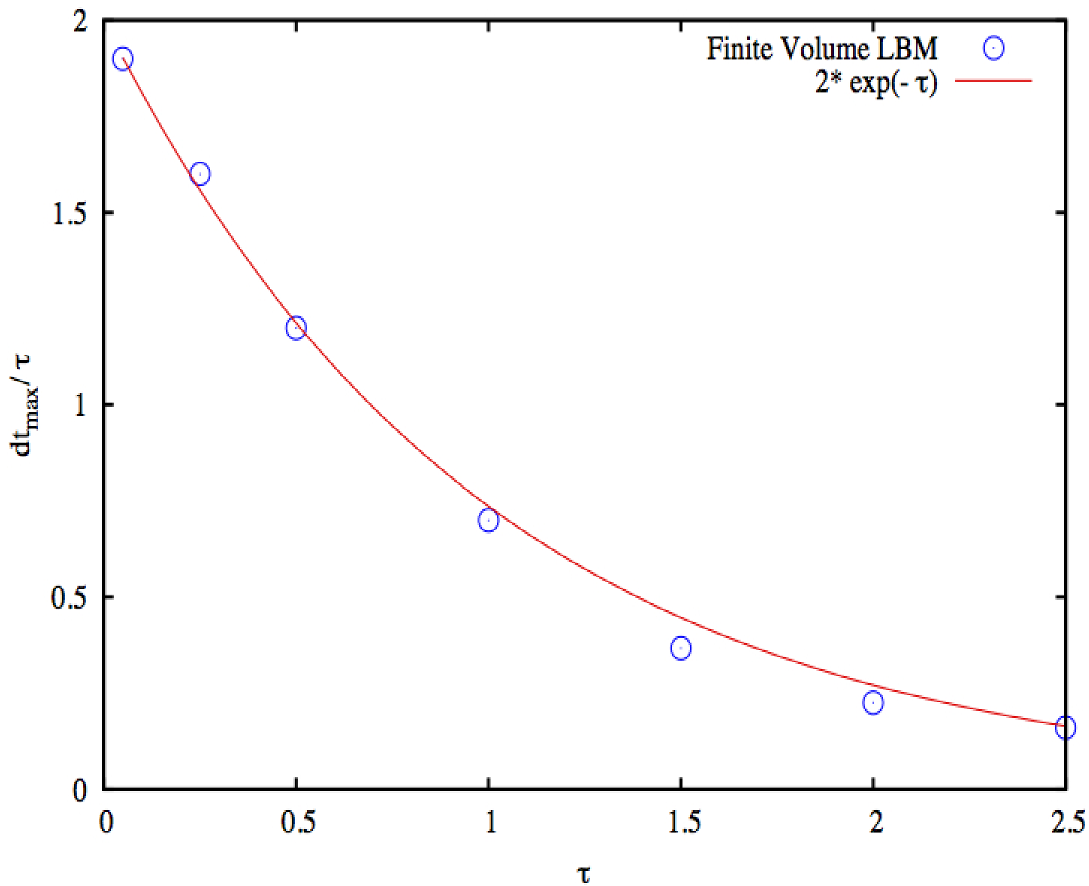


Fig: *Test of accuracy of different numerical methods for standard Poiseuille flow. Velocity profile comparison for mesh size 1X64.*

FVLBM algorithm

Major drawback :



$$2 > \frac{\Delta t}{\tau} > 0$$

Fig: *Decaying Kolmogorov f ow: (dtmax/tau vs tau) measure of maximum threshold stable time step for different values of tau for FVLBM*

FVLBM algorithm: Improvement

➢ Semi-implicit integration scheme

$$f_\alpha^{(t+\Delta t)} = f_\alpha - \Delta t \frac{S_j}{V} \boldsymbol{c}_\alpha \cdot \boldsymbol{n}_j \left[f_\alpha + \frac{\Delta t}{2\tilde{\tau}} (f_\alpha^{eq} - f_\alpha) \right]_j + \frac{\Delta t}{\tilde{\tau}} (f_\alpha^{eq} - f_\alpha) + \Delta t \; F_\alpha$$

where, $\tilde{\tau} = \tau + \frac{\Delta t}{2}$ $\quad \tilde{\tau} > \frac{\Delta t}{2}$

➢ Further improvement with Heun scheme for advection term

- Greater stability criteria
- Increased accuracy

$$f_\alpha^{(t+\Delta t)} = f_\alpha - \Delta t \frac{S_j}{V} \boldsymbol{c}_\alpha \cdot \boldsymbol{n}_j \left(\frac{\left[f_\alpha^* + \frac{\Delta t}{2\tilde{\tau}} (f_\alpha^{eq*} - f_\alpha^*) \right]_j + \left[f_\alpha + \frac{\Delta t}{2\tilde{\tau}} (f_\alpha^{eq} - f_\alpha) \right]_j}{2} \right) + \frac{\Delta t}{\tilde{\tau}} (f_\alpha^{eq} - f_\alpha) + \Delta t \; F_\alpha$$

$$\nu = \tau \; c_s^2 = \left(\tilde{\tau} - \frac{\Delta t}{2} \right) c_s^2$$

FV-LBM vs. standard LBM

stream-collision

- Well-known; highly tested
- Restricted to homogenous grid
- Streaming along characteristics performed at almost zero cost

FV-LBM algorithm

- Not well explored
- Allows non-homogenous grids
- Addition of a heavy advection term in the calculation

Comparison
efficiency, accuracy, performance, stability

Viscosity evaluation

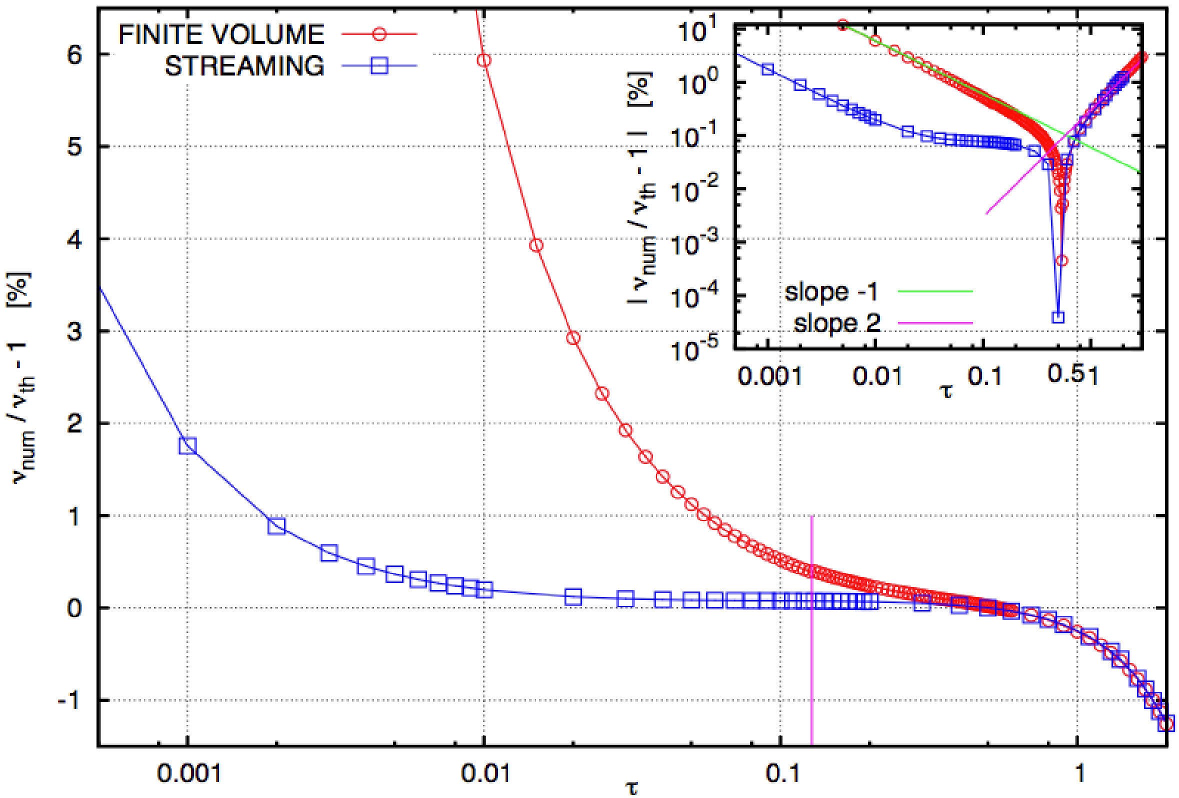


Fig: ***Decaying Kolmogorov f ow:*** *Relative error measurement. Finite Volume case: dt = 1 for tau >= 0.13 (marked with a vertical line) and dt = 0.1 for tau < 0.13. Streaming case: dt = 1 always.* ***Inset:*** *the absolute value of the same error in log-log scale.*

FVLBM is less accurate but there is an optimal region of application (tau = 0.5).

Accuracy of velocity (bounded flow)

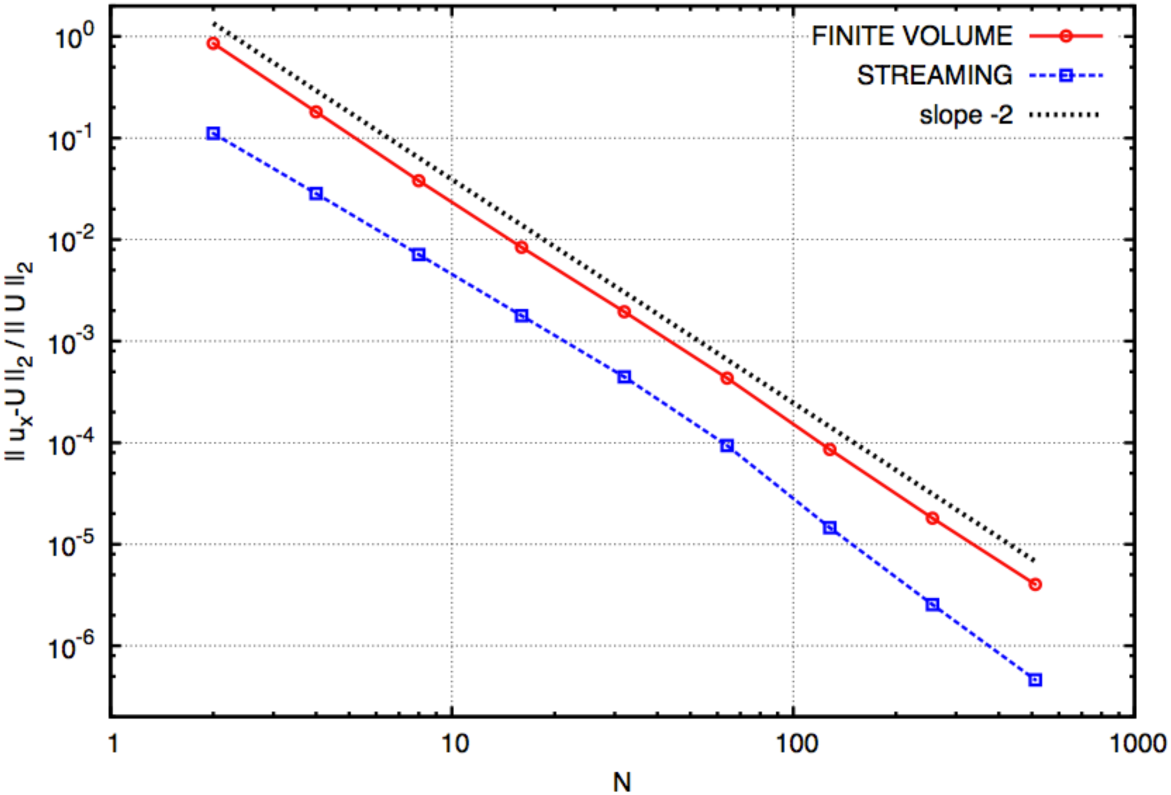


Fig: ***Poiseuille Flow:*** *Relative error on the Re = 10 velocity Poiseuille flow profile at changing the number of grid points (N) and keeping Δx = 1.*

FV is same order in space as stream -collision (i.e. second order accurate) but less accurate of a factor 8 to 10.

Comparative study of the accuracy of FV-LBM and standard LBM algorithms

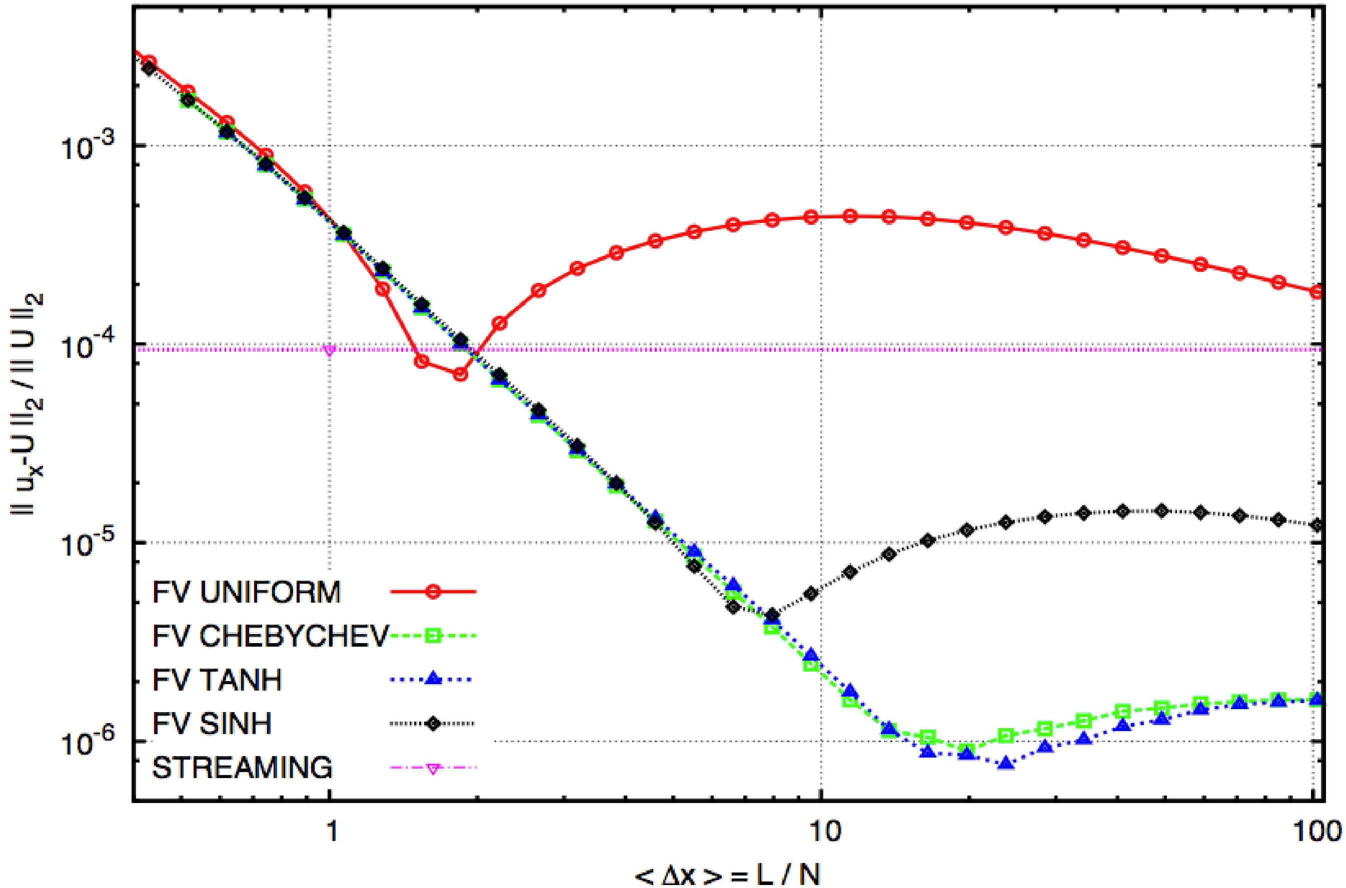
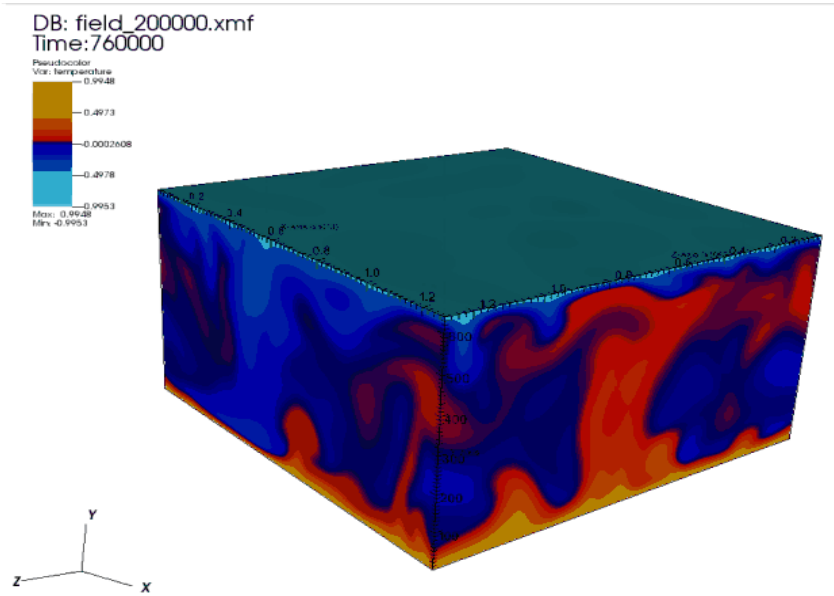


Fig: ***Poiseuille flow:*** *Relative error measurement on the Re = 10 velocity Poiseuille flow profile. Here domain size and forcing is varied keeping Re constant. Also grid points are kept constant .*
At average dt of 10, the FV can be even 100 times more accurate that stream-collision.

FVLBM algorithm: 3D flows

➢ 3D Rayleigh-Benard System



- Ra=2.5*10^6; Pr=1; Aspect ratio=2
- Grid : 128X128X64
- Hyperbolic sine grid refinement at the walls
- No-slip BC at walls
- Periodic BC in x,y directions
- Constant upper and lower wall temperatures to be -1 and 1 respectively

3D Validation: Rayleigh-Benard convection

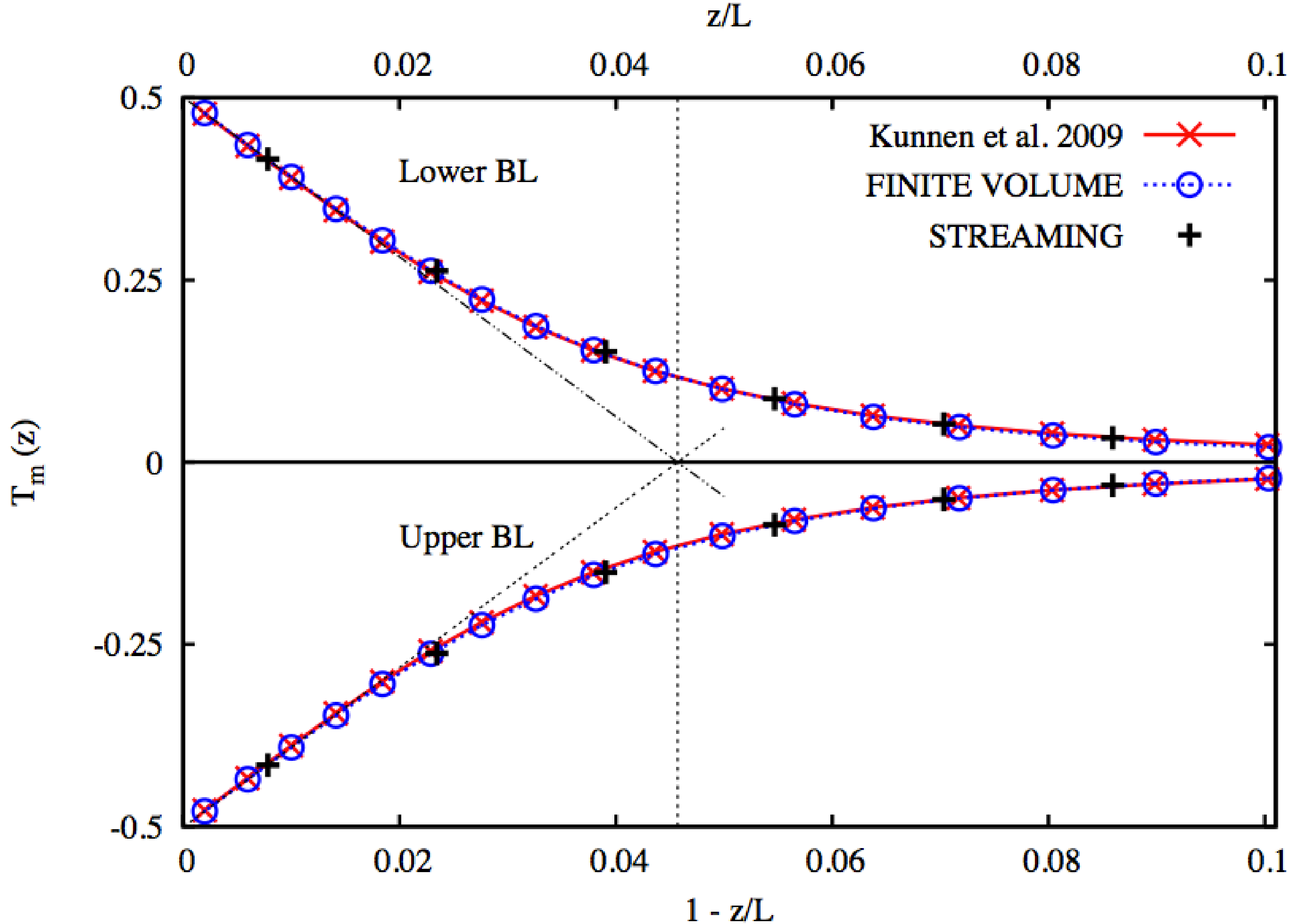


Fig: *Mean temperature prof le T_m - averaged over time and horizontal planes - as a function of the height z in the cell. Prof les only the lower/upper 10% of the cell.*

3D Validation: Rayleigh-Benard convection

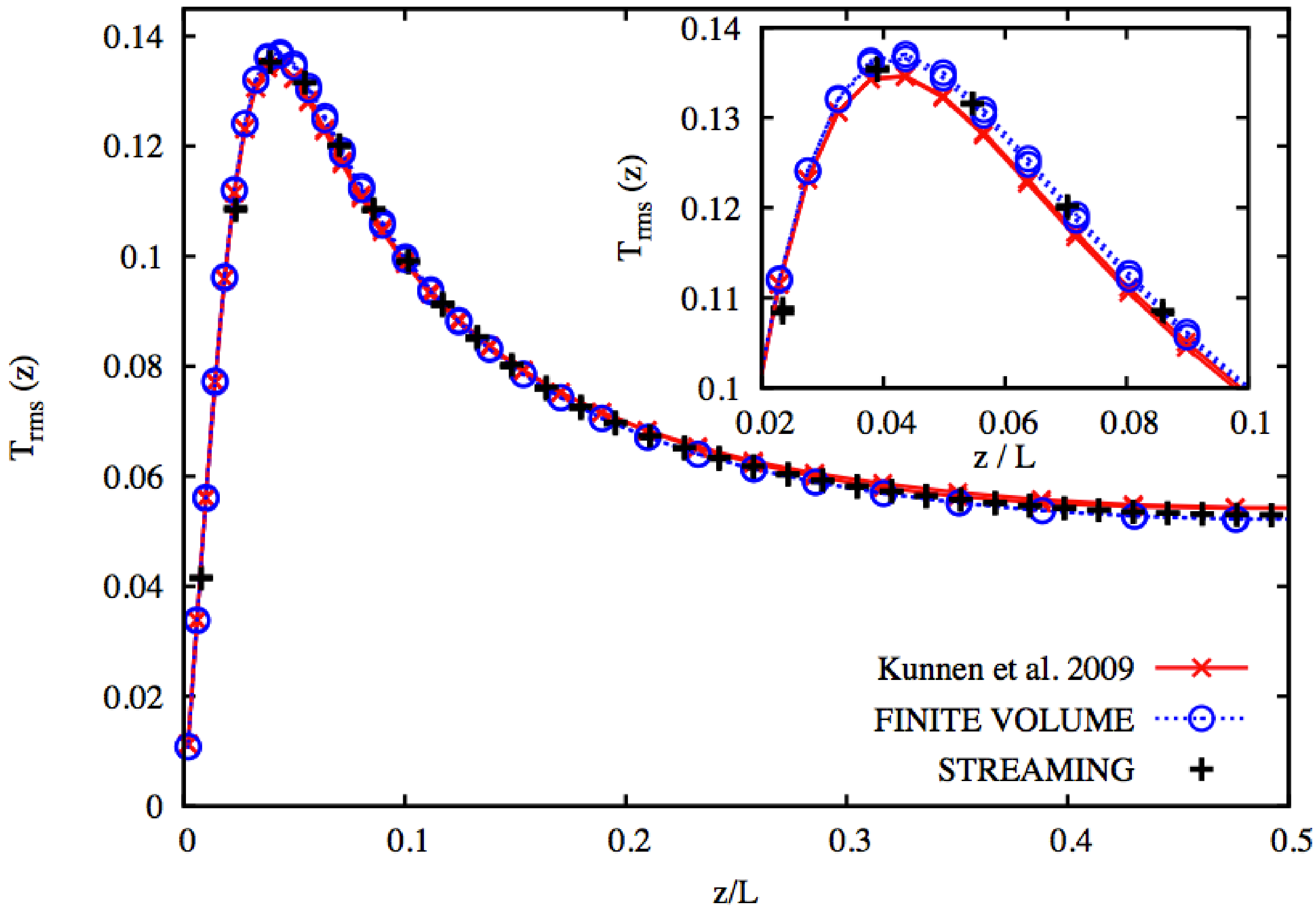


Fig: *Root-mean-square temperature prof les T_{rms}, averaged over time and horizontal planes, as a function of the cell height z. Prof les only the lower/upper 10% of the cell.*

3D Validation: Rayleigh-Benard convection

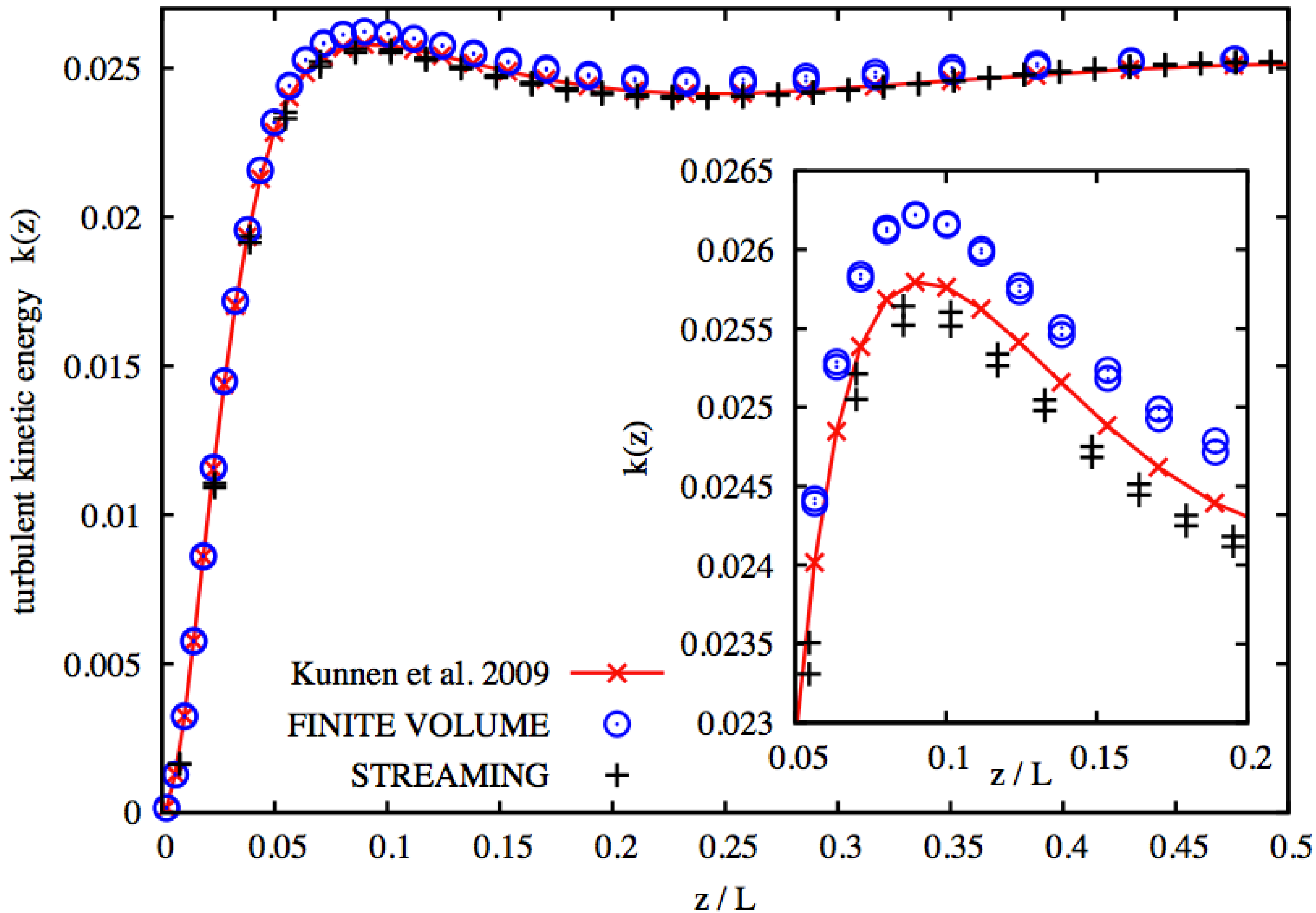


Fig: *Turbulent kinetic energy k = 0.5* <(u <u>)² >, averaged over time and horizontal planes, as a function of the cell height z.*

Perspectives

- Performance comparison
- Simulation of fully turbulent Channel flow by FVLBM

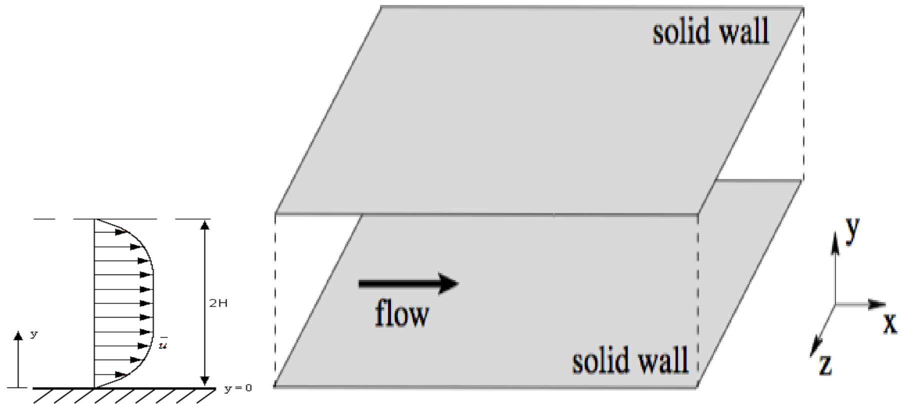


Fig: *Fully-developed turbulent channel flow simulation domain*

- Re_(tau) = 180
- Grid 128 X 128 X 256
- No-slip BC at top and bottom walls
- Periodic BC in x-y directions
- Inlet initiated with parabolic velocity profile

- Further enhancing the stability of the method

Questions?

Thank You